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Solving steel mill slab design problems

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ABSTRACT. The steel mill slab design problem from the CSPLIB is a combinatorial optimization problem motivated by an application of the steel industry. It has been widely studied in the constraint programming community. Several methods were proposed to solve this problem. A steel mill slab library was created which contains 380 instances. A closely related binpacking problem called the multiple knapsack problem with color constraints, originated from the same industrial problem, was discussed in the integer programming community. In particular, a simple integer program for this problem has been given by Forrest et al. [4]. The aim of this paper is to bring these different studies together. Moreover, we adapt the model of [4] for the steel mill slab design problem. Using this model and a state-of-the-art integer program solver *all* instances of the steel mill slab library can be solved efficiently to optimality. We improved, thereby, the solution values of 76 instances compared to previous results [13]. Finally, we consider a recently introduced variant of the steel mill slab design problem, where within all solutions which minimize the leftover one is interested in a solution which requires a minimum number of slabs. For that variant we introduce two approaches and solve all instances of the steel mill slab library with this slightly changed objective function to optimality.

1. INTRODUCTION

The *steel mill slab design problem* is motivated by a real world application from the steel industry. Mathematically, the problem consists of a set of $n \in \mathbb{N}$ orders, each order j coming with a size $s_j \in \mathbb{N}$ and color $c_j \in C$, where C is a finite set. Furthermore, we are given a set $K := \{k_1, \dots, k_m\} \subset \mathbb{N}$ of $m \in \mathbb{N}$ capacities. The task is to equip each used slab with one capacity and assign each order to exactly one slab with the requirements that the selected capacities are respected and that each slab only processes orders of at most two different colors. The objective is to minimize the *leftover* that is the total loss or equivalent the residual capacity. Recently, Schaus et al. [13] considered a variation of the problem by adding a second criteria to the objective. Within all solutions, which minimize the leftover, one searches for a solution with a minimal number of used slabs.

The steel mill slab design problem is problem number 38 of the CSPLIB¹. This library provides one instance which consists of 111 orders with 88 different colors, and 20 possible capacities. We call this instance the *original instance*. Furthermore, there exists a steel mill slab library [14]. This library contains 380 instances which are grouped into 19 classes each with 20 instances. These instances have been created by changing the set of possible capacities of the original instance. This means, the orders are the same as the one of the original instance. The capacities are generated uniformly and range between 10 and 50; the 19 classes are ranging from having 2 to 20 possible capacities. For more details about the generation of these instances and the library we refer to [13].

¹<http://www.csplib.org/>

Outline. In the following section, we give a brief overview on different approaches to solve the steel mill slab design problem and related binpacking problems. We recall among others a set packing formulation [4] to the so-called *multiple knapsack problem with color constraints* which can be perceived as a slight generalization of the steel mill slab design problem. In Section 3, we adapt this model to the steel mill slab design problem and the variant where also the number of used slabs is minimized. In Section 4 we report on our computational results. Using a state-of-the-art integer program solver, we solved *all* instances of the steel mill slab library and the original instance to optimality improving the solution value of 76 instances. Moreover, we solved all these instances for the above-mentioned modification of the problem to optimality as well.

2. RELATED WORK

In the past, several different models have been proposed to solve the steel mill slab design problem. A first set of constraint programming models was presented by Frisch et al. [5] and first computational results for a (small) subset of orders of the original instance were given by the same authors in [6]. Dawande et al. [3] presented an asymptotic polynomial time approximation scheme and two 3-approximation algorithms. Hnich et al. [9] introduced an integer programming formulation, a constraint programming formulation, and a hybrid model and solved also one instance which consists of a subset of orders of the original instance. A first optimal solution of the original instance (total loss of zero) was given by Gargani and Refalo [7] using a large neighborhood search heuristic. Van Hentenryck and Michel [15] introduced a constraint programming model which can be used to solve the original instance using a heuristic approach. Recently, Schaus et al. [13] presented a collection of different constraint-based solving techniques for this problem and introduced the steel mill slab library [14]. All previously used models and solving techniques are not capable of solving all instances of the steel mill slab library.

Kalagnanam et al. [10] and Forrest et al. [4] studied a closely related binpacking problem called the *multiple knapsack problem with color constraints*. The problem provides another view on the same industrial application as the steel mill slab design problem. The problem input consists of m slabs, each slab j coming with a capacity $k_j \in \mathbb{R}$, and n items, each item i coming with a size $s_i \in \mathbb{R}$, a color $c_i \in \mathbb{N}$, and a specification in form of a subset of slabs indicating from which slabs this item can be manufactured. We say that an item is *valid* for a slab if the item can be manufactured from it. The goal is to find an assignment such that each slab contains valid items of at most two different colors, the capacities of the slabs are respected, and the unused capacity of the used slabs is minimized. For this problem, Kalagnanam et al. [10] presented a compact integer programming formulation, while Forrest et al. [4] gave a set packing formulation and designed a simple column generation approach. Their computational results indicate that this method is superior in practice. They solved an instance with 439 orders, 347 different colors, and 24 slabs (two having the same capacity). Due to the additional assignment restrictions, a slab has a restricted set of items which can be manufactured from it. The number of different colors of the associated valid items is at most 222. This instance is called `mkc` and is part of the MIPLIB2010 [11]. Interpreting,² this instance w.r.t. the steel mill slab design problem, it consists of 439 orders, 23 different slab capacities, and 347 colors. Forrest et al. [4] tried to solve an even larger instance called `mkc7`. This instance has 74 slabs, 9484 orders, and 233 colors within the context of the multiple knapsack problem with color

²We ignored the assignment restrictions and allowed an arbitrary number of slabs of each capacity.

constraints. This boils down to 70 different slab capacities and 642 colors w.r.t. the steel mill slab design problem. See the appendix for a more detailed description of the transformation and the particular problem instances in the context of steel mill slab design.

One main reason why these two binpacking problems are hard to solve in practice is that the used models, with the exception of the set packing formulation of Forrest et al. [4], are symmetric. In these models, orders are explicitly assigned to slabs, and therefore, symmetry naturally arises by permuting slabs. It is well known, for instance, that symmetry causes branch-and-bound algorithms to perform poorly, since the resulting problems change only marginally after branching, see Barnhart et al. [2]. In principle, one can respond to this difficulty by either adding symmetry breaking constraints to the given model or by avoiding such a symmetric model in advance. The first strategy was pursued by several authors. Van Hentenryck and Michel [15] partly broke symmetry using a customized search routine. Other symmetry breaking techniques are discussed in [6]. The set packing formulation of Forrest et al. [4], however, provides a model that avoids this kind of symmetry, which is one explanation for the performance of their column generation algorithm.

3. INTEGER PROGRAMMING FORMULATION

Adapting the set packing formulation of Forrest et al. [4], we obtain an integer programming formulation for the steel mill slab design problem. This model does not contain the kind of symmetry mentioned in the previous section.

Let S be the set of all feasible slab designs. A slab design s is an assignment vector $\lambda_s \in \{0, 1\}^n$. This vector defines which orders belong to this particular slab design s . This means, order $j \in \{1, \dots, n\}$ belongs to slab design s if $(\lambda_s)_j$ is one. A slab design is *feasible* if the total order size is not greater than the largest available capacity and if s contains orders of at most two different color classes. Each slab design s comes with an unique leftover l_s which is given by

$$l_s = \min\{k \in K \mid k \geq \sum_{j=1}^n (\lambda_s)_j\} - \sum_{j=1}^n (\lambda_s)_j.$$

Introducing for each feasible slab design $s \in S$ a binary decision variable x_s which is one if s is used and zero otherwise, we can formulate the steel mill slab design problem as an integer program:

$$\begin{aligned} & \min \quad \sum_{s \in S} l_s x_s \\ & \text{subject to} \quad \sum_{s \in S} (\lambda_s)_j x_s = 1 \quad \forall j \in \{1, \dots, n\} \\ & \quad x_s \in \{0, 1\} \quad \forall s \in S. \end{aligned} \tag{1}$$

This is a set partitioning problem. The objective is to minimize the total leftover. The equalities are set partitioning constraints to ensure that for each order j exactly one slab design s is chosen. Finally, the last conditions state that all variables are binary.

