Konrad-Zuse-Zentrum
für Informationstechnik Berlin

Takustraße 7
D-14195 Berlin-Dahlem
Germany

ARMIN FÜGENSCHUH, BENJAMIN HILLER, JESCO HUMPOLA, THORSTEN KOCH,
THOMAS LEHMANN, ROBERT SCHWARZ, JONAS SCHWEIGER, JÁCINT SZABÓ

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Gas Network Topology Optimization for Upcoming Market Requirements

Armin Fügenschuh, Benjamin Hiller, Jesco Humpola, Thorsten Koch,
Thomas Lehmann, Robert Schwarz, Jonas Schweiger, Jácint Szabó

Konrad-Zuse-Zentrum für Informationstechnik Berlin
Takustraße 7, D-14195 Berlin, Germany
fuegenschuh@zib.de, hiller@zib.de, humpola@zib.de, koch@zib.de,
lehmann@zib.de, schwarz@zib.de, schweiger@zib.de, szabo@zib.de

***Abstract*—Gas distribution networks are complex structures that consist of passive pipes, and active, controllable elements such as valves and compressors. Controlling such network means to find a suitable setting for all active components such that a nominated amount of gas can be transmitted from entries to exits through the network, without violating physical or operational constraints. The control of a large-scale gas network is a challenging task from a practical point of view. In most companies the actual controlling process is supported by means of computer software that is able to simulate the flow of the gas. However, the active settings have to be set manually within such simulation software. The solution quality thus depends on the experience of a human planner.**

When the gas network is insufficient for the transport then topology extensions come into play. Here a set of new pipes or active elements is determined such that the extended network admits a feasible control again. The question again is how to select these extensions and where to place them such that the total extension costs are minimal. Industrial practice is again to use the same simulation software, determine extensions by experience, add them to the virtual network, and then try to find a feasible control of the active elements. The validity of this approach now depends even more on the human planner.

Another weakness of this manual simulation-based approach is that it cannot establish infeasibility of a certain gas nomination, unless all settings of the active elements are tried. Moreover, it is impossible to find a cost-optimal network extension in this way. In order to overcome these shortcomings of the manual planning approach we present a new approach, rigorously based on mathematical optimization. Hereto we describe a model for finding feasible controls and then extend this model such that topology extensions can additionally and simultaneously be covered. Numerical results for real-world instances are presented and discussed.

I. Introduction

In the past, the German gas supply companies were gas vendors and gas network operators at the same time: They purchased gas from other suppliers and set up and operated the necessary infrastructure to transport the gas from those suppliers to customers. In course of the liberalization of the German gas market, these roles and business units were separated by regulatory authorities. Now there are companies whose sole task is the transportation of gas and who operate gas transportation networks for this purpose. Several previously independent networks were aggregated into bigger units. A discrimination free access to these networks has to be granted to everyone. This increase in flexibility for gas vendors and customers requires a higher degree of operational flexibility from the gas network operators. Although the total amount of transported gas is approximately the same, today's gas networks cannot cope with this. Various congestions show up obstructing the desired flexibility. To overcome these shortcomings a massive investment in the networks is necessary in the future. Extension management becomes a crucial issue, since each single investment into a new compressor or a new pipe costs up to several hundreds of million Euros.

A gas network operator concludes transportation contracts with customers (usually gas suppliers and vendors), conceding the right to feed in gas at entry points and/or to feed out gas at exit points of his network. These contracts define limits on the amount of gas fed in or out, respectively. To initiate an actual gas transfer, the customer has to *nominate* the amount that is to be transferred some time before the actual feed-in or feed-out occurs. The customer has to ensure that the fed-in (fed-out) amount of gas is fed-out (fed-in) elsewhere in the network; one says that the nomination has to be *balanced*. To this end, customers conclude supply contracts among each others. The gas network operator has no knowledge of these contracts. He only provides the infrastructure needed to fulfill these contracts, and is obliged to fulfill each nomination. Note that it is totally up to the gas network operator how the gas is routed inside the network. All he has to ensure is that the amount of gas fed in or out matches the nominated amounts at those points.

