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Simulation of the Transient Heating in an Unsymmetrical Coated Hot–Strip Sensor with a Self–Adaptive Finite Element Method

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Abstract

The transient heating in an unsymmetrical coated hot–strip sensor was simulated with a self–adaptive finite element method. The first tests of this model show that it can determine with a small error the thermal conductivity of liquids, from the transient temperature rise in the hot–strip, deposited in a substrate and coated by an alumina spray.

Nomenclature

ρ	material density
λ	material thermal conductivity
a	thickness of the strip
b	width of the strip
C_p	material isobaric heat capacity
l	longitudinal dimension of the strip
q	experimental power supplied to the strip
Q	heat generated per unit volume
Q_{strip}	heat generated in the strip per unit volume
T	temperature
t	time

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1 Introduction

It is now well known that the measurement of the thermal conductivity of molten materials is very difficult, mainly because the mathematical modelling of heat transfer processes at high temperatures, with several different media involved, is far from being solved. However, the scatter of the experimental data presented by different authors using several methods is so high that without serious approximations any scientific or technological application is strongly limited.

The development of a new instrument for the measurement of the thermal conductivity of molten salts, metals and semiconductors, apart from the necessary electronic equipment for the data acquisition and processing, furnaces and gas/vacuum manifolds, implies the design of a sensor for the measurement of temperature profiles in the melt.

This paper describes the development of a powerful algorithm, based on the self–adaptive finite elements method to process the temperature profile in a sensor previously constructed and characterised [1, 2].

2 Theory

A planar, electrically conducting (metallic) element is mounted within an insulating substrate material, which is surrounded by a material whose thermal properties have to be determined. From an initial state of equilibrium, ohmic dissipation within the metallic strip (q) results in a temperature rise on the strip and a conductive thermal wave spreads out from it through the substrate into the testing material. The temperature history of the metallic strip, as indicated by its change of electrical resistance, is both determined by its own thermal conductivity and thermal diffusivity as well as by the properties of the substrate and by the material thermal conductivity and thermal diffusivity. The working equation used in transient techniques to obtain the thermal conductivity value of a viscous, isotropic and incompressible fluid with temperature independent properties, is the energy conservation equation that can be transformed to

$$\rho C_p \frac{DT}{Dt} = \nabla \cdot (\lambda \nabla T) + Q \; . \tag{1}$$

The values of the material properties are piecewise constant. This equation has to be applied to three distinct regions: to the strip, to the substrate, and

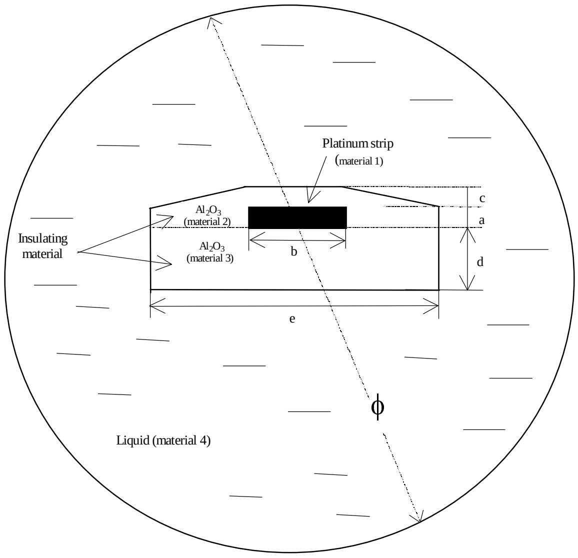

Figure 1: The system

to the material. Until now it has not been possible to solve this equation analytically. The complexity of the geometry of the sensor developed in Lisboa originates a 3–D problem. In a first attempt the end effects are neglected in the longitudinal dimension of the sensor. A sequence of linear heat transfer equations in 2–D is solved for the cross section of the metal strip to obtain the thermal conductivity parameter λ of the surrounding material. In equation (1) Q is zero for all materials except for the strip, where it is a non zero constant $Q_{\mathrm{strip}} = q/(abl)$. Because the experiment is done in short periods of time, we assume that no convection can occur.

Figure 1 shows a scheme of the metal strip (material 1), the alumina substrate (material 3), the alumina coating (material 2), and the molten material (material 4).

In the theoretical treatment examined so far it is assumed that the system is infinitely long in the z–direction, perpendicular to the page ($l_{\text{theoretical length}} \longrightarrow \infty$). Therefore, T depends only on x and y and $\partial T/\partial z = 0$.

This allows us to study only a two dimensional problem. All the other relevant geometrical data of the system are defined in figure 1.

It is assumed that, from an initial equilibrium state in which $\Delta T = 0$ or $T = T_0$ everywhere, heat is generated in the strip at a rate Q. We assume that $\Delta T = 0$ or $T = T_0$ on the boundary of the square domain. We place this artificial boundary far enough from the sensor that no significant part of the heat generated at the strip reaches the boundary during the time of the experiment. We use a variational formulation that fulfills the transmission conditions, i.e., we always have continuous solutions and fluxes at the material boundaries.