In contrast to the setting of Forrest et al. [4] we consider the case that all orders must be covered. This is simply reflected by the transition from packing to partitioning constraints. As a result we focus on pure minimizing of the total leftover whereas Forrest et al. [4] additionally consider to maximize satisfied orders, i.e., they combine both goals in one objective function. In general we would propose the same solution methodology as Forrest et al. [4] to cope with such formulations. Since the number of columns/slab designs can become quite large, an integer program like the one above is usually solved with a branch-and-price [2] algorithm.

Checking the instances of the steel mill slab library revealed, however, that these instances have between 7103 and 10011 feasible slab designs. Therefore, all variables can be generated, i.e., all feasible slab designs can be enumerated, in advance. In [8] a branch-and-price approach for the steel mill slab design problem is briefly discussed.

The set partitioning Model 1 can also be used for the problem of finding within the solutions with minimal leftover one which additionally uses a minimal number of slabs. To cope with this tie breaker rule for different optimal solutions, we can use a simple sequential approach or integrate that tie breaker rule directly in to the first model.

The sequential approach works as follows. First, we solve Model 1 to compute the minimal leftover, say L^* . Then we add a knapsack constraint to limit the total leftover by L^* , and change the objective function to $\min \sum_{s \in S} x_s$ which leads to:

$$\begin{aligned} \min \quad & \sum_{s \in S} x_s && (2) \\ \text{subject to} \quad & \sum_{s \in S} (\lambda_s)_j x_s = 1 && \forall j \in \{1, \dots, n\} \\ & \sum_{s \in S} l_s x_s \leq L^* \\ & x_s \in \{0, 1\} && \forall s \in S. \end{aligned}$$

Solving this integer program, knowing L^* , gives us a solution with minimal leftover which uses the minimum number of slabs. Note that the knapsack constraint can be replaced by an equation. Computational experiments showed that this variant performed with a similar efficiency as the above knapsack version.

As mentioned above, one also can integrate the additional optimization criterion directly into the first model. Since n orders require at most n slabs to be served, we can slightly manipulate the objective function by adding $\frac{1}{2n}$ to each coefficient. Then, each used slab design contributes a fixed amount to the objective value independently of the leftover. Therefore, a solution with minimum leftover which uses fewer slabs is cheaper than one which uses more slabs. Using this known technique results in the following integer program:

$$\begin{aligned} \min \quad & \sum_{s \in S} (l_s + \frac{1}{2n}) x_s && (3) \\ \text{subject to} \quad & \sum_{s \in S} (\lambda_s)_j x_s = 1 && \forall j \in \{1, \dots, n\} \\ & x_s \in \{0, 1\} && \forall s \in S. \end{aligned}$$

In the following section we present computational results for the steel mill slab library in its original formulation and the variant which also minimizes the number of slabs.

4. COMPUTATIONAL RESULTS

In this section we present our computational studies. We used IBM ILOG CPLEX 12.1.0 to solve the resulting integer programs. All computations were performed on computers with an Intel Core 2 Extreme CPU X9650 with 3 GHz, 6 MB cache, and 8 GB of RAM. We used the deterministic parallel mode with 4 threads of IBM ILOG CPLEX. The remaining parameters are kept at their default values. As test set we chose the recently established steel mill slab library [14]. This library contains 380 steel mill slab design instances. We first present the overall results for

TABLE 1. Optimal leftover for all instances of the steel mill slab library [14].

$ K $	Instance																			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	22	54	100	34	15	36	40	42	531	76	66	64	19	78	44	296	56	155	36	36
3	5	15	10	14	7	35	11	39	63	155	39	14	6	19	15	45	35	8	22	17
4	32	18	10	7	8	6	6	3	1	12	13	8	1	19	1	11	15	0	5	12
5	0	21	5	1	9	8	0	0	1	2	7	5	17	7	2	10	5	11	15	0
6	0	19	0	0	0	1	0	0	0	1	0	7	0	12	2	3	0	0	0	0
7	0	0	1	0	1	2	0	1	0	0	7	0	2	4	0	0	0	1	0	1
8	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

the original steel mill slab version and its variant. Second, we present more detailed performance results for the different models and selected instances.

4.1. Overall results. For each instance we generated all feasible slab designs in advance. For the instances of the steel mill slab library this took a negligible amount of time (at most 0.02 seconds). The overall results for the minimization of the leftover are summarized in Table 1. The rows represent capacity classes, each of them consisting of 20 problem instances. The first column, indexed by “ $|K|$ ”, states the number of available capacities in this class. The other columns, indexed from 0 to 19, list the *optimal* objective values (leftover) of the 20 (ordered) instances of the corresponding capacity class. Values written in bold italic font indicate an improvement to the previous best known solutions [13]. Overall we improved 76 instances and proved for *all* instances optimality. The running time, which does not include the time for generating all feasible slab designs, for these instances were around one second each, except for five instances of the capacity class 2. Instance 2 required 2.5 seconds, instance 5 took 101.5 seconds, instance 6 ran in 174.8 seconds, instance 8 needed 7.2 seconds, and instance 15 required 14.6 seconds (see Table 4). In these cases most of the time is spent for proving optimality.

Table 2 gives the overall results for all instances of the steel mill slab library for which we additionally minimized the number of slabs. The columns of this table are arranged in the same fashion as in the previous table. The values state for each instance the minimum number of used slabs (w.r.t. the minimum leftover). Again, all problems of the steel mill slab library are solved to optimality.

4.2. Detailed results. In what follows we give some more insights on the different models. Table 3 summarizes performance results for each capacity class and model. Thereby, the first column “ $|K|$ ” shows the capacity class by stating the number of available capacities. Followed by pairs of columns for the results of Model 1, Model 2, the sequential solving approach (Model 1+2) to find within all solutions with minimal leftover one which uses a minimal number of slabs, and the integrated Model 3 which solves the secondary slab minimization directly. For performance measures we choose the shifted geometric mean³ for the number of search “nodes”

³The shifted geometric mean of values t_1, \dots, t_n is defined as $(\prod(t_i + s))^{1/n} - s$ with shift s . We use a shift $s = 10$ for time and $s = 100$ for nodes.

TABLE 2. Minimal number of used slabs for all instances of the steel mill slab library [14].

$ K $	Instance																			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	58	51	48	50	50	48	50	52	47	48	52	54	51	56	56	47	50	47	49	55
3	59	55	52	53	52	51	51	53	58	47	55	50	53	52	48	53	53	56	57	59
4	54	53	50	50	47	50	49	51	52	51	51	52	51	48	55	57	55	50	52	52
5	57	56	56	51	51	53	55	49	51	53	49	50	51	54	52	52	52	52	48	51
6	51	50	51	47	47	51	54	48	49	50	52	51	50	52	53	52	52	48	47	53
7	56	48	53	47	50	49	47	50	47	51	54	49	48	51	47	48	50	55	47	47
8	47	50	52	47	51	48	48	47	51	49	51	47	50	49	47	49	48	49	50	47
9	51	50	47	48	47	48	47	49	48	47	50	47	51	47	47	50	49	47	55	48
10	47	47	49	47	48	47	48	47	47	47	51	47	47	47	49	48	49	50	47	47
11	47	47	47	47	47	47	48	51	48	47	48	47	47	49	47	47	47	48	48	47
12	47	48	47	47	47	50	49	47	56	47	47	47	47	47	48	47	47	47	47	47
13	47	47	47	47	47	47	47	50	47	47	47	47	47	47	47	47	47	47	47	47
14	47	47	47	47	47	48	47	47	47	47	47	47	47	47	47	47	47	49	47	47
15	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47
16	47	47	47	47	47	47	47	47	47	48	47	47	47	47	47	47	47	47	47	47
17	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47
18	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47
19	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	48	47	47	47	47
20	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47

TABLE 3. Number of search nodes and running times for all models summarized for the different capacity classes of the steel mill slab library [14].

