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Thomas Schlechte

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Abstract

This article gives an overview of the results of the author's PhD thesis Schlechte [2012]. The thesis deals with the mathematical optimization for the efficient use of railway infrastructure. We address the optimal allocation of the available railway track capacity - the track allocation problem. This track allocation problem is a major challenge for a railway company, independent of whether a free market, a private monopoly, or a public monopoly is given. Planning and operating railway transportation systems is extremely hard due to the combinatorial complexity of the underlying discrete optimization problems, the technical intricacies, and the immense sizes of the problem instances. Mathematical models and optimization techniques can result in huge gains for both railway customers and operators, e.g., in terms of cost reductions or service quality improvements. We tackle this challenge by developing novel mathematical models and associated innovative algorithmic solution methods for large scale instances. We made considerable progress on solving track allocation problems by two main features - a novel modeling approach for the macroscopic track allocation problem and algorithmic improvements based on the utilization of the bundle method. This allows us to produce for the first time reliable solutions for a real world instance, i.e., the Simplon corridor in Switzerland.

1 Micro-Macro Transformation of Railway Networks

A major challenge is modeling railway systems to allow for resource and capacity analysis. Railway capacity has basically two dimensions, a space dimension which are the physical infrastructure elements as well as a time dimension that refers to the train movements, i.e., occupation or blocking times, on the physical infrastructure. Railway safety systems operate on the same principle all over the world. A train has to reserve infrastructure blocks for some time to pass through. Two trains reserving the same block of the infrastructure within the same point in time is called *block conflict*. Therefore, models for railway capacity involve the definition and calculation of reasonable running and associated reservation and blocking times to allow for a conflict free allocation.

There are *microscopic* models that describe the railway system extremely detailed and thorough. Microscopic models have the advantage that the calculation of the running times and the energy consumption of the trains is very accurate. A major strength of microscopic models is that almost all technical details and local peculiarities are adjustable and are taken into account. We describe the railway system on a microscopic scale that covers the behavior of trains and the safety system completely and correctly. Those models of the railway infrastructure are already very large even for very small parts of the network. The reason is that all signals, incline changes, and switches around a railway station have to be modeled to allow for precise running time calculations of trains. In general microscopic models are used in simulation tools which are nowadays present at almost all railway companies all over the world. The most important field of application is to validate a single timetable and to decide whether a timetable is operable and realizable in practice. However, microscopic models are inappropriate for mathematical optimization because of the size and the high level of detail. Hence, most optimization approaches consider simplified, so called *macroscopic*, models. The challenging part is to construct a reliable macroscopic model for the associated microscopic model and to facilitate the transition between both models of different scale.

In order to allocate railway capacity significant parts of the microscopic model can be transformed into aggregated resource consumption in space and time. We develop a general macroscopic representation of railway systems which is based on minimal headway times for entering tracks of train routes and which is able to cope with all relevant railway safety systems. We introduce a novel bottom-up approach to generate a macroscopic model by an automatic aggregation of simulation data produced by any microscopic model. The transformation aggregates and shrinks the infrastructure network to a smaller representation, i.e., it conserves all resource and capacity aspects of the results of the microscopic simulation by conservative rounding of all times. The main advantage of our approach is that we can guarantee that our macroscopic results, i.e., train routes, are feasible after re-transformation for the original microscopic model. Because of the conservative rounding macroscopic models tend to underestimate the capacity. We can control the accuracy of our macroscopic model by changing the used time discretization. Finally, we provide a priori error estimations of our transformation algorithm, i.e., in terms of exceeding of running and headway times. We implemented our new transformation algorithm in a tool called NETCAST. The technical details can be found in Schlechte et al. [2011].

2 Railway Track Allocation

The main application of railway track allocation is to determine the best operational implementable realization of a requested timetable, which is the main focus of our work. But, we want to mention that in a segregated railway system the track allocation process directly gives information about the infrastructure capacity. Imaging the case that two trains of a certain type, i.e., two train slots, are only in conflict in one station. A potential upgrade of the capacity of that station allows for allocating both trains. This kind of feedback to the department concerning network design is very important. Even more long-term infrastructure decisions could be evaluated by applying automatically the track allocation process, i.e., without full details on a coarse macroscopic level but with different demand expectations. It is clear that suitable extensions or simplifications of our models could support infrastructure decisions in a quantifiable way. For example major upgrades of the German railway system like the high-speed route from Erfurt to Nürnberg or the extension of the main station of Stuttgart can be evaluated from a reliable resource perspective. The billions of euros for such large projects can then be justify or sorted by reasonable quantifications of the real capacity benefit with respect to the given expected demand.

