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The S-Bahn Challenge in Berlin

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ZIB Report 15-13 (March 2015)

Herausgegeben vom
Konrad-Zuse-Zentrum für Informationstechnik Berlin
Takustraße 7
D-14195 Berlin-Dahlem

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URL: http://www.zib.de

ZIB-Report (Print) ISSN 1438-0064
ZIB-Report (Internet) ISSN 2192-7782

The S-Bahn Challenge in Berlin

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March 5, 2015

Traveling an entire subway or S-Bahn network as fast as possible has become a kind of sport, which is gaining popularity. In fact, the Guinness Book of Records already has several entries in this category, all quite recent, including the "Fastest time to travel to all London Underground stations" (16 hr 20 min 27 sec, August 16, 2013), the "Fastest time to travel to all New York City Subway stations" (22 hr 26 min 02 sec, November 18/19, 2013), and the "Fastest time to travel to all the Berlin U-Bahn metro stations" (7 hr 33 min 15 sec, May 02, 2014) [4]. Of course, the S-Bahn network of Berlin had been a blatant gap in this illustrious list.

The origin of the S-Bahn Challenge, however, is not a mere hunt for the record. Indeed, it started as an Arts project at the Bard College Berlin, where David Kretz wanted to "reach back through history" to experience "a mode of traveling ... close to the one that both documented and opposed the changes that trains stood for originally in the 19th century". In this vein, the S-Bahn challenge is a means to get into this mind-set by creating a mixture of time pressure and "moments of repose, contemplation, wonder, and joyful observation" [8].

David, who is from Austria, immediately realized that determining the shortest tour is a combinatorial optimization problem and asked mathematician Martin Aigner of the Free University of Berlin, also born in Austria. He forwarded the request to the Zuse Institute Berlin, where it ended up at Isabel Beckenbach and Ralf Borndörfer. When Marc Uetz from the University of Twente suggested an Erasmus traineeship for his student Loes Knoben, the topic was clear – we wanted to find the optimum round trip through the S-Bahn network in a joint Austrian-Dutch-German Arts-and-Math project. In the meantime, David had tried by hand and computed a solution that took him 17 hr 1 min 00 sec. Would that be optimal?

The Rules of the S-Bahn Challenge There are several variants of the public transport challenge. Sometimes, like in the Guinness cases, it suffices to visit all stations, sometimes all lines must be visited, the tour can be closed or open, etc. In the S-Bahn Challenge, we look for an open tour subject to the following restrictions:

- Each S-Bahn station and each S-Bahn connection between two stations in the zones A,B,C must be visited at least once.
  - Each connection only has to be visited in one direction.
  - If there are multiple lines connecting two stations, visiting one line suffices.
- Walking between stations and the use of all other means of public transport that follow a fixed schedule is allowed.

That is, we need to cover all stations and all connections.

The Mathematical Model There is already some literature about the mathematics of public transport challenges, see Chapter 6 of the thesis of Welz [10] for an overview. However, as far as we are aware of, all of these models use identical, standardized transfer times, which seemed to be a big problem and a disadvantage that we wanted to overcome.

In a first attempt, the S-Bahn network can be modeled in terms of a directed graph with vertices corresponding to the stations and with arcs for all connections between two stations. If each arc gets a length that is equal to the travel time, the S-Bahn challenge becomes a Directed Chinese Postman problem except that there are groups of arcs (parallel lines between two stations) from which at least one arc has to be visited. This model, however, is not well suited to deal with transfer and waiting times. These are important, as some lines of the S-Bahn only have a frequency of 20 or even 40 minutes.

We therefore use a refined space-time network representation that is constructed in two steps. The first step is illustrated in Figure 1. It constructs a vertex for each combination of station, line, and direction. Between any of the vertices corresponding to the same station, we add a transfer arc whose length corresponds to the transfer time. Analogously, vertices of different stations are connected by arcs with lengths equal to the travel times, if lines exists between those vertices.