$ K $	Model 1		Model 2		Model 1+2		Model 3	
	nodes	time [s]	nodes	time [s]	nodes	time [s]	nodes	time [s]
2	1285	5.0	179	2.8	1656	8.2	2178	11.1
3	86	0.3	41	0.9	117	1.2	185	0.6
4	43	0.2	49	0.7	91	0.9	325	0.5
5	1	0.1	23	0.3	25	0.4	115	0.3
6	15	0.2	6	0.2	21	0.3	50	0.3
7	1	0.1	2	0.1	3	0.2	71	0.3
8	1	0.1	1	0.1	2	0.2	77	0.3
9	1	0.1	2	0.1	3	0.2	37	0.3
10	1	0.1	2	0.1	3	0.2	78	0.3
11	1	0.1	1	0.1	2	0.2	29	0.2
12	1	0.1	1	0.1	2	0.2	56	0.3
13	1	0.1	1	0.1	2	0.2	62	0.3
14	1	0.1	1	0.1	2	0.2	54	0.3
15	1	0.1	1	0.1	2	0.2	20	0.2
16	1	0.1	1	0.1	2	0.2	58	0.3
17	1	0.1	1	0.1	2	0.2	12	0.2
18	1	0.1	1	0.1	2	0.1	10	0.2
19	1	0.1	1	0.1	2	0.2	33	0.2
20	1	0.1	1	0.1	2	0.2	32	0.3

and for the running “time” in seconds. The *shifted geometric mean* has the advantage that it reduces the influence of outliers. The geometric mean ensures that hard instances are prevented of having a huge impact on the measures. Similar shifting reduces the bias of easy instances, those solved in less than 10 seconds and/or less than 100 nodes. Note that the measures for the columns related to Model 1+2 are computed by first adding the measure values of Model 1 and Model 2 for each instance and then applying the shifted geometric mean to these values. For a detailed discussion about different measures we refer to [1].

All instances belonging to a class with more than 2 capacities are easy to solve independently of the chosen model. Regarding the running times all models need less than one second w.r.t. the shifted geometric mean except for the sequential approach (Model 1+2) for capacity class 3 which takes 1.2 seconds. The number of visited search nodes reveals that for larger capacity classes almost no search is

TABLE 4. Individual results for the instances from capacity class $|K| = 2$ of the steel mill slab library [14] and the steel mill slab version of the `mkc` instance.

Inst.	Model 1		Model 2		Model 1+2		Model 3	
	nodes	time [s]	nodes	time [s]	nodes	time [s]	nodes	time [s]
0	150	0.2	1	0.5	151	0.7	1274	0.4
1	507	0.6	520	1.6	1027	2.2	555	0.7
2	541	2.5	1	0.6	542	3.1	544	3.0
3	531	1.1	529	5.5	1060	6.6	534	1.0
4	1	0.1	1	0.4	2	0.5	39	0.1
5	226 348	101.5	1	0.6	226 349	102.1	1 850 052	1149.5
6	367 617	174.8	530	2.3	368 147	177.1	310 277	195.7
7	520	0.8	1	0.7	521	1.5	527	0.8
8	3421	7.2	1	1.0	3422	8.2	275 385	402.5
9	367	0.6	1	0.2	368	0.8	1	0.4
10	375	0.3	88	1.0	463	1.3	496	0.7
11	1024	0.5	536	4.1	1560	4.6	66	0.3
12	526	0.9	1183	6.5	1709	7.4	1109	1.4
13	713	0.6	1	0.4	714	1.0	565	0.7
14	1589	1.0	540	3.1	2129	4.1	55 464	18.2
15	10 569	14.6	7906	41.2	18 475	55.8	555	2.9
16	530	0.7	157	2.0	687	2.7	1480	1.7
17	1012	2.0	1402	9.7	2414	11.7	830	2.1
18	517	2.3	1	0.5	518	2.8	21 165	14.9
19	509	0.2	1	0.2	510	0.4	206	0.2
<code>mkc</code> ⁴	13 041	119.2	496	136.1	13 537	255.3	207 003	639.7

required. That means problems are solved in the root node of the search tree. Concerning the sequential and integrated approaches for additionally minimizing the number of used slabs, it makes (almost) no difference regarding these two methods.

Next, we have a closer look at the results for the capacity class two which are given in Table 4. The columns of this table have almost the same meaning as in Table 3 except that we are stating the real number of search nodes and running times in seconds. The table shows that within this capacity class there are only a few instances which are slightly harder. For the sequential and integrated approaches to minimize, in addition, the number of used slabs, we note that for the instances 5, 8, 14, and 18 the sequential method is superior to the integrated model. In case of the instances 11, 12, 15 and 17, however, the integrated approach slightly dominates the sequential method.

Finally, we consider the two instances `mkc` and `mkc7` (see [4]). Our approach works for the smaller `mkc` instance which consists in the steel mill slab context of 439 orders, 23 (different) capacities, 347 colors. There exist 140 223 feasible slab designs which lead in our models to the same number of binary variables. It takes 0.2 seconds to generate these variables. Table 4 shows in the last line the performance results for this particular instance (excluding the problem generation time). The minimal leftover is 48.32 which requires at least 191 slabs. In case of the much larger instance `mkc7` our static approach already failed to generate all feasible slab designs. There exist more than 12 billion slab designs. To cope with that issue a branch-and-price approach using the introduced set partitioning models as basis could be a promising approach, see for example [8].

⁴Note that in this paper the `mkc` instance is not equivalent to the correspond instance in the MIPLIB2010 [11]. The MIPLIB2010 version just gave the data input for the steel mill slab design problem which is consider in this paper. See appendix for more details.

5. CONCLUSION

We utilized a standard integer programming model to solve the steel mill slab design problem. An advantage of the proposed model is that the naturally arising symmetries are removed. We solved all instances of the steel mill slab library efficiently. This approach is superior to all previous techniques applied to this problem. Furthermore, we showed that the recently introduced variant of that problem, which additionally minimizes the number of used slabs, can be easily incorporated into our integer programming approach. Again, all instances of the steel mill slab library with that secondary objective criteria are solved efficiently to optimality. Besides these instances which all consists of 111 orders we solved the steel mill slab version of `mkc` which contains 439 orders.

All results state that the current steel mill slab library needs an update. Instances with 111 orders can be solved with the introduced integer programming models using a general purposes state-of-the-art solver efficiently.

The introduced models, however, are not scalable. They have the drawback that the number of required variables increases rapidly if there are more orders to be placed. For example, in case of the `mkc7` instance, which has more than 12 billion feasible slab designs, we were not able to create all feasible slab designs in advance. This challenge, however, can be overcome using a branch-and-price approach which generates feasible slab designs only on demand and by reducing the number of required slab designs by considering dominance between one slab design and m others. For the latter one we refer to Prestwich and Beck [12].

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APPENDIX A. CONVERTING MKC AND MKC7

The `mkc` instance is part of the MIPLIB2010 [11] and `mkc7` is provided by the Coin-OR project.⁵ Both instances are given as assignment formulation of the multiple knapsack problem with color constraints as described in [4] Section 2. Thereby, the variable names introduced by the authors are the same as the ones used in both files. The assignment formulation consists basically of 4 constraint classes. That are the constraints ensuring that the capacity of each slab is not exceeded, the constraints taking care that a item is manufactured at most once, the set of constraints which check if a color class is used within a slab, and the set of constraints which enforce that at most two different colors are present within a slab. To construct a steel mill slab design problem out of these instances, we only need to consider the constraints ensuring that the slab capacities are respected and the set of constraints which check if a color class is used within a slab. The first constraint class provides for each item its size and for each slab its capacity. The later class states for each item the corresponding color class. In both cases the item sizes and capacities are rational numbers. By multiplying all numbers with 100, we are left with integer values for the sizes and capacities. The leftover we stated in that paper for the `mkc` instance is 48.32 which belongs to the instance without multiplying the sizes and capacities.