There are several types of contracts a gas network operator can offer his customers. There are so-called RAC-type contracts (restrictively allocable capacity), which offer the right to feed in up to a certain amount of gas at a specified entry and to feed out the same amount at a specified exit. Obviously, any nomination on a RAC-type contract is itself balanced. More important are so-called FAC-type contracts, which constitute either the right to feed in at a certain entry or to feed out at a certain exit. The same amount of gas may be fed out or fed in at an arbitrary set of exits or entries; this is why this type of contract is referred to as "freely allocable capacity". FAC-type contracts are very flexible and ultimately allow gas consumers to purchase gas from various gas vendors. This possibility is a consequence of the liberalization of the German gas market; it is therefore desirable to offer a large share of the transportation capacity by FAC-type contracts. To achieve this, the gas transportation network must be flexible enough to meet the possible nominations resulting from the FAC-type contracts sold. A gas network operator may only sell FAC-type contracts such that he can guarantee fulfillment of the customers' nominations.

The existing German gas network has grown over time. It was built by gas supply companies such that it can ensure the transportation of exactly the required/planned amounts of gas to their customers. In addition, there are large pipe systems, which are used on the one hand for the transfer of gas through Germany and on the other hand to supply smaller gas distribution networks. This structure is indeed appropriate for the original requirements, but it does not necessarily facilitate fulfillment of the very different nominations that may arise from many FAC-type contracts. It is thus a limiting factor for increasing the availability of FAC-type contracts. Extending the network by suitable means allows to use the network in a more flexible way and thus to fulfill more potential nomination situations. Note that these extension measures are not meant to increase the overall amount of gas that may be transported but the flexibility of the gas supply. In fact, Germany's future gas consumption is estimated to decrease.

A gas network may be extended in several ways to increase the local transportation capacity. It is possible to build new pipes and to extend the capabilities of compressor stations and control valves or to build new ones. A special case of building a new pipe is *looping*: A *loop* is a pipe that follows an existing one; loops are somewhat cheaper to build than a new pipe somewhere in the country. Other extension measures include adjusting existing contracts or concluding new contracts. For instance, higher feed-in pressures or lower feed-out pressures may be established. In addition, the gas network operator may require a minimum feed-in at some entries or to conclude load flow commitments. A load flow commitment with a supplier allows the gas network operator to require a certain minimum feed-in at an entry if this helps to operate the network. Since load flow commitments usually lead to a tighter coupling between gas vendors and gas network operators they are undesirable from a regulatory perspective.

Several approaches to improve the topology of a gas network are reported in the literature. Mainly various heuristic and local optimization methods are in use. Boyd et al. [1] apply a genetic algorithm to solve a pipe-sizing problem for a network with 25 nodes and 25 pipes, each of which could have one of six possible diameters. Castillo and Gonzaleza [2] also apply a genetic algorithm for finding a tree topology solution for a network problem with up to 21 nodes and 20 arcs. In addition to pipes, also compressors can be placed into the network. Mariani et al. [3] describe the design problem of a natural gas pipeline. They present a set of parameters to evaluate the quality of the transportation system. Based on these they evaluate a number of potential configurations to identify the best among them. Osiadacz and Gorecki [4] formulate a network design problem for a given topology as a nonlinear optimization problem, for which they iteratively compute a local optimum. For a given topology the diameter of the pipes is a free design variable. Their method is applied to a network with up to 108 pipes and 83 nodes. De Wolf and Smeers [5] also use a nonlinear formulation and apply a local solver. They distinguish the operational problem (running the network) from the strategical investment problem (extending the network). For a given topology with up to 30 arcs and nodes they can determine optimized pipe diameters.