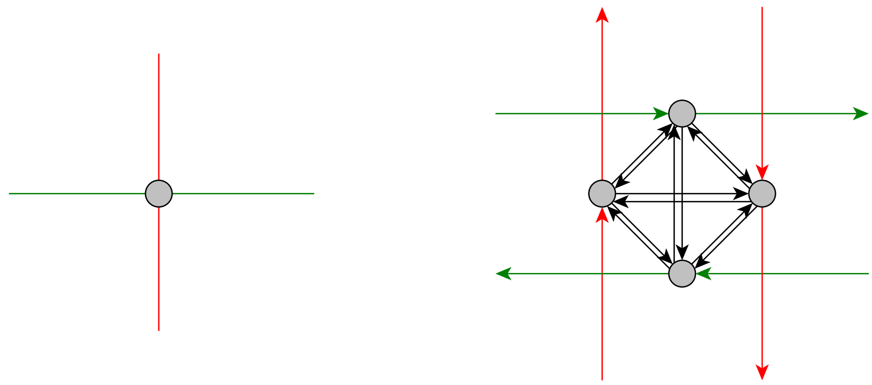

Figure 1: Modeling transfers between lines.

This model takes transfer times between different lines into account, but not the time of the day. It better should, because if one changes from a line with a frequency of say 10 minutes to a line with a frequency of 20 minutes, it could be that one can make the transfer in in only say 5 minutes, but then 10 minutes later one might have to wait 15 minutes. This means that we really have to use the exact travel times. This can be accomplished by constructing one vertex for every departure of every line at every station. Of course, all these vertices blow up the graph.

In case of the S-Bahn, we are lucky and can shrink the graph back by exploiting the regularity of the train schedule. Namely, the departure times modulo 10 are always the same at each line and each station, only the frequencies change during the day (we excluded the night service). It therefore suffices to consider a period time equal to the least common multiple of all frequencies, copying the nodes according to their frequency in this interval. For example, if the period time is one hour, vertices corresponding to lines with frequency 10 will appear six times whereas vertices corresponding to lines with frequency 20 will only be copied three times, see Figure 2. If the period time is small enough, this produces a graph of reasonable size. Likewise, adding

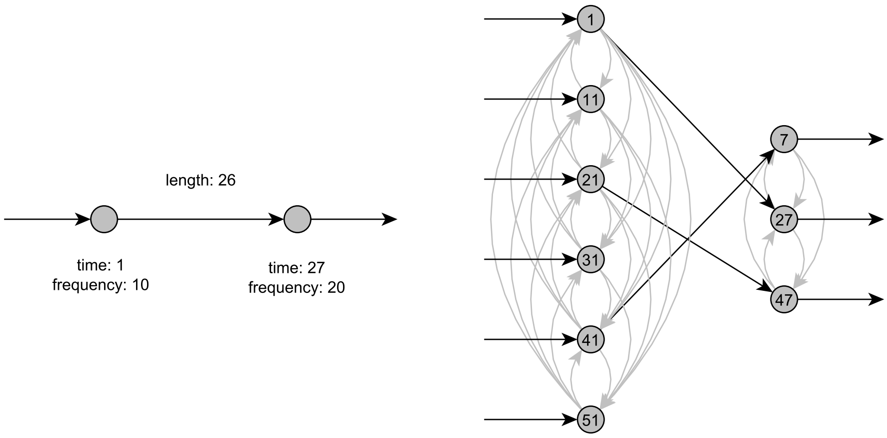

Figure 2: Taking frequencies into account (time modulo 60 minutes).

all connections of the Berlin public transport system would again result in a network that is too big; we therefore add only some extra connections, that have been constructed by hand and careful inspection.

The final result of this work is a directed graph, in which a station corresponds to several vertices and in which the arcs can be partitioned into several groups. The first group contains all transfer arcs and all extra connections; these arcs can, but don't need to be used. Every connection between two stations also gives rise to a group of arcs. The problem is to find a path in this directed graph, visiting at least one arc of every group except the first. This problem is known in the literature as a Generalized Directed Rural Postman Problem (GDRPP).

Solution Approach We have chosen to transform the GDRPP into a Traveling Salesman Problem, relying on the software Concorde by David Applegate, Bob Bixby, Vasek Chvátal, and Bill Cook [1] to do the real hard work. The transformation goes in several steps. The Generalized Directed Rural Postman Problem is first transformed into a Generalized ATSP, then into an ATSP, and finally into a TSP, see [3], [9] and [6], respectively. Doing this is actually a pretty good idea, as the number of nodes and arcs increases only moderately.