In the following two sections we explicitly state for `mkc` and `mkc7` the steel mill slab design problem input data.

A.1. Instance `mkc`. Using the instance provided in the MIPLIB2010 [11] we detected 23 different capacities and 439 orders belonging to 347 color classes. Below we list the capacities and order tuples which give the size and the color. These tuples are listed in the form

$$s_1 \ c_1; s_2 \ c_2; \dots; s_n \ c_n;$$

List of capacities.

2120; 2274; 1955; 1982; 2257; 2322; 1807; 2088; 1971; 1467; 2346; 2458; 2805; 2872; 2971; 3213; 3388; 3615; 3742; 3831; 3418; 3636; 3660;

List of order tuples.

1000 0; 1500 1; 1500 2; 1000 3; 1000 4; 1000 5; 1000 6; 500 7; 1200 8; 1500 9; 1000 10; 500 11; 1000 5; 1000 12; 500 7; 500 13; 1000 14; 1000 15; 720 16; 1000 17; 2000 18; 1500 19; 500 20; 1000 21; 1000 21; 500 22; 1000 23; 500 24; 800 25; 1500 26; 1000 27; 900 28; 1500 29; 1500 30; 1500 31; 1200 6; 500 32; 1000 17; 500 33; 1500 34; 1500 2; 1000 35; 1500 36; 900 37; 500 38; 1000 21; 1000 39; 500 40; 1000 41; 1500 42; 1000 43; 500 33; 1500 44; 1500 45; 1500 45; 500 46; 500 47; 500 38; 1500 48; 2000 49; 1000 50; 1000 51; 2000 52; 1000 14; 1250 53; 1200 54; 1000 43; 1000 55; 1750 56; 1500 34; 500 57; 1490 58; 500 59; 500 60; 500 61; 1000 62; 1500 63; 500 46; 500 46; 1000 64; 1200 65; 890 66; 1500 67; 1500 68; 530 69; 1500 31; 1000 70; 1000 55; 1500 71; 1750 56; 1000 72; 1800 73; 1000 74; 1000 74; 1370 75; 720 76; 1000 77; 500 78; 1800 79; 500 78; 1500 80; 1400 81; 1500 82; 1500 83; 2000 84; 1500 85; 1750 86; 1750 86; 1500 87; 1000 88; 1500 89; 1500 89; 1750 86; 1500 87; 1750 90; 1750 90; 2000 91; 2000 91; 500 92; 1500 89; 1750 90; 2000 93; 1500 94; 500 95; 2000 96; 2000 97; 1500 94; 2000 96; 1500 94; 500 95; 500 98; 1490 0; 500 99; 1320 100; 840 101; 810 102; 810 103; 1200 104; 840 101; 810 105; 840 101; 1200 104; 810 105; 1500 106; 810 103; 1200 104; 800 107; 1500 108; 1500 109; 1500 110; 1430 111; 540 112; 400 112; 1000 113; 1000 113; 1300 114; 1500 115; 1000 116; 1500 117; 1500 118; 2000 119; 2000 120; 1300 121; 1300 121; 930 122; 930 123; 930 123; 1500 124; 1500 117; 900 125; 900 22; 1300 126; 900 127; 1300 126; 900 126; 900 126; 900 128; 1500 129; 1500 130; 1500 125; 2000 131; 2000 132; 900 22; 1500 133; 1500 134; 1500 135; 1500 136; 1500 137; 1500 138; 1500 139; 1500 119; 1500 140; 2000 141; 1000 142; 1700 143; 1500 144; 1500 144; 1500 145; 1450 146; 869 147; 1500 148; 390 149; 990 150; 990 151; 990 152; 990 153; 990 154; 990 155; 990 156; 990 157;

⁵<http://www.coin-or.org/>

990 158; 990 6; 990 159; 1090 160; 1090 160; 990 161; 990 162; 990 28; 990 163; 990 164; 990 165; 990 166; 990 10; 990 167; 990 168; 990 169; 1190 170; 1190 171; 1190 170; 1190 172; 1490 173; 1490 174; 290 175; 284 176; 990 177; 187 178; 1190 179; 187 178; 990 180; 819 181; 560 182; 796 183; 994 184; 994 184; 994 185; 894 186; 496 187; 994 188; 990 189; 990 164; 990 165; 690 190; 990 177; 990 191; 990 192; 990 193; 990 194; 990 195; 990 196; 890 197; 890 198; 890 199; 890 200; 890 201; 990 202; 341 203; 471 204; 463 205; 740 206; 480 207; 990 208; 560 182; 386 209; 580 210; 490 211; 994 212; 994 213; 1194 214; 894 215; 776 216; 557 217; 994 218; 994 184; 994 184; 994 185; 994 219; 894 220; 994 221; 994 188; 1500 222; 1650 223; 1000 224; 1650 223; 1500 222; 1500 225; 1000 226; 1000 227; 1500 228; 1000 229; 1270 230; 600 231; 1200 232; 1200 232; 1200 233; 1200 233; 1200 234; 1200 234; 1200 234; 1390 235; 600 233; 600 234; 1200 232; 1000 236; 700 237; 600 233; 1200 232; 1000 238; 1500 239; 1500 240; 1500 240; 1000 241; 1100 242; 1000 243; 1000 244; 1000 245; 1000 246; 1000 247; 1270 248; 1390 249; 600 250; 1100 251; 1050 252; 1050 253; 1050 254; 1390 255; 205 256; 1250 257; 1200 258; 600 259; 994 260; 994 261; 696 262; 497 263; 497 264; 497 265; 497 266; 596 267; 190 268; 994 269; 497 270; 497 271; 994 272; 994 273; 497 274; 497 275; 994 276; 497 277; 994 278; 500 279; 650 280; 640 281; 1500 282; 1500 283; 1500 284; 1500 285; 1500 286; 1500 287; 1300 288; 1000 289; 1000 289; 1300 290; 1500 291; 1500 292; 1000 293; 1500 294; 1500 295; 497 296; 497 297; 497 298; 497 299; 1000 300; 497 301; 497 302; 497 303; 500 304; 1000 305; 700 306; 1470 307; 500 308; 1500 309; 1500 309; 1220 310; 1000 311; 1410 312; 700 313; 600 314; 1000 315; 1000 315; 1000 316; 600 317; 1000 318; 1000 319; 1100 320; 1320 321; 1140 322; 990 323; 1480 324; 1440 325; 990 326; 994 327; 994 328; 497 329; 994 330; 994 331; 994 332; 496 333; 994 334; 700 335; 600 336; 600 337; 700 335; 800 338; 800 338; 800 338; 700 339; 700 340; 700 340; 990 341; 890 342; 890 343; 500 344; 1800 345; 1800 345; 1000 346; 1000 346;

A.2. Instance `mkc7`. The instance `mkc7` can be collected from the Coin-OR project. It contains w.r.t. the steel mill slab design problem 70 different slab capacities and 9484 orders. These orders are spread out over 642 color classes. Below we list the capacities and order tuples which give the size and the color. These tuples are listed in the form

$$s_1 \ c_1; s_2 \ c_2; \dots; s_n \ c_n;$$

List of capacities.

1707; 1695; 1622; 1978; 2054; 1782; 1736; 1832; 1605; 1904; 1395; 1668; 1717; 998; 2230; 2277; 2145; 928; 2241; 1970; 1763; 1630; 1222; 1634; 1995; 1085; 1972; 2045; 1554; 1523; 1456; 1989; 1988; 1682; 1657; 1641; 1003; 4155; 1764; 1747; 2311; 3481; 2451; 3008; 1186; 1163; 1060; 1587; 2763; 831; 814; 827; 817; 809; 811; 2360; 1042; 1046; 1011; 743; 737; 746; 740; 833; 2710; 2662; 2689; 2910; 2892; 4391;

List of order tuples.