Our contribution in this field is to apply exact optimization methods that can converge to and prove global optimality. Moreover we allow for larger networks with up to 600 nodes and arcs. Our methods were developed in close cooperation with Open Grid Europe GmbH (OGE), a large gas transportation company. Section II presents a mathematical programming model that is used in our network extension procedure to model feasible nominations. However, it may also be used on its own to quickly check a nomination for feasibility. In Section III, we present our main contribution, a procedure for computing suitable cost-optimal network extensions based on rigorous mathematical programming methods. We show some initial results obtained using this procedure in Section IV.

II. Gas Network Nominations

A gas network consists of passive and active components. While the behavior of passive components, i.e., pipes, is completely determined by physics, active components allow to control the network to some extent. For instance, a compressor may increase the pressure, while a control valve may reduce and regulate the pressure. Controlling a gas network means to find a suitable setting for all active components such that the nominated gas can be transmitted through the network, without violating physical or operational constraints. The control of a large-scale gas network is a challenging task from a practical point of view. In most companies the actual controlling process is supported by means of computer software that is able to simulate the flow of the gas. However, the active settings have to be set manually within such simulation software. We present a mathematical model to identify settings for all active elements simultaneously. It is solved using numerical optimization techniques outlined at the end of this section.

Given is a gas pipe network as a directed graph $D = (V, A)$, where the set of nodes V is the disjoint union of entry nodes V^{entry}, exit nodes V^{exit}, and intermediate nodes V^{inter}. The set of arcs A is the disjoint union of passive arcs, or pipes, A^{pipe}, and active arcs, which are compressor stations A^{cs}, control valves A^{cv}, or valves A^{va}.

For each node $i \in V$ the amount of gas entering or leaving the network is given by $d_i \in \mathbb{R}$. We have $d_i > 0$ for all $i \in V^{\text{entry}}$, $d_i < 0$ for all $i \in V^{\text{exit}}$, and $d_i = 0$ for all $i \in V^{\text{inter}}$. The vector $d \in \mathbb{R}^V$ specifies a nomination, which the gas transportation company needs to deliver through their network. If this is possible, the nomination is said to be feasible, otherwise it is infeasible. Later, we will treat infeasible nominations as input data for determining suitable topology extensions, so that they also become feasible.

In order to answer the question of feasibility for a nomination we need to model the behavior of natural gas in pipes, i.e., the physics of gas. To this end, we introduce two families of variables. One are flow-per-arc variables $q_{i,j} \in \mathbb{R}$ for all $(i,j) \in A$. If $q_{i,j} > 0$ then gas flows from i to j, and for $q_{i,j} < 0$ the gas flows in the opposite direction. The other are pressure-per-node variables $p_i \in \mathbb{R}_+$, which are bounded by minimum and maximum node pressures:

$$\forall i \in V : \underline{p}_i \leq p_i \leq \overline{p}_i. \tag{1}$$

The upper bound relates to physical limitations (if violated the network might be damaged), the lower bound is due to contracts with the customers, or 0 for intermediate nodes.

Since we study a network flow problem, we have the usual flow conservation or continuity constraint in our model:

$$\forall j \in V : \sum_{k:(j,k)\in A} q_{j,k} - \sum_{i:(i,j)\in A} q_{i,j} = d_j, \tag{2}$$

which means that all gas that enters a node has to leave the node, only plus or minus the gas that is nominated at this node. This constraint occurs in any network flow model. In addition to that we have special constraints that model the physical properties of the flow of gas. Here we use the Weymouth equation [6], a quadratic pressure loss equation of the form

$$\forall (i,j) \in A^{\text{pipe}} : q_{i,j}\,|q_{i,j}| = \alpha_{i,j}(p_i^2 - \beta_{i,j} p_j^2). \tag{3}$$

Here $\alpha_{i,j}$ is a pipe-dependent constant that models all physical properties of the gas, such as its compressibility and its (estimated average) Reynolds number, and the pipe, such as its length $L_{i,j}$, diameter, and roughness. A potential height difference of nodes i and j is expressed by parameter $\beta_{i,j}$. Despite being almost 100 years old now, it is still considered to be a good approximation among all empirical formulas for computing the flow in medium to high pressure pipes [7].

