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Optimizing Toll Enforcement in Transportation Networks: a Game-Theoretic Approach

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Abstract

We present a game-theoretic approach to optimize the strategies of toll enforcement on a motorway network. In contrast to previous approaches, we consider a network with an arbitrary topology, and we handle the fact that users may choose their Origin-Destination path; in particular they may take a detour to avoid sections with a high control rate. We show that a Nash equilibrium can be computed with an LP (although the game is not zero-sum), and we give a MIP for the computation of a Stackelberg equilibrium. Experimental results based on an application to the enforcement of a truck toll on German motorways are presented.

keyword Game Theory; Stackelberg Equilibrium; Mixed Integer Programming

1 Introduction

In 2005 Germany introduced a distance-based toll for trucks weighing twelve tonnes or more in order to fund growing investments for maintenance and extensions of motorways. The enforcement of the toll is the responsibility of the German Federal Office for Goods Transport (BAG), who has the task to carry out a network-wide control. To this end, 300 vehicles make control tours on the entire highway network. In this paper, we present some theoretical work obtained in the framework of our cooperation with the BAG, whose final goal is to develop an optimization tool to schedule the control tours of the inspectors. This real-world problem is subject to a variety of legal constraints, which we handle by mixed integer programming [2]. In a follow-up work, we plan to use the results of the present article as a target for the real-world problem.

In this paper, the problem of allocating inspectors to spatial locations of a transportation network in order to enforce the payment of a transit toll is studied from a game-theoretic point of view. This problem presents several similarities with recent studies on the application of game theory to a class of problems where the goal is to randomize different kind of inspections, in a strategical way; this includes a work on the optimal selection of checkpoints and patrol routes to protect the LA Airport towards adversaries [5], a study of the scheduling and allocation of air marshals to a list of flights in the US [3], or the problem of optimally scheduling fare inspection patrols in LA Metro [6].

The core of this work is to handle the difficulties arising from the large number of available paths for the network users, while taking into account the additional traveling costs when users make a detour. In contrast, previous approaches used the trivial topology of a single metro line [6], or assumed that each user takes the shortest path [1]. A similar network security game, where the defender has an exponential number of actions was studied in [3] with the help of a branch-and-price algorithm. In this article, we represent user strategies by network flows, which allows us to give a compact LP formulation for the computation of a Nash equilibrium of the game. We next use some ideas of [5] to formulate the problem of finding a Stackelberg equilibrium of the game as a mixed integer program (MIP). Experimental results based on real traffic data (averaged over time) are given in section 4, and suggest that the Nash equilibrium strategy is a good trade-off between computation time and efficiency of the controls.

2 A Spot-checking game

In this section we extend the game theoretic model presented in [1], which studies the interaction between the fare inspectors and the users of a transportation network, to handle the case where every user is free to choose its path in the network to reach its destination.

We first recall the notion of *best strategy* in game theory, which is central in this article, since it is used in the definitions of Nash and Stackelberg equilibria. Consider a game where each player $i = 1, \ldots, N$ can commit to a strategy $\boldsymbol{p^i}$ in a set Δ_i, and wishes to maximize his own payoff $u_i(\boldsymbol{p_i}, \boldsymbol{p_{-i}})$. We say that player i's strategy $\boldsymbol{p_i}$ is a *best response* to the others' strategies $\boldsymbol{p_{-i}}$ if his payoff cannot increase when $\boldsymbol{p_{-i}}$ is fixed:

$$\forall \boldsymbol{p_i'} \in \Delta_i, \qquad u_i(\boldsymbol{p_i'}, \boldsymbol{p_{-i}}) \leq u_i(\boldsymbol{p_i}, \boldsymbol{p_{-i}}).$$

Model settings The transportation network is represented by a weighted directed graph $G(V, E, \boldsymbol{w})$, where weight w_e represents the traveling cost on edge $e \in E$. We assume that the users of the network are distributed over a set of commodities $\mathcal{K} = \{k_1, \ldots, k_m\}$, which represent Origin-Destination pairs of the network $k = \big(\mathrm{src}(k), \mathrm{dst}(k)\big)$. We denote by S the set of commodity sources $\{s : \exists (s,t) \in \mathcal{K}\}$ and for $s \in S$ we define $D_s := \{t : (s,t) \in \mathcal{K}\}$.