The optimal track allocation problem for macroscopic railway models can be formulated with a graph-theoretic model. In that model optimal track allocations correspond to conflict-free paths in special time-expanded graphs.

More specifically, we studied four types of integer programming model formulations for the track allocation problem: two standard formulations that model resource or block conflicts in terms of packing constraints, and two novel coupling or "configuration" formulations. In both cases we considered variants with either arc variables or with path variables. The classical way of formulating the train pathing problem is by using single flow formulations for the trains which are coupled by additional packing constraints to exclude conflicts between those trains, see Brännlund et al. [1998] or Caprara et al. [2006]. In Borndörfer & Schlechte [2007] we introduced an alternative formulation for the track allocation problem. The key idea of the new formulation is to use additional "configuration" variables that are appropriately coupled with the standard "train" flow variables to ensure feasibility. We show that these models are a so called "extended" formulations of the standard packing models. This arc coupling formulation (ACP) formulation is based on the concept of feasible arc *configurations*, i.e., sets of time-expanded arcs on a track without occupation conflicts. Given is a standard time expanded scheduling graph D, in which time expanded paths represent train routes and departure and arrival times for the individual train requests $i \in I$ in the subgraph $D_i \subseteq D$. Operational feasiblilty with respect to headway times between two consecutive trains is handled by artificial digraphs D_j for each $j \in J$. Each two train moves on track j which respect the minimal headway time are connected by artificial arcs in D_j. Table 1 lists all relevant objects for the ACP formulation.

Figure 1 shows the construction of a track digraph respecting headway times for succesions of trains. On the left hand side the requested

Table 1: Sets or Parameters of formulation ACP for problem TPP

Symbol	**Description**
$D_i = (V_i, A_i)$	The time expanded graph of train i
s_i	The artificial source node of train digraph D_i
t_i	The artificial sink node of train digraph D_i
$D_j = (V_j, A_j)$	The artificial track digraph of track j
s_j	The artificial source node of track digraph D_j
t_j	The artificial sink node of track digraph D_j
w_a	The cost weight of arc a
C	The set of all station conflict sets
A_c	The set of arcs belonging to station conflict $c \in C$
κ_c	The upper limit or capacity of station conflict $c \in C$
A_S	The set of arcs (representing train moves) to couple

train moves on track $j = (u, v)$ are shwown in a time expanded setting,, i.e., 6 trains belonging to set A^j_Ψ with different 3 different running times. Note that for all these arc we use binary variables x_a. On the right hand side we introduce artificial dotted arcs which determine the first or last departure of a train on track j. In addition, each feasible consecutive succession of two trains are connected by an artificial dashed arc, i.e., if the difference between both departure times is larger or equal to the given headway time of that train combination. Hence, graph D_j models all potential orderings of the departures of trains on track j. Note that for all arcs of D_j we use binary variables y_a. In case of a transitive headway matrix paths in D_j represent valid consecutive successions of discrete train moves on track j. Note that each arc representing a train move is contained in exactly one D_j and D_i and induces two variables x_a and y_a. More details on the construction of D_i and D_j can be found in Schlechte [2012].

In the ACP formulation variables x_a, $a \in A_i$, $i \in I$ control the use of arc a in D_i and y_b, $b \in A_j$, $j \in J$ in D_J, respectively. Note that only arcs which represent track usage, i.e., $A_S = A_I \cap A_J$, have to be coupled in that formulation by two variables x_a and y_b. The objective, denoted in ACP (i), is to maximize the weight of the track allocation, which can be expressed as the sum of the arc weights w_a. Equalities (ii) and (iv) are well-known *flow conservation constraints* at intermediate nodes for all trains flows $i \in I$ and for all flows on tracks $j \in J$, (iii) and (v) state that at most one flow, i.e., train and track, unit is realized. Equalities (vi) link arcs used by train routes and track configurations to ensure a conflict-free allocation on each track individually. Packing constraints (vii) ensures that no station capacity is violated in its most general form. Finally, (viii) states that all variables are binary.