Compressor stations can increase the pressure up to a certain level. The possible pressure increase depends in a nonlinear way on both the incoming pressure and the flow through the compressor. For simulation purposes, this relationship is described using the characteristic diagram of a compressor. We use a linear inequality system $F(p_i, q_{i,j}, p_j) \geq 0$ that couples the incoming and outgoing pressure and the flow throughput. This linear inequality system is derived from the characteristic diagram to be a tight approximation to it.

Control valves are working in the opposite direction. They decrease the pressure level. When going through such a control valve the pressure is reduced by at least $\underline{\Delta}_{i,j}$ and by at most $\overline{\Delta}_{i,j}$, where these two are given values depending on the actual type of control valve:

$$\forall (i,j) \in A^{\text{cv}} : \underline{\Delta}_{i,j} \leq p_i - p_j \leq \overline{\Delta}_{i,j}. \tag{4}$$

Valves are to open or shut a pipe completely. We introduce a binary decision variable $x_{i,j}$ for each valve $(i,j) \in A^{\text{va}}$, where $x_{i,j} = 0$ means the valve is shut and $x_{i,j} = 1$ represents an open valve. The pressures p_i and p_j before and after the valve coincide if and only if the valve is open:

$$\forall (i,j) \in A^{\text{va}} : \begin{cases} (\underline{p}_i - \overline{p}_j)(1 - x_{i,j}) \leq p_i - p_j, \\ p_i - p_j \leq (\overline{p}_i - \underline{p}_j)(1 - x_{i,j}). \end{cases} \tag{5}$$

Flow is going through the pipe if and only if the valve is open:

$$\forall (i,j) \in A^{\text{va}} : x_{i,j} \cdot \underline{q}_{i,j} \leq q_{i,j} \leq x_{i,j} \cdot \overline{q}_{i,j}. \tag{6}$$

The nomination problem now asks for a feasible solution in the variables p, q, x that simultaneously fulfills the constraints (1)–(6). Mathematically, this problem is a mixed-integer nonlinear feasibility problem. It cannot be solved by today's commercial or freely available solvers off-the-shelf. We implemented our own special tailored solver that combines techniques of mixed-integer linear programming, nonlinear optimization and constraint programming in a unique way. More details of this solver are described in [8]. Our method is currently able to control networks of industrial relevant size with several hundreds of pipes, about hundred entries and exits and about hundred active elements in short time.

III. Gas Network Extensions

If the above model does not have a feasible solution, and assuming that our model is a meaningful representation of the real gas network and the contractual situation, the network needs to be extended. If not, the gas transportation company would not be allowed to make such contracts with its customers. From an algorithmic point of view, finding suitable extensions is even more difficult than solving the nomination problem alone, since it contains the latter as a subproblem. In the subsequel we describe our approach in this respect.

The extension planning starts with an infeasible nomination that needs to be made feasible. This means that there is no way to switch the active elements of the gas network system, so that a gas flow exists, which satisfies the technical and physical constraints. The extension is done by adding further pipes, loops, compressors or control valves to the network. In principle it is possible to install these extensions at any location into the network. In addition to identifying a suitable location, the technical layout is to be determined, i.e., the exact thickness of a new pipe or the power of a new compressor. So there is an infinitely large continuum of possibilities for extending the network. Each single measure is associated with certain construction costs. For example, the construction costs of a new pipe can be estimated from the length of the pipe, its diameter, and its exact course in the landscape. The course of the pipe is related to the cost of different soil conditions that influence the construction process, and costs for the purchase of land use rights. The determination of the optimal course of a single pipe to given linkages to the existing network is already an optimization problem in itself.

If we discretize this infinite set of possible extensions by permitting only certain discrete pipe thickness and compressor strengths, and allow the connection to the existing network only at selected locations, the resulting set will become finite. But even then it is way too large to be used in an optimization process within the limitations of the current state of computer technology and algorithmic development.