For all $k \in \mathcal{K}$, we denote by $\mathcal{R}_k$ the set of all paths from $\mathrm{src}(k)$ to $\mathrm{dst}(k)$. In particular, $R_k^* \in \mathcal{R}_k$ is the shortest path through k (with respect to weights w_e). In addition, we are given the demand x_k of commodity k, i.e., the number of users who make a trip on commodity k during a given period of time.

The users of commodity k are expected to pay a toll fee T_k. If a user evades the toll, he takes the risk to pay a penalty $P >> T_k$ in case of a control. If an inspector is present on edge e, we denote by σ_e the probability that an individual passing on e is controlled.

Inspectors' strategy The set of edges is partitioned as $E = E_{\text{pay}} \cup E_{\text{free}}$, where E_{free} represent some *toll-free* edges, where users shall not be controlled. There are κ teams of inspectors over the network, who can each control an edge $e \in E_{\text{pay}}$. Their pure strategies hence correspond to the subsets of E_{pay} of cardinality κ. A mixed strategy is a probability distribution over those subsets, but we will see that our model only depends on the marginal probabilities q_e that some inspector is present on e,

$$\sum_{e \in E_{\text{pay}}} q_e = \kappa \qquad \text{and} \qquad \forall e \in E_{\text{pay}},\ 0 \leq q_e \leq 1. \tag{1}$$

Conversely, if we are given a vector $\boldsymbol{q}$ satisfying Equation (1), we point out that we can find a probability distribution over the subsets of cardinality κ whose marginal equals $\boldsymbol{q}$. To simplify the notation, we assume hereafter that $\sigma_e = 0$, and $q_e = 0$ is a constant for every toll-free edge $e \in E_{\text{free}}$.

Network users, fare evasion and path selection We associate the users of commodity k with a single player (called player k). Player k can either pay the toll and take the shortest path, or try to evade the toll by taking any path $R \in \mathcal{R}_k$ (which might be a detour). His mixed strategy can be interpreted as the proportion of $k-$users who pay or evade on a particular path $R \in \mathcal{R}_k$. For the sake of simplicity we create an artificial edge e_k^* with weight $w_{e_k^*} := \sum_{e \in R_k^*} w_e + T_k$ which goes directly from the origin to the destination of k, and we define $\bar{E}_k := E \cup \{e_k^*\}$; the interpretation is that player k pays the toll if he takes e_k^*. Our model depends only on the probabilities p_e^k that player k uses edge e, that must form a flow of value one through commodity k:

$$\forall v \in V, \sum_{\{u:(v,u) \in \bar{E}_i\}} p^k_{(v,u)} - \sum_{\{u:(u,v) \in \bar{E}_k\}} p^k_{(u,v)} = \begin{cases} 1 & \text{if } \ v = \text{src}(k); \\ -1 & \text{if } \ v = \text{dst}(k); \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The probability to be controlled on an edge $e \in R$ is $q_e \sigma_e$, and hence the expected number of times player k is subjected to a control during his trip is $\sum_{e \in R} p_e^k q_e \sigma_e$. We approximate the total expected cost of player k by

$$\text{Payoff}_k(\boldsymbol{p}, \boldsymbol{q}) = -\left(\sum_{e \in \bar{E}_k} p_e^k w_e + \sum_{e \in E} p_e^k q_e \sigma_e P \right), \tag{3}$$

where the first term accounts for travel and toll costs, while the second is the expected fine, i.e. we do as if evaders could be fined several times (for a realistic number of controllers, our results show that the risk of being controlled more than once is very small; a similar approximation has been used in [6] and [1]). A consequence of Equation (3) is that the best response of player k to the inspectors' strategy $\boldsymbol{q}$ is to take the shortest path for commodity k in the graph $G(V, \bar{E}_k, \boldsymbol{w'})$ with modified edge weights $w'_e = w_e + q_e \sigma_e P$, where $q_{e_k^*}$ and $\sigma_{e_k^*}$ are constants set to 0 for the artificial edge e_k^*.