103 0; 111 0; 268 0; 116 0; 234 0; 117 0; 210 0; 113 0; 273 0; 112 0; 115 0; 117 0; 116 0; 104 0; 100 0; 108 0; 255 0; 205 0; 107 0; 114 0; 116 0; 274 0; 291 0; 269 0; 258 0; 117 0; 101 0; 243 0; 218 0; 114 0; 106 0; 281 0; 114 0; 109 0; 277 0; 105 0; 283 0; 235 0; 110 0; 112 0; 252 0; 113 0; 298 0; 261 0; 201 0; 289 0; 115 0; 221 0; 209 0; 115 0; 104 0; 107 0; 299 0; 219 0; 267 0; 117 0; 116 0; 297 0; 265 0; 214 0; 103 0; 275 0; 223 0; 110 0; 215 0; 102 0; 290 0; 106 0; 109 0; 225 0; 115 0; 101 0; 116 0; 246 0; 251 0; 222 0; 112 0; 285 0; 282 0; 104 0; 114 0; 272 0; 242 0; 296 0; 232 0; 111 0; 266 0; 242 1; 292 1; 256 1; 100 1; 239 1; 115 1; 117 1; 264 1; 115 1; 244 1; 255 1; 117 1; 114 1; 279 1; 106 1; 268 1; 272 1; 117 1; 232 1; 103 1; 229 1; 116 1; 105 1; 254 1; 106 1; 284 1; 288 1; 274 1; 101 1; 104 1; 114 1; 269 1; 103 1; 221 1; 226 1; 250 1; 215 1; 110 1; 104 1; 243 1; 273 1; 267 1; 282 1; 107 1; 258 1; 112 1; 293 1; 230 1; 241 1; 238 1; 112 1; 114 1; 113 1; 285 1; 299 1; 116 1; 225 1; 116 1; 289 1; 116 1; 217 1; 108 1; 111 1; 112 1; 115 1; 223 1; 110 1; 116 1; 102 1; 113 1; 104 1; 219 1; 296 1; 224 1; 211 1; 107 1; 109 1; 208 1; 286 1; 117 1; 111 1; 115 1; 259 1; 109 1; 116 1; 114 1; 101 1; 112 2; 258 2; 209 2; 117 2; 244 2; 103 2; 214 2; 113 2; 110 2; 294 2; 100 2; 116 2; 112 2; 114 2; 291 2; 106 2; 114 2; 114 2; 211 2; 267 2; 215 2; 116 2; 235 2; 111 2; 101 2; 117 2; 283 2; 282 2; 287 2; 104 2; 259 2; 102 2; 288 2; 225 2; 116 2; 109 2; 227 2; 243 2; 205 2; 265 2; 299 2; 115 2; 206 2; 111 2; 116 2; 101 2; 218 2; 249 2; 261 2; 272 2; 107 2; 223 2; 115 2; 116 2; 275 2; 285 2; 238 2; 109 2; 256 2; 107 2; 234 2; 113 2; 106 2; 105 2; 232 2; 668 3; 544 3; 641 3; 562 3; 568 4; 600 4; 698 4; 329 5; 279 5; 343 5; 289 5; 311 6; 343 6; 279 6; 329 6; 220 7; 233 7; 275 7; 270 7; 240 7; 352 8; 465 8; 379 8; 699 9; 554 9; 552 10; 564 10; 654 10; 629 10; 591 10; 701 10; 684 10; 575 10; 1000 11; 947 11; 1163 11; 699 12; 554 12; 686 13; 659 13; 578 13; 558 13; 698 14; 568 14; 600 14; 554 15; 587 15; 639 15; 568 15; 666 15; 699 15; 558 16; 686 16; 578 16; 659 16; 465 17; 379 17; 400 17; 699 18; 554 18; 699 19; 554 19; 443 20; 559 20; 663 21; 700 21; 814 21; 218 22; 200 22; 189 22; 814 21; 700 21; 663 21; 915 23; 770 23; 879 23; 745 23; 400 24; 379 24; 465 24; 769 25; 800 25; 674 25; 652 25; 700 26; 643 26; 681 26; 803 26; 788 26; 663 23; 746 23; 778 23; 685 23; 646 23; 816 23; 554 27; 639 27; 699 27; 568 27; 587 27; 666 27; 279 28; 227 28; 240 28; 379 22; 400 22; 465 22; 457 29; 388 29; 426 29; 439 29; 372 29; 468 29; 412 29; 385 29; 460 29; 367 29; 397 29; 373 29; 400 21; 465 21; 379 21; 457 30; 385 30; 372 30; 439 30; 656 31; 600 31; 629 31; 597 31; 586 31; 680 31; 697 31; 693 31; 553 31; 560 31; 573 31; 552 31; 702 31; 669 31; 560 31; 576 31; 627 31; 414 32; 442 32; 387 32; 392 32; 466 32; 428 32; 411 32; 464 32; 374 32; 370 32; 404 32; 461 32; 383 32; 395 32; 399 32; 368 32; 434 32; 368 32; 452 32; 444 32; 378 32; 374 32; 468 32; 396 32; 424 32; 370 32; 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266 57; 281 57; 222 57; 240 57; 258 58; 326 58; 183 59; 229 59; 225 59; 204 59; 194 59; 163 60; 172 60; 161 60; 136 60; 204 60; 115 60; 176 60; 147 60; 111 60; 183 60; 201 60; 186 60; 203 60; 193 60; 133 60; 154 60; 143 60; 129 60; 140 60; 197 60; 190 60; 158 60; 151 60; 122 60; 179 60; 126 60; 118 60; 189 61; 200 61; 232 61; 232 62; 189 62; 200 62; 232 63; 200 63; 189 63; 257 64; 305 64; 327 64; 268 64; 263 64; 319 64; 276 64; 293 64; 305 65; 268 65; 257 65; 293 65; 276 65; 263 65; 319 65; 327 65; 293 66; 327 66; 263 66; 319 66; 276 66; 305 66; 372 67; 385 67; 367 67; 426 67; 388 67; 397 67; 373 67; 460 67; 468 67; 457 67; 439 67; 412 67; 462 68; 583 68; 462 69; 519 69; 578 69; 537 69; 558 69; 583 69; 495 69; 476 69; 483 69; 462 69; 166 70; 138 70; 142 70; 145 70; 152 70; 175 70; 164 70; 173 70; 140 70; 164 70; 144 70; 169 70; 146 70; 171 70; 146 70; 173 70; 162 70; 168 70; 158 70; 141 70; 148 70; 172 70; 146 70; 139 70; 153 70; 160 70; 174 70; 138 70; 171 70; 143 70; 144 70; 151 70; 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800 119; 639 133; 554 133; 699 133; 568 133; 133; 379 110; 400 110; 465 110; 583 115; 462 115; 500 127; 486 127; 563 127; 459 127; 573 127; 699 133; 554 133; 200 134; 232 134; 189 134; 463 135; 379 135; 400 135; 600 136; 554 136; 513 137; 406 137; 404 137; 495 137; 428 137; 504 137; 440 137; 349 138; 284 138; 300 138; 515 139; 410 139; 440 139; 407 139; 432 139; 483 139; 489 139; 816 140; 646 140; 816 141; 663 141; 746 141; 685 141; 646 141; 778 141; 346 142; 379 142; 284 143; 300 143; 406 144; 513 144; 488 145; 406 145; 513 145; 469 145; 417 145; 384 146; 305 146; 364 146; 322 146; 328 146; 303 146; 360 146; 600 147; 698 147; 568 147; 684 147; 591 147; 552 147; 701 147; 564 147; 575 147; 629 147; 654 147; 319 148; 277 148; 284 148; 293 148; 350 148; 333 148; 284 149; 293 149; 350 149; 319 149; 277 149; 333 149; 284 150; 293 150; 319 150; 277 150; 350 150; 333 150; 688 151; 391 151; 444 151; 369 151; 466 151; 426 151; 874 152; 693 152; 289 153; 343 153; 329 153; 279 153; 583 154; 462 154; 699 155; 554 155; 300 153; 350 153; 279 156; 329 156; 289 156; 343 156; 583 157; 462 157; 750 158; 868 158; 690 158; 741 158; 811 158; 873 158; 710 158; 702 158; 329 158; 734 159; 799 159; 693 159; 833 159; 711 159; 874 159; 566 160; 702 160; 672 160; 630 160; 585 160; 672 160; 586 160; 570 160; 558 160; 552 160; 554 160; 597 160; 644 160; 696 160; 614 160; 693 160; 698 161; 568 161; 600 161; 352 162; 379 162; 465 162; 300 153; 349 153; 284 153; 628 163; 511 163; 540 163; 376 164; 456 164; 467 164; 419 164; 383 164; 436 164; 394 164; 368 164; 383 165; 436 165; 376 165; 419 165; 368 165; 394 165; 467 165; 456 165; 693 110; 874 110; 693 166; 874 166; 379 167; 400 167; 465 167; 591 163; 540 163; 511 163; 629 163; 499 163; 436 168; 419 168; 467 168; 383 168; 456 168; 394 168; 368 168; 437 169; 400 169; 379 169; 554 170; 699 170; 639 170; 666 170; 587 170; 686 171; 558 171; 578 171; 659 171; 587 172; 666 172; 639 172; 554 172; 568 172; 699 172; 634 173; 580 173; 549 173; 379 174; 400 174; 465 174; 400 175; 465 175; 379 175; 469 176; 592 176; 299 177; 367 177; 316 177; 699 178; 554 178; 275 179; 220 179; 270 179; 233 179; 240 179; 369 180; 434 180; 388 180; 390 180; 411 180; 375 180; 368 180; 377 180; 397 180; 452 180; 454 180; 466 180; 466 180; 423 180; 293 181; 316 181; 265 181; 333 181; 274 181; 287 181; 335 181; 337 181; 267 181; 300 181; 329 181; 277 181; 326 181; 282 181; 272 181; 268 181; 265 181; 305 181; 800 182; 465 183; 379 183; 400 183; 424 184; 439 184; 394 184; 367 184; 351 184; 351 184; 376 184; 362 184; 443 184; 408 184; 356 185; 414 185; 337 185; 392 186; 456 186; 371 186; 457 187; 385 187; 439 187; 372 187; 379 187; 400 187; 465 187; 656 188; 568 188; 600 188; 565 189; 593 189; 553 189; 688 189; 690 189; 681 189; 605 189; 629 189; 700 189; 555 189; 558 189; 610 189; 647 189; 701 189; 669 189; 702 189; 661 189; 575 189; 565 189; 557 189; 562 189; 650 189; 676 189; 693 189; 589 189; 702 189; 569 189; 675 189; 627 189; 578 189; 594 189; 597 189; 618 189; 662 189; 684 189; 622 189; 699 189; 583 189; 655 189; 586 189; 702 189; 554 189; 697 189; 595 189; 696 189; 560 189; 552 189; 563 189; 551 189; 553 189; 588 189; 580 189; 633 189; 666 189; 560 189; 576 189; 551 189; 591 189; 598 189; 686 189; 552 189; 600 189; 691 189; 644 189; 616 189; 607 189; 581 189; 585 189; 573 189; 556 189; 567 189; 680 189; 552 189; 656 189; 599 189; 671 189; 570 189; 572 189; 699 189; 702 189; 554 189; 638 189; 694 189; 639 189; 1140 190; 1036 190; 997 190; 986 190; 1155