Yet to solve the problem, we developed a multi-stage approach. Starting from an infeasible nomination this process consists of these four main steps:

A) An analysis of possible congestion points in the network,
B) preselection of extensions that are appropriate to remedy the shortage,
C) a final selection among the candidates in the preselection, which is cost-optimal,
D) a subsequent refinement of the measures in the final round, to reduce costs further.

A. Gas flow congestion analysis

The first of the four steps is to identify possible congestions of the network. There is a well-developed mathematical network flow theory for networks where the capacity of an arc does not depend on the flow. In such networks, it is relatively easy to identify a capacity-limiting congestion, which is a cut (literally, a bottleneck) in the network graph. However, in gas networks the amount of flow across an arc depends on the pressure difference between its end nodes, which in turn depends on the flow on neighboring arcs. Thus this elegant network flow theory is not applicable; we know of no suitable generalization that allows identifying bottlenecks in gas networks. For the time being we have to compute congestions numerically, and interpret the values accordingly.

The key constraint that links the different network elements (pipes, compressors, valves, control valves) is the flow conservation equation (2), reading

$$\forall j \in V : \sum_{k:(j,k)\in A} q_{j,k} - \sum_{i:(i,j)\in A} q_{i,j} = d_j. \qquad (2)$$

In case of an infeasible nomination we want to identify the constraints of the model that hinder the flow from being feasible. To do this, we relax the flow conservation constraints (2) of the gas network control model by introducing so-called slack variables. This means we allow more flow as the physical conditions would actually permit. We developed two variants which we call node slack and arc slack, to be presented below. In any of these two cases, we aim to keep the additional (unphysical) slack flow as low as possible. For this reason, we introduce an objective function to minimize the deviation from the physical reality.

In the case of the arc-based slack, we allow an additional amount of flow to be transported on each arc, without being subject to any physical laws. Denoting this additional gas flow through arc (i, j) by $s_{i,j}$, the modified constraint (2) then is:

$$\forall j \in V : \sum_{k:(j,k)\in A} (q_{j,k}+s_{j,k}) - \sum_{i:(i,j)\in A} (q_{i,j}+s_{i,j}) = d_j. \qquad (7)$$

A natural interpretation of this modified constraint is that we have a loop next to each existing pipe, i.e., we have a whole "unphysical" network parallel to the existing network. It is for the numerical solver of the model more "attractive" (in a certain sense), to transport the gas without pressure loss across the parallel network. For this reason, we introduce an objective function in which the amount of gas transported on these loops is minimized. The goal is thus to minimize the "unphysical moment of transportation":

$$\sum_{(i,j)\in A} L_{i,j} \cdot |s_{i,j}| \to \min . \qquad (8)$$

For the node-based slack, we allow that some gas may leave or enter the network at each node; in a sense, this gas may be "beamed" from one node to another. If we denote by $s_j^+ \geq 0$ the amount of flow that leaves the network and by $s_j^- \geq 0$ the amount of flow that re-enters the network via node $j \in V$, then the relaxed node-slack version of (2) is:

$$\forall j \in V : \sum_{k:(j,k)\in A} q_{j,k} - \sum_{i:(i,j)\in A} q_{i,j} = d_j - s_j^+ + s_j^- . \qquad (9)$$

An alternative way of viewing this is that there is a single additional node connected by an "unphysical" pipe (i.e., one allowing an arbitrary flow, disregarding any pressure) to each original node. If the flow then encounters a congestion in the network, it can therefore escape into another dimension (to the new node), to overcome the congestion in an unphysical way, and return into the network behind the congestion. Again, the use of these new pipes, and thus the deviation from the true physics, needs to be minimized, so that the solutions remain interpretable. This raises the question of what may be appropriate cost factors for each unit of flow on such a virtual pipe. Due to the fact that the additional node has no current geographical equivalent, we cannot use the same "unphysical moment of transportation" as above. At least in the case of an additional RAC-type contract there is a way to set the cost factors being proportional to the distance from the respective entry and exit nodes of the contract. A generic objective is:

$$\sum_{j\in V} (c_j^+ \cdot s_j^+ + c_j^- \cdot s_j^-) \to \min . \qquad (10)$$

As a result, the gas flow congestion analysis gives the slack values for nodes and for arcs (by solving two separate problems). These values indicate the points at which the gas bypasses the physical network since it is too limited. In the next step, these values must now be translated into meaningful extension measures.