Inspectors' payoff We introduce two parameters α and $\boldsymbol{\beta}$, where $\alpha \in [0, 1]$ indicates the fraction of the revenue from penalties to take into account, and

β_e is a reward for each user who takes edge e. Hence, the total payoff for the controllers is:

$$\text{Payoff}_C(\boldsymbol{p}, \boldsymbol{q}) = \sum_k x_k \sum_{e \in \bar{E}_k} p_e^k(\alpha \sigma_e q_e P + \beta_e). \tag{4}$$

If we set $\alpha = 1$ (resp. $\alpha = 0$) and $\beta_e = 0$ for all edges except the artificial ones, where $\beta_{e_k^*} = T_k$, the payoff defined in (4) corresponds to the total revenues from toll and penalties (resp. the toll revenues only). We denote this setting as MAXPROFIT (resp. MAXTOLL). Another interesting case, called MAXPAYERS, is $\alpha = 0$ and $\beta_{e_k^*} = 1$, where the goal is to maximize the number of users who have an incentive to pay the toll.

3 Computation of Equilibria

The notion of equilibrium is essential in game theory. Depending on the ability of the players to observe the others' actions, committing to a Nash or a Stackelberg equilibrium may be better suited [4]. We first show that in the case of MAXPROFIT, our game can be transformed into a *zero-sum game* which has the same Nash equilibria. As a consequence, a Nash equilibrium strategy can be computed by linear programming.

Computation of a Nash equilibrium for MAXPROFIT A Nash equilibrium is a collection of mixed strategies ($\boldsymbol{p}$,$\boldsymbol{q}$) such that every player plays with best response to the others' strategies. As seen in the last section, this means that $\lambda_k := -\text{Payoff}_k(\boldsymbol{p}, \boldsymbol{q})$ equals the length of the shortest path for commodity k in the graph $G(V, \bar{E}_k, \boldsymbol{w'})$, where $w'_e = w_e + q_e \sigma_e P$. Now, for a fixed strategy $\boldsymbol{p}$ of the network users, the goal of the controller is to maximize his total revenue $\sum_k x_k(\sum_{e \in E} p_e^k \sigma_e q_e P + p_{e_k^*}^k T_k)$ with respect to $\boldsymbol{q}$. Equivalently, the controller's goal is to maximize

$$\sum_k x_k (\sum_{e \in E} p_e^k \sigma_e q_e P + p_{e_k^*}^k T_k) + \sum_k x_k (\sum_{e \in E} p_e^k w_e + p_{e_k^*}^k (w_{e_k^*} - T_k)) = \sum_k x_k \lambda_k,$$

because the term which was added does not depend on $\boldsymbol{q}$. We can now formulate a linear program (LP) which computes a Nash equilibrium strategy:

$$\max_{\boldsymbol{q}, \boldsymbol{\lambda}, \boldsymbol{y}} \quad \sum_k x_k \lambda_k \tag{5a}$$

$$\text{s.t.} \quad y_v^s - y_u^s \leq w_{(u,v)} + \sigma_{(u,v)} q_{(u,v)} P, \qquad \forall s \in S,\ \forall (u,v) \in E; \tag{5b}$$

$$y_s^s = 0, \qquad \forall s \in S; \tag{5c}$$

$$\lambda_k \leq y_{\text{dst}(k)}^{\text{src}(k)} \qquad \forall k \in \mathcal{K} \tag{5d}$$

$$\lambda_k \leq w_{e_k^*}, \qquad \forall k \in \mathcal{K}; \tag{5e}$$

$$0 \leq q_e \leq 1, \qquad \forall e \in E; \tag{5f}$$

$$\sum_{e \in E} q_e = \kappa. \tag{5g}$$

The constraints (5b)-(5c) are from the classical linear programming formulation of the single-source shortest path problem, and bound y_v^s from above by the the shortest path length from s to v in the graph $G(V, E, \boldsymbol{w'})$. Constraints (5d) and (5e) further bound λ_k from above by the shortest path length for commodity k in the augmented graph $G(V, \bar{E}_i, \boldsymbol{w'})$. Finally the constraints (5f)-(5g) force $\boldsymbol{q}$ to be a feasible strategy for a set of κ inspectors.