190; 940 190; 925 190; 1160 190; 938 190; 1090 190; 921 190; 919 190; 1169 190; 1129 190; 928 190; 971 190; 985
 190; 1082 190; 1015 190; 1170 190; 1049 190; 955 190; 1070 191; 1125 191; 1149 191; 1125 191; 977 191; 922 191;
 920 191; 929 191; 1011 191; 1162 191; 1165 191; 1147 191; 1061 191; 957 191; 1028 191; 949 191; 919 191; 1078
 191; 1095 191; 919 191; 1058 191; 921 191; 1170 191; 1116 191; 974 191; 945 191; 969 191; 1019 191; 927 191;
 984 191; 1028 191; 1036 191; 981 191; 924 191; 1171 191; 927 191; 999 191; 981 191; 936 191; 938 191; 1170 191;
 1131 191; 1071 191; 935 191; 919 191; 953 191; 1137 191; 925 191; 1084 191; 1143 191; 923 191; 1003 191; 1053
 191; 1045 191; 929 191; 942 191; 988 191; 961 191; 965 191; 1153 191; 1106 191; 1158 191; 973 191;
 987 191; 991 191; 1154 191; 1165 191; 1133 191; 1087 191; 1167 191; 1095 191; 993 191; 1010 191; 1140 191; 996
 191; 1168 191; 996 191; 1169 191; 941 191; 998 191; 961 191; 1162 191; 921 191; 1169 191; 932 191; 1118 191;
 1103 191; 1044 191; 933 191; 956 191; 1110 191; 920 191; 950 191; 968 191; 945 191; 931 189; 758 189; 800 189;
 587 192; 680 192; 696 192; 560 192; 625 192; 571 192; 649 192; 548 192; 870 189; 931 189; 749 189; 865 189; 736
 189; 758 189; 800 189; 926 189; 790 189; 333 193; 365 193; 379 193; 320 193; 343 193; 397 193; 327 193; 406 193;
 400 194; 379 194; 465 194; 391 195; 318 195; 336 195; 379 196; 465 196; 400 196; 374 197; 357 197; 349 197; 433
 197; 414 197; 398 197; 364 197; 444 197; 328 198; 407 198; 348 198; 395 198; 400 198; 320 198; 333 198; 345 198;
 372 198; 323 198; 373 198; 349 199; 405 199; 329 199; 405 200; 329 200; 348 200; 931 201; 736 201; 790 201; 870
 201; 758 201; 926 201; 800 201; 865 201; 749 201; 778 202; 900 202; 735 202; 800 202; 918 202; 800 203; 758 203;
 931 203; 800 204; 769 204; 652 204; 674 204; 681 205; 700 205; 803 205; 643 205; 788 205; 814 206; 663 206; 700
 206; 758 207; 931 207; 800 207; 421 208; 467 208; 456 208; 459 208; 371 208; 389 208; 421 208; 382 208;
 400 208; 371 208; 397 208; 367 208; 379 208; 442 208; 383 209; 419 209; 456 209; 394 209; 368 209; 467 209; 376
 209; 436 209; 1100 210; 924 211; 1166 211; 178 212; 132 212; 200 212; 135 212; 113 212; 184 212; 201 212; 128
 212; 188 212; 127 212; 198 212; 114 212; 164 212; 143 212; 199 212; 175 212; 181 212; 150 212; 165 212; 192 212;
 136 212; 166 212; 161 212; 156 212; 148 212; 157 212; 189 212; 159 212; 134 212; 185 212; 128 212; 153 212; 121
 212; 172 212; 158 212; 207 212; 182 212; 110 212; 203 212; 152 212; 137 212; 117 212; 208 212; 119 212; 183 212;
 147 212; 171 212; 141 212; 126 212; 116 212; 173 212; 197 212; 193 212; 122 212; 131 212; 205 212; 186 212; 163
 212; 187 212; 154 212; 140 212; 149 212; 124 212; 145 212; 191 212; 133 212; 196 212; 209 212; 130 212; 142 212;
 111 212; 179 212; 202 212; 155 212; 151 212; 195 212; 120 212; 139 212; 195 212; 206 212; 144 212; 180 212; 176
 212; 129 212; 170 212; 204 212; 112 212; 194 212; 138 212; 169 212; 168 212; 161 212; 125 212; 174 212; 160 212;
 123 212; 190 212; 146 212; 177 212; 118 212; 167 212; 162 212; 115 212; 150 213; 113 213; 157 213; 181 213; 155
 213; 151 213; 144 213; 209 213; 126 213; 143 213; 164 213; 140 213; 128 213; 165 213; 124 213; 204 213; 139 213;
 162 213; 149 213; 176 213; 169 213; 199 213; 193 213; 166 213; 172 213; 146 213; 207 213; 175 213; 154 213; 117
 213; 145 213; 125 213; 180 213; 205 213; 197 213; 120 213; 194 213; 173 213; 179 213; 191 213; 136 213; 128 213;
 167 213; 168 213; 170 213; 163 213; 184 213; 129 213; 182 213; 111 213; 187 213; 121 213; 156 213; 189 213; 203
 213; 195 213; 185 213; 183 213; 206 213; 122 213; 198 213; 130 213; 116 213; 192 213; 115 213; 135 213; 148 213;
 174 213; 159 213; 208 213; 119 213; 177 213; 123 213; 195 213; 118 213; 171 213; 112 213; 186 213; 190 213; 138
 213; 158 213; 153 213; 132 213; 131 213; 196 213; 133 213; 127 213; 114 213; 201 213; 142 213; 137 213; 200 213;
 141 213; 134 213; 161 213; 160 213; 110 213; 173 213; 202 213; 152 213; 147 213; 188 213; 161 213; 414 214; 356
 214; 337 214; 329 215; 415 215; 327 216; 400 216; 408 216; 356 216; 346 216; 395 217; 337 217; 415 217; 329 217;
 348 217; 379 217; 296 218; 320 218; 298 218; 314 218; 351 218; 374 218; 355 218; 374 219; 369 219; 317 219; 373
 219; 295 219; 298 219; 370 219; 343 219; 327 219; 344 219; 354 219; 309 219; 301 219; 306 219; 366 219; 296 219;
 336 219; 373 219; 312 219; 294 219; 360 219; 297 219; 313 219; 320 219; 319 219; 305 219; 302 219; 329 219; 294
 219; 363 219; 352 219; 370 220; 343 220; 358 220; 371 220; 327 220; 318 220; 297 220; 313 220; 304 220; 294 220;
 374 220; 336 220; 295 220; 301 220; 312 220; 358 220; 1200 221; 223 222; 207 222; 233 222; 193 222; 184 222; 215
 222; 198 222; 190 222; 231 222; 185 222; 971 223; 1222 223; 1216 223; 1229 223; 1192 223; 1191 223; 1129 223;
 1006 223; 985 223; 977 223; 1229 223; 1021 223; 1046 223; 1048 223; 969 223; 1186 223; 996 223; 1205 223; 1041