B. Determining a list of extension measures

The second step of four in the topology planning is the determination of extension measures. Its goal is to reduce the infinite number of possible extensions to a small finite selection of useful measures. To this end, results of the previous gas flow congestion analysis are used.

Assume that the arc-based gas flow congestion analysis has identified an arc (i, j) with a non-zero slack $s_{i,j}$. In case the flow $q_{i,j}$ on the original network arc and that on the hypothetical loop go in the same direction, i.e., $q_{i,j}s_{i,j} > 0$, this indicates that more flow needs to traverse this arc than it is physically possible. This suggests a loop as an extension measure, or a compressor, if the pressure difference along (i, j)

is too small. If the loop flow is directed opposite to that of the original arc, there are two cases. If the net flow $q_{i,j}+s_{i,j}$ along arc (i,j) is non-negative, the pressure difference enforces too much flow. Therefore, a control valve reducing the pressure along (i,j) is a reasonable extension. In the remaining case that the net flow $q_{i,j}+s_{i,j}$ is indeed negative, it might help to install a compressor increasing the pressure in the opposite direction of the physical flow.

The node slacks from the node-based gas flow congestion analysis may be turned into extension measures similarly. Consider a node i with $s_i^+ > 0$ and another node j with $s_j^- > 0$. This means that s_i^+ units of gas leave the network at node i, whereas s_j^- units of flow re-enter the network at node j. It is thus natural to consider extension measures that can physically realize a flow between these nodes. If $p_i > p_j$ and this pressure difference is sufficiently high, it suffices to build a pipe between the nodes. In case the pressure difference is small or even $p_i < p_j$, we may still build a pipe, but we need an additional compressor on that pipe as well. If a new pipe creates a connection between two parts of the network with different pressure levels, it must also have a control valve such that the necessary pressure reduction can be handled.

Of course, we also need to define costs for all proposed extension measures. Valves and control valves incur only fixed construction costs and are relatively cheap. A compressor incurs both construction and operating costs depending on its power. Both cost types need to be combined into a single value. Finally, to determine the construction cost for a new pipe one needs to know its geographical route. This is known for pipes that are loops of existing pipes. For completely new pipes, we use Dijkstra's algorithm for computing shortest paths in graphs with positive arc weights to compute the geographical route. Here the underlying graph consists of a fine discretized map of the country where the properties of soils and land uses are registered, which influence the cost of construction in each particular area. Our project partner OGE provided us with numerical data to obtain realistic costs for each proposed extension measure.

The result of this third step is a candidate list of pipes, loops, control valves and compressors, denoted by A^{new}, from which suitable extensions can be selected. At this point, all elements have default dimensions (diameter for pipes, power of compressors); the exact dimensions for the elements to be build are computed in the fourth step of the topology planning.

C. Extension selection

The measures on the list of candidate extensions are still many. It is too expensive and even not necessary to build all of them. In this third step of topology planning, called extension selection, a subset is selected from the list of measures, which makes the nomination feasible, and at the same time is the most cost-effective among all other such subsets.

To determine this extension subset, we solve the model (1)–(6) from Section II on an extended network that additionally contains the extension elements from A^{new}. Each of these extension measures features an additional valve; opening this valve means building the corresponding extension. If the valve remains closed, the network behaves just as it would without the extension arc. A family of binary decision variables $y_{i,j} \in \{0,1\}$ for all $(i,j) \in A^{\text{new}}$ is introduced for these valves. The constraints for this new valve are the same as for the original valves presented in (5) and (6) (simply replace v by x). The opening of the valve is provided with cost equivalent to the construction costs $c_{i,j}$ for building this extension measure:

$$\sum_{(i,j)\in A^{\text{new}}} c_{i,j}\, y_{i,j} \to \min\,. \tag{11}$$

The solution of this model contains a subset of the extension arcs, which represent the most cost-effective selection from the candidate extensions ensuring feasibility of the initial nomination.