For the used geometry we can profit from the vertical symmetry axis on the middle of the recipient and simulate the evolution of temperature only in the right half of the domain. On the symmetry axis we impose a homogeneous Neumann boundary condition $\partial T/\partial n = 0$.

The heat equation can be solved analytically on simple geometries. However, when the model is more complex we only expect to obtain numerically approximate solutions. In our case the transient problem (1) is reduced to a sequence of elliptic problems. Each of these problems is solved using a finite element method. Finite element methods are based on an integral formulation of the equations and are very well suited to problems on complex geometries or when different materials occur.

3 Adaptive Numerical Algorithm

We are interested in solving the linear heat transfer equation (1) in two space dimensions, in the absence of the fluid movement:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (\lambda \nabla T) + Q_{\text{strip}} \tag{2}$$

$$\frac{\partial T}{\partial n}(0, y, t) = 0, \quad T = 0 \text{ elsewhere on the boundary} \tag{3}$$

$$T(x, y, 0) = 0 \tag{4}$$

for different thermal conductivities λ. Due to the strongly localised source Q and the different properties of the involved materials, we observe at the beginning steep gradients of the temperature profiles that decrease in time. To ensure a good resolution in space and an efficient time integration, in such a situation a method with automatic control of spatial and temporal discretization is recommended. We use the programming package KARDOS [3, 4, 5] providing these desired features. Starting with a guess, we improve the parameter iteratively until the computational result is a good approximation of the measured data. For a fixed value of λ the computation of

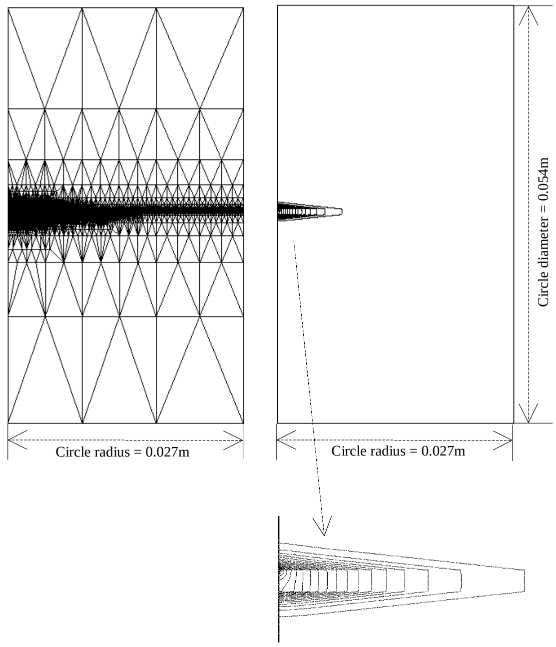

Figure 2: Adaptive grid and isotherms of the corresponding solution at time $t = 1.0$.

the heat transfer in the sensor and the surrounding liquid is based on the self–adaptive finite element code KARDOS developed at the Konrad–Zuse–Zentrum in Berlin. An implicit time integrator of Rosenbrock type is coupled with a multilevel approach in space. Local a posteriori estimates based on higher order solutions are computed to assess the spatial discretization. The arising elliptic equations are solved by an adaptive multilevel finite element method. Because of the very different sizes of the involved elements (sensor components, liquid) it is necessary to provide an appropriate initial grid in order to guarantee numerical stability during the adaptive mesh refinement. These estimations are used to decide where further refinement is necessary and where the mesh can be coarsened without loosing accuracy. Such adaptive strategies, including step size control in time, spatial refinement or coarsening combined with multilevel solvers, allow us to compute efficiently an accurate solution. In figure 2 we present a typical adaptive grid with the corresponding picture of the solution.

A special difficulty arises from the multiscaling character of the configuration: the tremendous differences in the sizes of the involved elements (sensor components, liquid). Taking a very coarse triangulation (see figure 3, left)

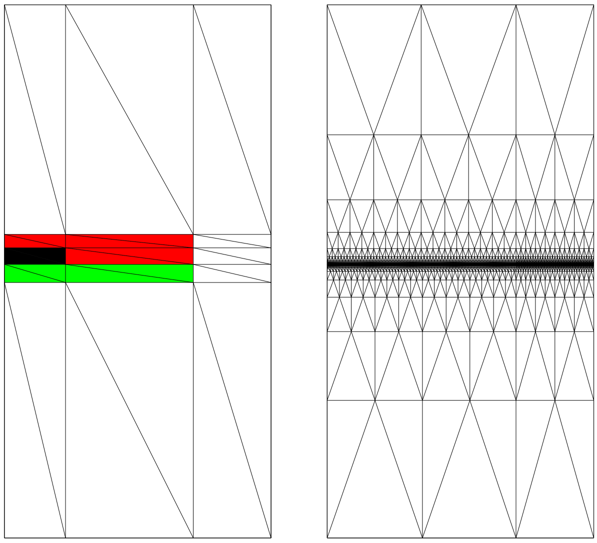

Figure 3: Left: Scheme of strip, isolating material, and liquid. Right: Initial grid.

of these elements as initial grid for our computations would result in large discretization errors and lead to instability during the time integration. No further refinement of such a bad grid (because of the obtuse angles) can remedy the instability. Consequently, we have to provide a reasonable initial grid (see figure 3, right) to start our adaptive method, i.e. we have to avoid obtuse angles and to generate a conform transient from fine to coarse mesh size [6].