We point out that the optimal dual variables of constraints (5b) and (5e) define a flow in the graph $G(V, \cup_k \bar{E}_k)$, from which a Nash equilibrium strategy $\boldsymbol{p^k}$ for player k can be inferred.

Computation of a Stackelberg equilibrium In a Stackelberg game, it is assumed that a player is the leader (in our case, the controller), who plays first, and the other players (called followers) react with a best response to the leader's action. Stackelberg games are arguably more adapted to the present spot-checking game because of the asymmetry between controllers and network users, and have already been used in several applications [5, 3, 6]. A Stackelberg equilibrium is a profile of strategies $(\boldsymbol{p}, \boldsymbol{q})$ which maximizes the leader's payoff, among the set of all the profiles where the followers' strategies $\boldsymbol{p^k} \in \boldsymbol{p}$ are best responses to the leader's action $\boldsymbol{q}$. Note that the definition implicitly implies that when a follower has several best response actions available, he will select one that favors the leader most. This can be justified in our spot-checking game, since strategies that favor the controller correspond to shorter paths (more penalties, less travel charges).

Using ideas similar as in [5], a mixed integer program (MIP) can be formulated for the computation of a Stackelberg equilibrium $(\boldsymbol{p}, \boldsymbol{q})$. We reduce drastically the number of required variables, by using a single-source multi-sink flow $\boldsymbol{\rho^s}$ for each $s \in S$ instead of using a flow $\boldsymbol{p^k}$ for every commodity. This however requires attention, since only the users of commodity k are allowed to take the artificial edge e_k^*:

$$\max_{\boldsymbol{q},\boldsymbol{y},\boldsymbol{\lambda},\boldsymbol{\mu},\boldsymbol{\rho}} \quad \sum_k x_k\big(\alpha\lambda_k + \mu^k(\beta_{e_k^*} - \alpha w_{e_k^*})\big) + \sum_{s\in S}\sum_{e\in E} \rho_e^s(\beta_e - \alpha w_e) \tag{6a}$$

$$\text{s.t.} \quad 0 \leq w_{(u,v)} + \sigma_{(u,v)} q_{(u,v)} P - (y_v^s - y_u^s) \leq M_{(u,v)}(1 - \mu_{(u,v)}^s), \quad \forall s \in S,\ \forall (u,v) \in E; \tag{6b}$$

$$y_s^s = 0, \qquad \forall s \in S; \tag{6c}$$

$$0 \leq y_{\mathrm{dst}(k)}^{\mathrm{src}(k)} - \lambda_k \leq M_1^k \mu^k, \qquad \forall k \in \mathcal{K}; \tag{6d}$$

$$0 \leq w_{e_k^*} - \lambda_k \leq M_2^k (1 - \mu^k), \qquad \forall k \in \mathcal{K}; \tag{6e}$$

$$0 \leq q_e \leq 1, \qquad \forall e \in E; \tag{6f}$$

$$\sum_{e\in E} q_e = \kappa; \tag{6g}$$

$$\sum_{\{u:(v,u)\in E\}} \rho_{(v,u)}^s - \sum_{\{u:(u,v)\in E\}} \rho_{(u,v)}^s = \delta_v^s(\boldsymbol{\mu}), \qquad \forall s \in S,\ \forall v \in V; \tag{6h}$$

$$0 \leq \rho_e^s \leq M^s \mu_e^s, \qquad \forall s \in S,\ \forall e \in E; \tag{6i}$$

$$\mu_e^s \in \{0,1\},\ \mu^k \in \{0,1\}, \qquad \forall (s,e,k) \in S \times E \times \mathcal{K}. \tag{6j}$$