 223; 1219 223; 994 223; 1155 223; 1058 223; 967 223; 988 223; 973 223; 1004 223; 1215 223; 1173 223; 1171 223;
 966 223; 1075 223; 1000 223; 965 223; 1012 223; 1228 223; 969 223; 1037 223; 981 223; 975 223; 1019 223; 1182
 223; 970 223; 974 223; 1162 223; 1025 223; 1084 223; 1163 223; 1000 223; 1198 223; 1137 223; 1224 223; 1111
 223; 966 223; 978 223; 1126 223; 1033 223; 1096 223; 1211 223; 985 223; 1227 223; 1229 223; 1093 223; 967 223;
 1151 223; 1061 223; 965 223; 1080 223; 1120 223; 1050 223; 1192 223; 1066 223; 1111 223; 1043 223; 990 223;
 992 223; 1049 223; 1029 223; 1013 223; 1210 223; 1205 223; 1220 223; 1026 223; 1009 223;
 1146 223; 1178 223; 1139 223; 1225 223; 1226 223; 1040 223; 1017 223; 981 223; 1034 223; 466 224; 369 224; 232
 225; 189 225; 200 225; 233 226; 184 226; 189 227; 193 227; 196 227; 185 227; 199 227; 222 227; 188 227; 207 227;
 184 227; 200 227; 232 227; 233 227; 186 227; 184 227; 193 227; 228 227; 219 227; 215 227; 228 227; 208 227; 234
 227; 183 228; 194 228; 200 228; 229 228; 225 228; 222 187; 186 187; 200 187; 219 187; 234 187; 185 187;
 256 229; 203 229; 371 186; 456 186; 392 186; 405 230; 415 230; 352 230; 424 230; 434 230; 404 230; 345 230; 438
 230; 431 230; 428 230; 375 230; 358 230; 366 230; 395 230; 361 230; 346 230; 385 230; 352 230; 386 230;
 366 230; 438 230; 436 230; 347 230; 374 230; 371 230; 348 230; 357 230; 344 230; 417 231; 404 231; 405 231; 352
 231; 385 231; 348 231; 428 231; 415 231; 352 231; 346 231; 371 231; 438 231; 386 231; 366 231; 344 231; 357 231;
 358 231; 431 231; 375 231; 434 231; 374 231; 345 231; 361 231; 438 231; 436 231; 395 231; 366 231; 347 231; 424
 231; 349 220; 349 220; 430 220; 388 220; 353 220; 346 220; 346 220; 356 220; 436 220; 375 220; 398 220;
 374 220; 407 220; 422 220; 437 220; 421 220; 432 220; 344 220; 364 220; 370 220; 386 220; 410 220; 364 220; 439
 220; 874 223; 693 223; 214 223; 230 223; 200 223; 234 223; 184 223; 198 223; 185 223; 214 223; 227 223; 188 223;
 191 233; 137 234; 124 234; 118 234; 176 234; 125 234; 195 234; 209 234; 160 234; 190 234; 154 234; 171 234; 119
 234; 150 234; 184 234; 143 234; 132 234; 170 234; 146 234; 161 234; 164 234; 157 234; 129 234; 111 234; 156 234;
 144 234; 151 234; 200 234; 174 234; 198 234; 145 234; 134 234; 191 234; 142 234; 120 234; 181 234; 112 234; 140
 234; 149 234; 189 234; 186 234; 127 234; 113 234; 196 234; 204 234; 173 234; 116 234; 167 234; 139 234; 178 234;
 201 234; 131 234; 115 234; 182 234; 168 234; 133 234; 198 234; 193 234; 208 234; 153 234; 192 234; 158 234; 135
 234; 117 234; 141 234; 206 234; 179 234; 169 234; 185 234; 110 234; 203 234; 175 234; 214 234; 227 233; 188 233;
 194 234; 159 234; 202 234; 165 234; 123 234; 187 234; 183 234; 166 234; 162 234; 177 234; 128 234; 136 234; 148
 234; 126 234; 233 235; 213 235; 184 235; 189 235; 222 235; 195 235; 549 236; 572 236; 465 236; 481 236; 217 233;
 198 233; 205 233; 195 233; 187 233; 211 233; 184 233; 194 233; 233 233; 227 233; 233 233; 226 233; 188 233; 184
 233; 219 237; 186 237; 192 237; 228 237; 474 238; 555 238; 489 238; 533 238; 583 238; 462 238; 153 239; 115 239;
 118 239; 136 239; 171 239; 209 239; 138 239; 166 239; 174 239; 168 239; 148 239; 125 239; 119 239; 182 239; 172
 239; 188 239; 150 239; 204 239; 156 239; 121 239; 123 239; 187 239; 191 239; 134 239; 132 239; 134 239; 152 239;
 144 239; 127 239; 178 239; 177 239; 203 239; 142 239; 194 239; 124 239; 206 239; 116 239; 110 239; 126 239; 151 239;
 159 239; 185 239; 147 239; 133 239; 195 239; 207 239; 141 239; 132 239; 175 239; 162 239; 183 239;
 130 239; 112 239; 163 239; 198 239; 186 239; 197 239; 113 239; 169 239; 151 239; 145 239; 157 239; 200 239; 180
 239; 165 239; 194 239; 140 239; 127 239; 179 239; 112 239; 142 239; 199 239; 164 239; 184 239; 162 239; 124 239;
 116 239; 154 239; 165 239; 131 239; 176 239; 177 239; 153 239; 137 239; 190 239; 135 239; 111 239; 117 239; 200
 239; 204 239; 201 239; 178 239; 123 239; 160 239; 192 239; 132 239; 157 239; 134 239; 132 239; 126 239; 151 239;
 169 239; 166 239; 174 239; 145 239; 182 239; 161 239; 129 239; 202 239; 128 239; 119 239; 148 239; 133 239; 191
 239; 198 239; 183 239; 207 239; 186 239; 186 239; 144 239; 149 239; 189 239; 193 239; 152 239; 209 239;
 156 239; 115 239; 208 239; 113 239; 143 239; 121 239; 206 239; 171 239; 150 239; 195 239; 146 239; 203 239; 118
 239; 196 239; 187 239; 181 239; 173 239; 168 239; 170 239; 136 239; 159 239; 167 239; 125 239; 185 239; 175 239;
 158 239; 141 239; 436 240; 456 240; 419 240; 467 240; 383 240; 376 240; 394 240; 368 240; 466 241; 369 241; 900
 242; 800 243; 933 243; 739 243; 500 244; 473 244; 582 244; 486 245; 563 245; 500 245; 459 245; 573 245; 582 246;
 473 246; 500 246; 698 247; 600 247; 568 247; 658 247; 698 247; 600 247; 600 248; 554 248; 379 249; 400 249; 465
 249; 415 250; 524 250; 375 251; 816 252; 646 252; 800 253; 793 254; 935 254; 828 254; 754 254; 809 254; 743 254;
 761 254; 750 254; 806 254; 934 254; 735 254; 914 254; 930 254; 790 254; 897 254; 886 254; 746 254; 917 254; 786