4 Results

From measurement we know the average temperatures in the platinum strip over the time [7]. In our first numerical study we try to simulate the heat conduction. Starting with a guess for the parameter λ we compute the temperature up to time $t = 1.0$. If the error between measurement and computed data is too large, we choose a new value for λ and repeat the calculation. Here we present our results for the determination of the thermal conductivity of water (see figures 4 and 5).

Figure 4 shows the results obtained for the sensor described in references [1],[2] not considering the very thin titanium layer between the substrate and the platinum thin film. The agreement is quite satisfactory and it gener-

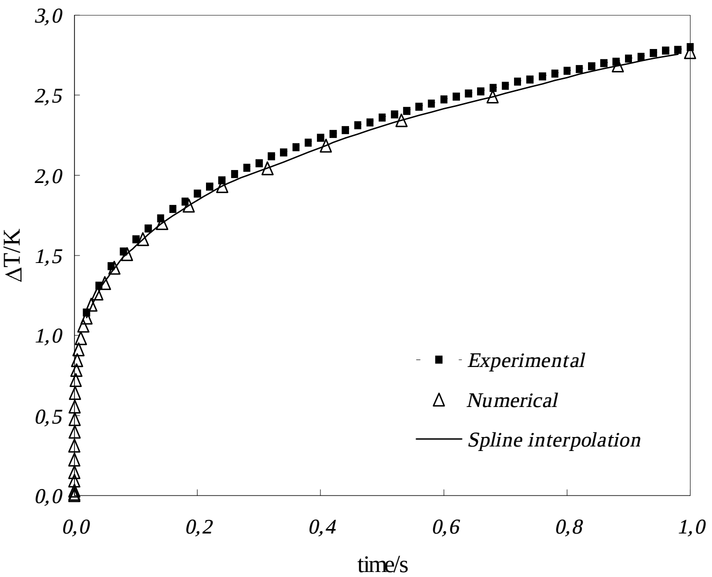

Figure 4: Simulation for water at 25° C; computation at 12V, 37.5 Wm^{-1}, λ_1=72.0, λ_2=25.5, λ_3=32.3, in $\mathrm{Wm}^{-1}\mathrm{K}^{-1}$ and c=4.7μm.

ates the water thermal conductivity to be 0.606 $\mathrm{Wm}^{-1}\mathrm{K}^{-1}$ at 25°C, a value within 0.1 % of the recommended thermal conductivity for this liquid at this temperature [2].

Figure 5 shows the deviation between the experimental points and a spline interpolation of the numerical points. The deviation is almost systematic, although with a slightly different slope. This might be caused by the fact that the numerical calculation did not consider the power necessary to heat the thin layer of titanium ($\approx$ 9,8 nm).

Preliminary numerical calculations with mercury, NaCl aqueous solutions, and toluene show the same trend.

5 Conclusions

A very powerful self–adaptive finite element method was applied to solve numerically the heat transfer in a resistive temperature sensor to be used in the measurement of the thermal conductivity of molten materials at high temperatures. The results so far obtained look very promising and can form the basis for the application of these metal thin film sensors at temperatures up to 1500°K.

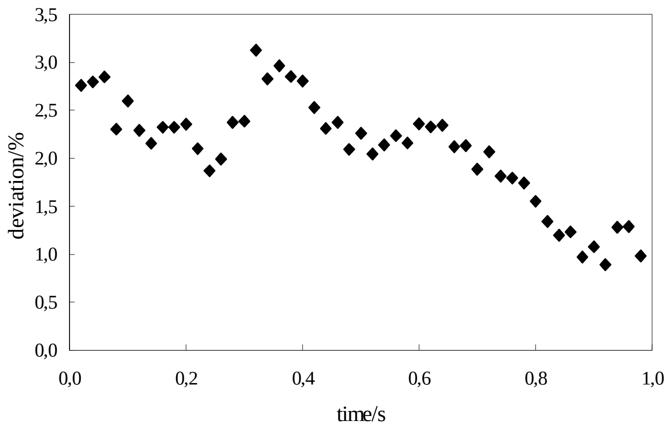

Figure 5: The deviation between the experimental points and a spline interpolation of the numerical points.

The program KARDOS can also be exploited for the simulation of more complicated models, e.g., using heat conduction in three space dimensions or non–linear dependencies. We hope to present the solution for the 3–D sensor in a near future.

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