As in Problem (5), constraints (6b)-(6e) bound λ_k from above by the shortest path length for k in the graph $G(V, \bar{E}_k, \boldsymbol{w'})$ with modified weights, and constraints (6f)-(6g) force $\boldsymbol{q}$ to be a feasible strategy for the κ inspectors. We introduce a binary variable μ_e^s which can take the value 1 only if edge e belongs to a shortest path tree rooted in s (second inequality in (6b)), and a binary variable μ^k which indicates whether player k's best response is to pay the toll (second inequalities in (6d)-(6e), μ^k is free when $\lambda_k = y_{\mathrm{dst}(k)}^{\mathrm{src}(k)} = w_{e_k^*}$). The right hand side in (6h) is defined as

$$\delta_v^s(\boldsymbol{\mu}) = \begin{cases} \sum_{d \in D_s} x_{(s,d)}(1-\mu^{(s,d)}) & \text{if } s = v; \\ -x_{(s,v)}(1-\mu^{(s,v)}) & \text{if } v \in D_s; \\ 0 & \text{otherwise,} \end{cases}$$

so that $\boldsymbol{\rho^s}$ defines a single-source multi-sink flow rooted in s, whose demand in $d \in D_s$ corresponds to the number of evaders on the commodity (s, d). Constraint (6i) ensures that the flow $\boldsymbol{\rho^s}$ only uses edges from a shortest path tree rooted in s. Now, $\boldsymbol{\rho^s}$ can be decomposed as $\sum_{d \in D_s} x_{(s,d)} \boldsymbol{p^{(s,d)}}$, where $\boldsymbol{p^{(s,d)}}$ is a flow from s to d of value $1 - \mu^{(s,d)}$. If (s, d) is the k^{th} commodity, i.e., $k = (s, d)$, we set $p_{e_k^*}^k := \mu^k$, and $\boldsymbol{p^k}$ becomes a flow of value one from $\mathrm{src}(k)$ to $\mathrm{dst}(k)$ in the augmented graph $G(V, \bar{E}_k, \boldsymbol{w'})$. By construction, $\boldsymbol{p^k}$ is a flow of minimal cost $\lambda_k = \sum_{e \in \bar{E}_k} p_e^k (w_e + q_e \sigma_e P)$, and it follows that $\boldsymbol{p^k}$ is a best response to $\boldsymbol{q}$. Finally, the objective function (6a) rewrites to the controller's payoff (4) when replacing λ_k and ρ_e^s by their values as a function of p_e^k. We point out that the *big-M* constants M_e, M_1^k, M_2^k and M^s can all be chosen in the same order of magnitude as the other coefficients of the problem.

Note that the problem becomes easier for MAXTOLL or MAXPAYERS, where $\alpha = 0$ and $\beta_e = 0$ for all non artificial edges $e \in E$, and the flows of network users $\boldsymbol{\rho^s}$ are not involved anymore. In this case, the second inequality of (6b) vanishes, as well as (6h) and (6i).

4 Experimental results on German motorways

We have solved the models presented in this paper for several instances based on real data (averaged over time) from the German motorways network.We present here a brief analysis of our results.

In Figure 1, a near-Stackelberg equilibrium strategy of the inspectors on the whole German network is represented, for the MAXPROFIT case. Here it was assumed that $\kappa = 50$ controllers are simultaneously present on the network, which consists of 319 nodes, 2948 edges and 5013 commodities (the dotted edges on the figure are toll-free edges $e \in E_{\text{free}}$). For this problem, we first computed a Nash Equilibrium with the LP (5); this took 29s on a PC with 8 processors at 3.2GHz. Then, we computed the shortest path through k in $G(V, \bar{E}_k, \boldsymbol{w'})$ for all $k \in \mathcal{K}$, which yields a feasible solution for the MIP (6) that can be used for a *warm start*. We used CPLEX, and an optimality gap of 1.5% was reached after 350s. We point out that the Nash Equilibrium strategy differs only with the MAXPROFIT Strategy on a few edges, and captures 99.7% of the profit from MAXPROFIT.