To transform an instance of the GDRPP in which at least one arc of every arc-group has to be visited, to an instance of the GATSP in which exactly one vertex of each vertex-group has to be visited, a new directed graph is constructed. This graph contains a vertex for each arc that belongs to an arc-group, except for the first group containing the transfer arcs and extra connections. This means that the vertex-groups of the new graph correspond exactly to the arc-groups of the original graph except of the first group whose arcs will be treated implicitly. Between every two vertices a, a' of different vertex-groups, corresponding to arcs (u, v) and (u', v') in the original graph, arcs (a, a') and (a', a) are added in the new graph. The arc from a to a' and the opposite arc from a' to a' receive as weights the lengths of the shortest path from v to u' including the weight of the original arc (u, v) and the length of the shortest path from v' to u including the weight of original arc (u', v'), see Figure 3.

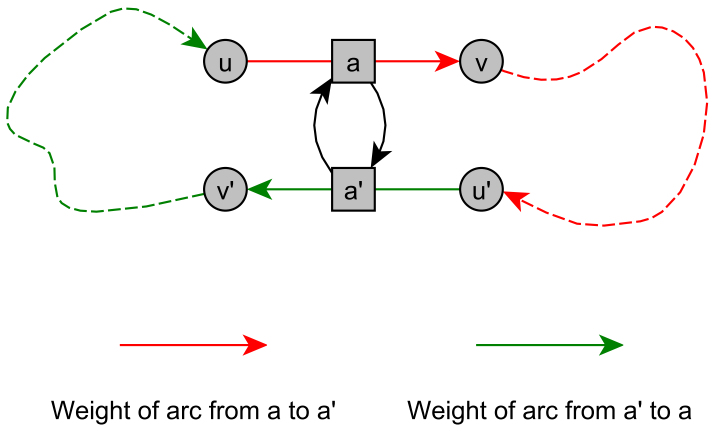

Figure 3: Transforming the GDRPP to the GATSP.

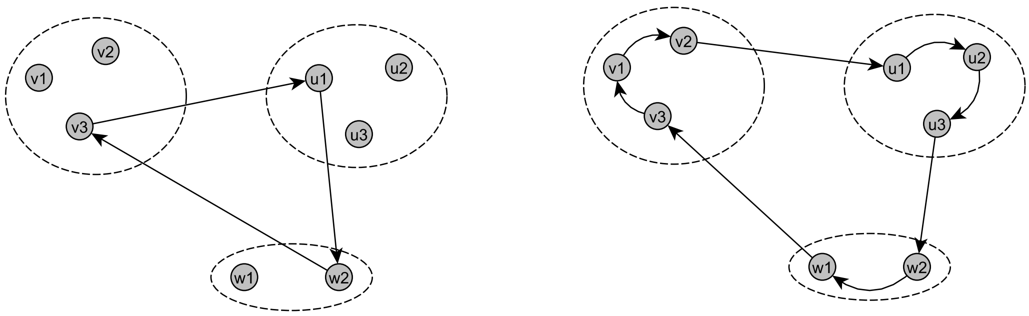

Figure 4: Transforming the GATSP to the ATSP.

In the transformation from the GATSP to the ATSP the vertices remain unchanged but the group structure disappears, the arc set changes, and the weight of the arcs is changed, see Figure 4. First, we add arcs of weight zero such that all vertices of the same group are connected by a directed cycle. The original ATSP graph contains only arcs between two different groups. Each such arc (u, v) is replaced by the arc (u', v) where u' is the predecessor of u in the directed cycle of the group containing u. The new arc (u', v) receives the weight of the original arc (u, v) increased by a large constant M which can be chosen as the maximum weight of an arc in the original graph times the number of groups. Adding M ensures that every group is entered and left exactly once.

Theoretical and Practical Solution For the timetable of January 2015, the best solution takes only 13 hours and 17 minutes. However, this itinerary contains several super-tight transfers that would be difficult to make in practice, and, even worse, the last of these has a frequency of 40 minutes. Missing this connection, which is highly likely, would result in a delay of 40 minutes. Adding some bounds on minimal transfer times at larger stations and fixing the starting station to Straußberg Nord produces a schedule that takes 13 hours and 44 minutes,