The success of an integer programming approach usually depends on the strength of the linear programming (LP) relaxation. Hence, we

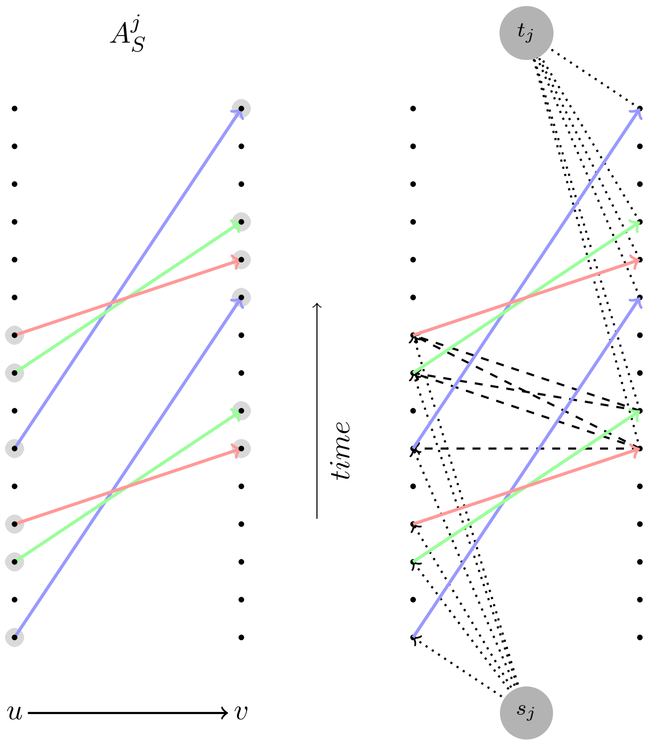

Figure 1: Example for the construction of a track digraph.

analyze the LP relaxations of our model formulations.

We show, that in case of block conflicts, the packing constraints in the standard formulation stem from cliques of an interval graph and can therefore be separated in polynomial time. It follows that the LP relaxation of a strong version of this model, including all clique inequalities from block conflicts, can be solved in polynomial time. We prove that the LP relaxation of the extended formulation for which the number of variables can be exponential, can also be solved in polynomial time, and that it produces the same LP bound. The path variant of the coupling model provides a strong LP bound, is amenable to standard column generation techniques, and therefore suited for large-scale computation.

$$
\begin{array}{llcll}
\text{(ACP)} & & & & \\
\max & \sum\limits_{a \in A} w_a x_a & & & \text{(i)} \\
s.t. & \sum\limits_{a \in \delta^i_-(v)} x_a - \sum\limits_{a \in \delta^i_+(v)} x_a & = 0, & \forall\, i \in I, v \in V_i \backslash \{s_i, t_i\} & \text{(ii)} \\
 & \sum\limits_{a \in \delta^i_-(s_i)} x_a & \leq 1, & \forall\, i \in I & \text{(iii)} \\
 & \sum\limits_{a \in \delta^i_-(v)} y_a - \sum\limits_{a \in \delta^i_+(v)} y_a & = 0, & \forall\, j \in J, v \in V_j \backslash \{s_j, t_j\} & \text{(iv)} \\
 & \sum\limits_{a \in \delta^i_-(s_j)} y_a & \leq 1, & \forall\, j \in J & \text{(v)} \\
 & x_a - y_a & = 0, & \forall\, a \in A_S & \text{(vi)} \\
 & \sum\limits_{a \in A_c} x_a & \leq \kappa_c, & \forall\, c \in C & \text{(vii)} \\
 & x_a, y_a & \in \{0, 1\}, & \forall\, a \in A_I, a \in A_J. & \text{(viii)}
\end{array}
$$

Furthermore, we present a sophisticated solution approach that is able to compute high-quality integer solutions for large-scale railway track allocation problems in practice. Our algorithm is a further development of the rapid branching method introduced in Borndörfer et al. [2008] (see also the thesis Weider [2007]) for integrated vehicle and duty scheduling in public transport. The method solves a Lagrangean relaxation of the track allocation problem as a basis for a branch-and-generate procedure that is guided by approximate LP solutions computed by the bundle method. This successful second application in public transportation provides evidence that the rapid branching heuristic guided by the bundle method is a general heuristic method for large-scale path packing and covering problems. All models and algorithms are implemented in a software module `TS-OPT`, see Borndörfer et al. [2010] and Borndörfer et al. [2010].

3 Computational Results

Finally, we go back to practice and present several case studies using the tools `NETCAST` and `TS-OPT`. In Schlechte [2012] we provide a computational comparison of our new models and standard packing models used in the literature. Our computational experience indicates that our approach, i.e., "configuration models", outperforms other models. Moreover, the rapid branching heuristic and the bundle method enables us to produce high quality solutions for very large scale instances, which has not been possible before.

The highlights are results for the Simplon corridor in Switzerland, see Borndörfer et al. [2010]. We optimized the train traffic through this tunnel using our models and software tools. To the best knowledge of the author and confirmed by several railway practitioners this was the first time that fully automatically produced track allocations on a macroscopic scale fulfill the requirements of the originating microscopic model, withstand the evaluation in the microscopic simulation tool `OpenTrack`, and exploit the infrastructure capacity. This documents the success of our approach in practice and the usefulness and applicability of mathematical optimization to railway track allocation.

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