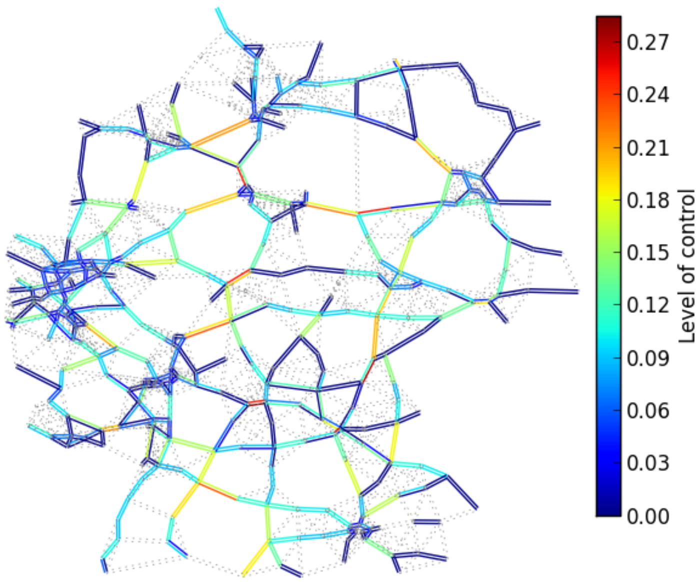
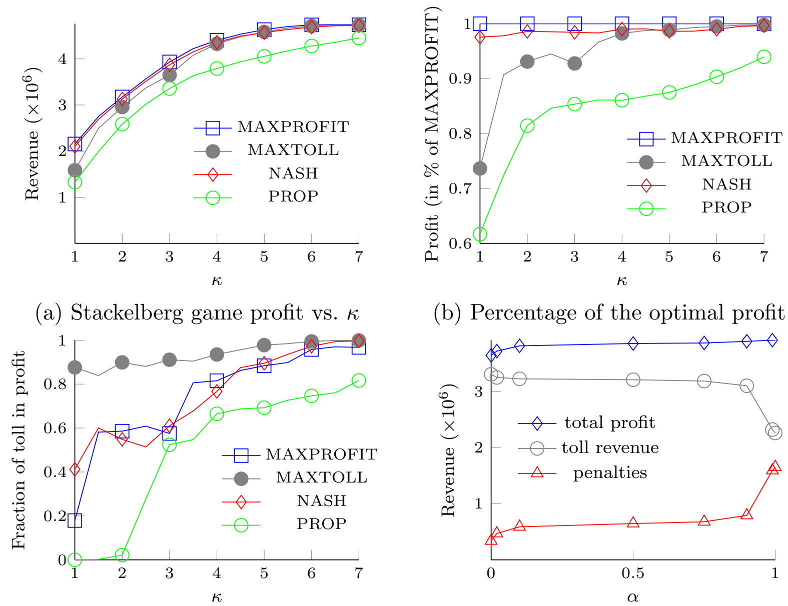


Figure 1: Stackelberg strategy of the controllers $\boldsymbol{q}$ on the German Motorways, for $\kappa = 50$, in the MAXPROFIT setting

Further tests on a smaller network representing the region of Berlin-Brandenburg (45 nodes, 130 edges, 596 commodities) confirm that the Nash Equilibrium strategy might be a good trade-off between the computation time and the efficiency of the controls. Figures 2(a)-2(c) compare 4 strategies in function of the number of controllers κ: the Stackelberg strategies MAXPROFIT and MAXTOLL, the Nash equilibrium strategy computed by LP (5), and a strategy in which the control intensities are proportional to the traffic volume on each edge. Plot (a) shows the profit collected when committing to one of these strategies (in the Stackelberg model, i.e. drivers select a best response which favors the controller most). We see on Plot (b) that the Nash strategy is always near-optimal in terms of profit; we want to investigate this fact in future research. However, we point out that the MAXTOLL strategy outperforms the others in terms of toll enforcement (Plots (c)), at the price of a small loss in total profit (7% for $\kappa = 2$ and 2% for $\kappa = 4$). In another experiment, we have set $\kappa = 3$ and we have played with the parameter α, which allows to join MAXPROFIT ($\alpha = 1$) to MAXTOLL ($\alpha = 0$). We see in Plot (d) that setting a value of α around 0.75 allows one to find a solution with almost the same total profit as in MAXPROFIT, but with a much higher fraction coming from the toll, and hence a lower evasion rate.



(a) Stackelberg game profit vs. κ

(b) Percentage of the optimal profit

(c) Fraction of toll in profit

(d) Toll and penalties vs. α

Figure 2: Experimental Results for Berlin-Brandenburg (a)-(d).

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