373 444; 376 444; 398 444; 363 444; 323 444; 327 444; 322 444; 354 444; 331 444; 322 444; 329 444; 391 444; 405 444; 342 444; 326 444; 360 444; 398 444; 333 444; 408 444; 404 444; 345 444; 338 444; 409 444; 346 444; 382 444; 408 444; 291 445; 300 445; 344 445; 337 445; 275 445; 400 446; 465 446; 379 446; 344 447; 291 447; 337 447; 275 447; 300 447; 277 448; 333 448; 293 448; 319 448; 350 448; 284 448; 322 449; 277 449; 335 449; 297 449; 277 449; 350 449; 311 449; 347 449; 289 449; 286 449; 1200 450; 444 451; 369 451; 379 451; 391 451; 466 451; 426 451; 1512 452; 1232 452; 1300 452; 1476 453; 1219 453; 1267 453; 1335 453; 1517 453; 1226 453; 1374 453; 1261 453; 1516 453; 1292 453; 1471 453; 1196 453; 1410 453; 1201 453; 1508 453; 1237 453; 1396 453; 1271 453; 1503 453; 1196 453; 1458 453; 1200 453; 1293 453; 1226 453; 1331 453; 1457 453; 1268 453; 1366 453; 1210 453; 1522 453; 1487 454; 1428 454; 1252 454; 1210 454; 1513 455; 1300 455; 1232 455; 1450 456; 1696 457; 1683 457; 1440 457; 1649 457; 1340 457; 1538 457; 1379 457; 1336 457; 1450 457; 1361 457; 1623 457; 1402 457; 1538 457; 1334 458; 1350 458; 1469 458; 1263 458; 1562 458; 1461 458; 1242 458; 1571 458; 1279 458; 1565 459; 1431 459; 1350 459; 1446 459; 1683 459; 1661 459; 1416 459; 1370 459; 1510 459; 1684 459; 1336 459; 1377 459; 1496 459; 1434 459; 1630 459; 1696 459; 1696 459; 1333 459; 1359 459; 1345 459; 1397 459; 1593 459; 1397 459; 1619 459; 1659 459; 1469 459; 1552 459; 1338 459; 1546 459; 1581 459; 1447 459; 1612 459; 1450 459; 1341 459; 1438 459; 1693 459; 1333 459; 1698 459; 1335 459; 1481 459; 1549 459; 1515 459; 1336 459; 1662 459; 1351 459; 1682 459; 1395 459; 1420 459; 1487 459; 1682 459; 1364 459; 1410 459; 1694 459; 1548 459; 1394 459; 1664 459; 1425 459; 1373 459; 1355 459; 1343 459; 1632 459; 1640 459; 1379 459; 1691 460; 1340 460; 419 461; 456 461; 383 461; 376 461; 436 461; 394 461; 368 461; 467 461; 1035 462; 1306 462; 406 463; 465 463; 379 463; 279 464; 351 464; 333 464; 295 464; 300 464; 329 464; 278 464; 464 465; 375 465; 380 465; 455 465; 448 465; 369 465; 397 465; 467 465; 386 465; 424 465; 368 465; 400 465; 424 465; 372 466; 385 466; 439 466; 457 466; 1300 467; 1386 468; 1516 468; 1444 468; 1272 468; 1201 468; 1232 468; 1246 467; 1479 467; 1227 467; 1391 467; 1300 467; 1521 467; 1196 467; 1288 467; 1394 467; 1496 467; 1207 467; 1264 469; 1195 469; 1491 469; 1300 469; 1464 469; 1618 470; 1369 470; 1624 470; 1320 470; 1471 470; 1288 470; 1433 470; 1639 470; 1303 470; 1504 470; 1569 470; 1332 470; 1365 470; 1393 470; 1570 470; 1293 470; 1316 471; 1432 471; 1526 471; 1380 471; 1464 471; 1288 471; 1636 471; 1597 471; 379 472; 465 472; 400 472; 329 473; 279 473; 343 473; 289 473; 929 474; 789 474; 765 474; 768 474; 797 474; 757 474; 840 474; 893 474; 745 474; 934 474; 806 474; 902 474; 767 474; 798 474; 796 474; 764 474; 915 474; 910 474; 903 474; 760 474; 850 474; 737 474; 759 474; 850 474; 925 474; 737 474; 764 474; 934 474; 782 474; 920 474; 770 474; 790 474; 800 474; 736 474; 736 474; 896 474; 896 474; 890 474; 787 474; 880 474; 762 474; 751 474; 866 474; 799 474; 736 474; 926 474; 736 474; 936 474; 785 474; 830 474; 888 474; 932 474; 874 474; 749 474; 803 474; 892 474; 936 474; 906 474; 740 474; 775 474; 776 474; 868 474; 844 474; 745 474; 839 474; 755 474; 833 474; 936 474; 768 474; 919 474; 816 474; 737 474; 922 474; 757 474; 739 474; 793 474; 861 474; 746 474; 793 474; 861 474; 826 474; 836 474; 926 474; 750 474; 936 474; 735 474; 773 474; 740 474; 857 474; 877 474; 743 474; 798 474; 739 474; 747 474; 819 474; 887 474; 847 474; 754 474; 923 474; 738 474; 813 474; 930 474; 794 474; 914 474; 804 474; 933 474; 907 474; 786 474; 899 474; 747 474; 787 474; 875 474; 748 474; 871 474; 933 474; 740 474; 864 474; 743 474; 921 474; 744 474; 756 474; 753 474; 750 474; 928 474; 781 474; 741 474; 778 474; 819 474; 871 474; 854 474; 912 474; 900 474; 935 474; 738 474; 918 474; 772 474; 764 474; 928 474; 917 474; 910 474; 809 474; 931 474; 736 474; 742 474; 780 474; 936 474; 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