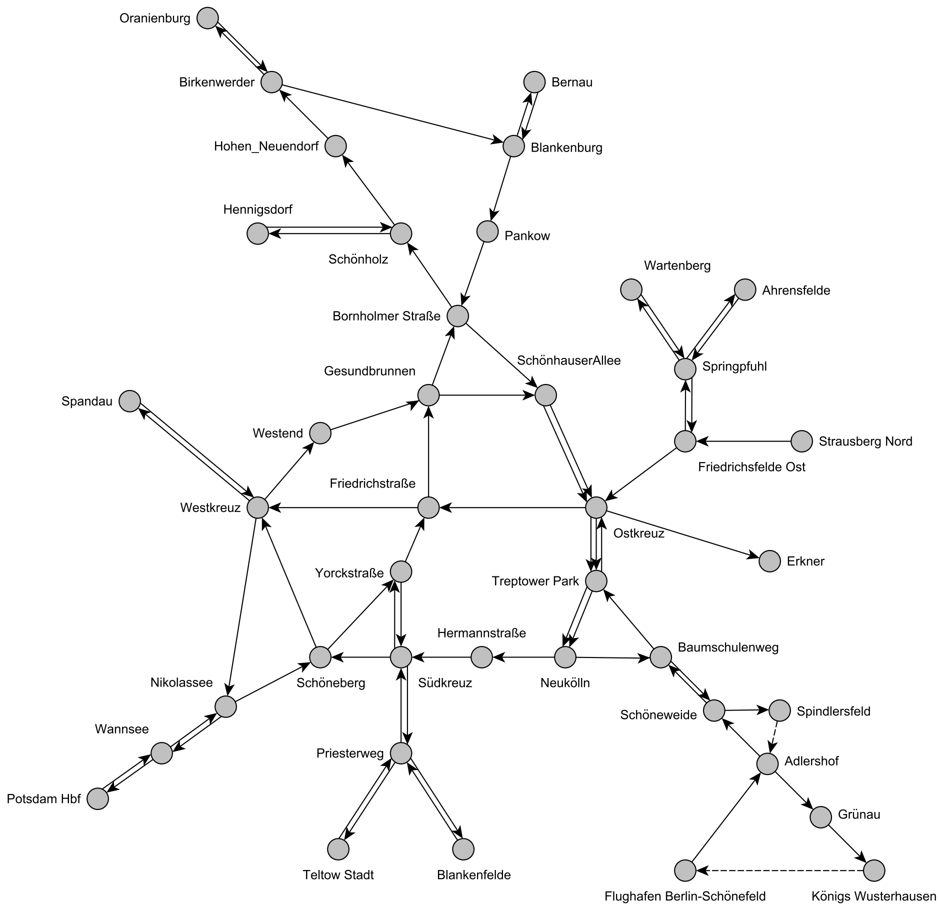

Figure 5: The final itinerary.

see Figure 5. All transfers of this itinerary are possible according to the BVG trip planner. This solution is also better from a robustness point of view, as it starts at the terminal station of the unique line with a frequency of 40 minutes (all other lines run every 10 or 20 minutes). Thus, at most 20 minutes of delay can be caused by missing one connection.

The Record Attempt The calculated itinerary was put into practice on January 10, 2015 by a team consisting of Isabel Beckenbach, Loes Knoben, David Kretz, and two friends, see [2] for the photo documentation. Starting in Straußberg Nord at 9:55, everything went well for almost 7 hours, until thunderstorm "Felix" started to destroy the carefully calculated plan. "Felix" let several trees fall onto the tracks, many connections were blocked, some stations even closed. We used all the information we could get from the Internet to adjust our tour, taking several bus and train replacement services. In the end we were able to finish our journey in 15 hr 04 min 00 sec at 0:59 in Erkner with a delay of 80 minutes – a new world record, almost 01 hr 57 min 00 sec faster than the old, despite the storm! Unfortunately, two stations that had been closed were visited using a regional train, which didn't stop. It is not clear if this is against the Guinness Book rules or not. According to the guidelines for the New York City

Figure 6: Loes Knoben, Isabel Beckenbach, Niels Lindner, and David Kretz at the starting station in Straußberg Nord

Subway Challenge [5], a non-stop pass through pass through of a temporarily closed station is acceptable, but maybe not with a regional train ....

So we still do not know if our record will be officially acknowledged. However, it should be possible to travel the entire Berlin S-Bahn system in 13 hours and 44 minutes, or even in 13 hours and 24 minutes, if you are lucky, or maybe even less, if you choose another starting point or find a bus shortcut that we overlooked. If anyone wants to try, you can download Loes's program TopTrack from our S-Bahn Challenge homepage `www.zib.de/projects/s-bahn-challenge-berlin` – the race is on!

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