D. Refined Planning

Although the selected network extension measures are cost-optimal among all other possible subsets of the measures in A^{new}, there may still be room for improvement. Since the computations were restricted to relatively few potential measures – especially compared to the continuum of all possible measures – it is expected that slightly better solutions can be found in the neighborhood of the selected extension measures by small local alterations. This is the goal of the fourth and last step of the topology planning: the detailed or refined planning.

Based on the output of the third step, the extension selection, the measures selected are altered by local variations. In the case of a pipe there will be some thicker and some thinner pipes and there are variations in the locations where the pipe is connected to the network. In the case of compressors, there will be several copies with more and with less power. In this way we obtain a new set of measures that is of the same kind of list that was generated in step two. Hence we may now use this list as input data for the extension selection again. The result of this fourth step is then a fine-planned, cost-effective network extension.

IV. Results

The topology planning was applied to a real-world scenario. Data for the network was provided by our project partner Open Grid Europe, Germany's largest gas network operator and a 100% subsidiary of the energy company e.on. The network is located in the north-western part of Germany on an area of about 100 km width and 300 km height. Our model of the network consists of 575 nodes, among them 26 entries and 87 exits, and 605 arcs, 542 of which are passive (mainly pipes). The remaining 63 arcs are active: There are 6 compressor stations, 92 valves and 23 control valves.

We solved the models described above using SCIP 2.0.1 [9], [10] as branch-and-cut/constraint programming framework, CPLEX 12.1 as LP-solver for the resulting linear programming relaxations, and ipopt 3.8.1 [11] as NLP-solver. The data is processed using the Lamatto++ framework [12]. GoogleEarth is used for visualization purposes. The software runs on a common single CPU core desktop personal computer.

Figure 1 shows a feasible nomination in this network. The color indicates the pressure level. Orange and red indicates high pressures (60-100 bar), whereas blue and green indicate low pressures (1-40 bar). The diameter of the pipes in Figure 1 corresponds to the amount of gas transported through the pipes (and not to the physical diameter).

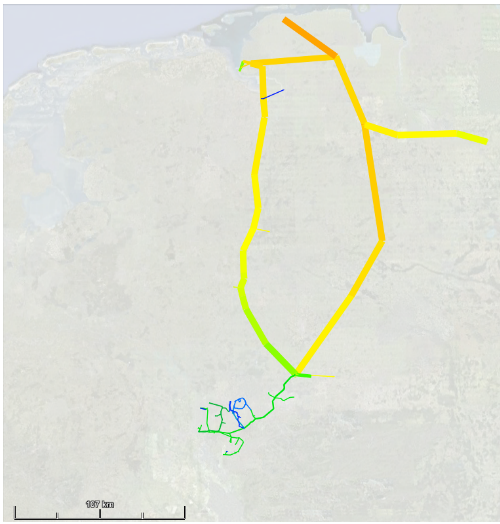

Fig. 1. A feasible nomination.

We now add a large RAC-type contract between one entry node in the north and one exit node in the south of this network. Then with the current network it is no longer possible to transfer this additional amount of gas, on top of the already nominated gas. The extension pipes that were determined are shown in Figure 2 in green; the original network is shown in blue. There are 144 extension pipes, 4 potential sites for new compressors, and no further control valves as candidate extensions.

If the additional RAC has a capacity of 40 GW then three additional loops should be built as shown in red in Figure 3 (left). If the capacity is 60 GW, then some more loops and also one pipe should be build, and additionally one compressor in the north-western part of the network, see Figure 3 (right).

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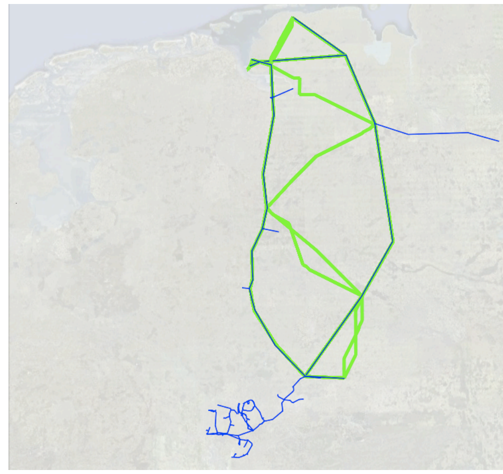

Fig. 2. Network extension pipes.

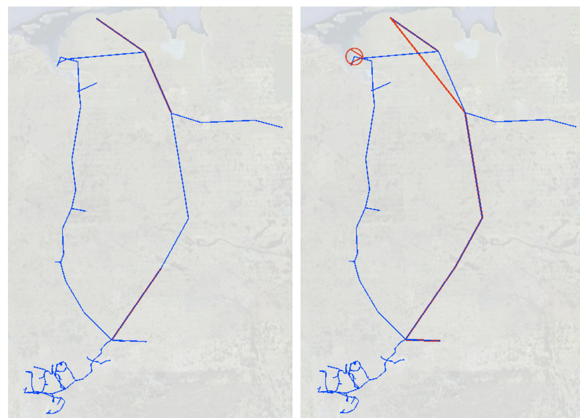

Fig. 3. Selected extensions for 40 GW (left) and 60 GW (right).

References

[1] I. D. Boyd, P. D. Surry, and N. J. Radcliffe, "Constrained gas network pipe sizing with genetic algorithms," Edinburgh Parallel Computing Centre, Tech. Rep. EPCC-TR94-11, 1994.

[2] L. Castillo and A. Gonzáleza, "Distribution network optimization: Finding the most economic solution by using genetic algorithms," *European Journal of Operational Research*, vol. 108, no. 3, pp. 527–537, 1998.

[3] O. Mariani, F. Ancillai, and E. Donati, "Design of a gas pipeline: Optimal configuration," Pipeline Simulation Interest Group, Tech. Rep. PSIG 9706, 1997.

[4] A. Osiadacz and M. Górecki, "Optimization of pipe sizes for distribution gas network design," Pipeline Simulation Interest Group, Tech. Rep. PSIG 9511, 1995.

[5] D. D. Wolf and Y. Smeers, "Optimal dimensioning of pipe networks with application to gas transmission networks," *Operations Research*, vol. 44, no. 4, pp. 596 – 608, July-August 1996.

[6] T. R. Weymouth, "Problems in Natural Gas Engineering," *Transactions of the American Society of Mechanical Engineers*, vol. 34, pp. 185–231, 1912.

[7] R. C. King, *Piping Handbook*. McGraw-Hill Inc., 1968.

[8] A. Fügenschuh, H. Homfeld, H. Schülldorf, and S. Vigerske, "Mixed-Integer Nonlinear Problems in Transportation Applications," in *Proceedings of the 2nd International Conference on Engineering Optimization (+CD-rom)*, 2010, h. Rodrigues et al. (Hrsg.), ISBN 978-989-96264-3-0.

[9] T. Achterberg, "SCIP: Solving Constraint Integer Programs," *Mathematical Programming Computation*, vol. 1, no. 1, pp. 1–41, 2009.

[10] T. Berthold, S. Heinz, and S. Vigerske, "Extending a CIP framework to solve MIQCPs," Zuse Institute Berlin, ZIB-Report 09-23, 2009.

[11] A. Wächter and L. T. Biegler, "On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming," *Mathematical Programming*, vol. 106, pp. 25–57, 2006.

[12] B. Geißler, A. Martin, and A. Morsi, "Lamatto++," information available at URL http://www.am.uni-erlangen.de/optimization/edom/research/projects/lamatto.html, February 2011.