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Optimization of energy supply systems by MILP branch and bound method in consideration of hierarchical relationship between design and operation

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Abstract

To attain the highest performance of energy supply systems, it is necessary to rationally determine types, capacities, and numbers of equipment in consideration of their operational strategies corresponding to seasonal and hourly variations in energy demands. In the combinatorial optimization method based on the mixed-integer linear programming (MILP), integer variables are used to express the selection, numbers, and on/off status of operation of equipment, and the number of these variables increases with those of equipment and periods for variations in energy demands, and affects the computation efficiency significantly. In this paper, a MILP method utilizing the hierarchical relationship between design and operation variables is proposed to solve the

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optimal design problem of energy supply systems efficiently: At the upper level, the optimal values of design variables are searched by the branch and bound method; At the lower level, the values of operation variables are optimized independently at each period by the branch and bound method under the values of design variables given tentatively during the search at the upper level; Lower bounds for the optimal value of the objective function are evaluated, and are utilized for the bounding operations at both the levels. This method is implemented into open and commercial MILP solvers. Illustrative and practical case studies on the optimal design of cogeneration systems are conducted, and the validity and effectiveness of the proposed method are clarified.

Keywords: Energy supply systems, Optimal design, Optimal operation, Mixed-integer linear programming, Branch and bound method, Hierarchical approach

1. Introduction

In designing energy supply systems, it is important to rationally determine their structures by selecting energy producing and conversion equipment from many alternatives so that they match energy demand requirements. It is also important to rationally determine capacities and numbers of selected equipment in consideration of their operational strategies such as on/off status of operation and load allocation corresponding to seasonal and hourly variations in energy demands.

In recent years, distributed energy supply systems have been widespread and diversified, and many types of equipment have been installed into them, which means that many alternatives for system design and operation have arisen. Thus, it has become more and more difficult for designers to design the systems properly in consideration of their operational strategies only with their experiences. In addition, not only reliability in energy supply but also economics, energy saving, and environmental impact have become more and more important criteria for system design and operation, with which designers have been burdened more heavily. For the purpose of assisting designers in system design and operation, therefore, it is necessary to develop a tool for providing rational design and operation solutions flexibly and automatically.

One of the ways to rationally determine the aforementioned design and operation items of energy supply systems is to use combinatorial optimization methods, which are based on the mathematical programming such as mixed-integer linear programming (MILP) [1–17] and mixed-integer nonlinear programming [18] as well as the meta heuristics such as simulated annealing [19] and genetic algorithm [20, 21]. Among these, the method based on the MILP has been proposed and utilized widely as one of the effective approaches. It leads to a natural expression of decision variables. For example, in this method, the selection, numbers, and on/off status of operation of equipment are expressed by integer variables, and the capacities and load allocation of equipment by continuous ones.

In earlier years, the conventional solution algorithm for the MILP which combines the branch and bound method with the simplex one has not been so efficient, and the optimal design problem has often been treated in consideration of single-period operation [1], or multi-period one for a small number of periods [2], to avoid excessive difficulty of the problem. This is because the number of integer variables increases with those of equipment and periods, and it becomes difficult to obtain the optimal solution in a practical computation time using the conventional solution algorithm. Afterwards, some efforts have been made to treat the optimal design problem in consideration of multi-period operation for a larger number of periods [3–5]. Nevertheless, equipment capacities have still been treated as continuous variables, and correspondingly performance characteristics and capital costs of equipment have been assumed to be continuous functions with respect to their capacities. This is because if equipment capacities are treated discretely, the number of integer variables increases drastically, and the problem becomes too difficult to solve. As a result, the treatment of equipment capacities as continuous variables causes discrepancies between existing and optimized values of capacities, and expresses the dependence of performance characteristics and capital costs on capacities with worse approximations.

In recent years, commercial MILP solvers have become more efficient, and many applications to the optimal design of distributed energy supply systems have been conducted in consideration of multi-period operation for a large number of periods. However, only the types of equipment with fixed capacities have been determined in [6, 7], the types and capacities of equipment have been determined, but the capacities have been treated as continuous variables in [8–10], and the types and numbers of equipment with fixed capacities have been determined in [11–13]. In addition, the dependence of performance characteristics of equipment on their capacities or part load levels has not been taken into account in [14, 15]. On the other hand, an optimal design method has been proposed in consideration of discreteness of equipment capacities to resolve the aforementioned insufficiency of equipment models [16]. In this method, a formulation for keeping the number of integer variables as small as possible has been presented to solve the optimal design problem efficiently. However, the aforementioned difficulty in the MILP method still exists essentially. Even commercial MILP solvers which are recently available may not derive the optimal solutions in practical computation times.

Recently, a MILP method utilizing the hierarchical relationship between design and operation variables has been proposed to solve the optimal design problem of energy supply systems efficiently [17]: At the upper level, the optimal values of design variables are searched by the branch and bound method; At the lower level, the values of operation variables are optimized independently at each period by the branch and bound method under the values of design variables given tentatively during the search at the upper level. This method has been implemented into an open MILP solver at the initial stage [22]. In addition, an illustrative case study on the optimal design of a gas engine cogeneration system has been conducted, and the validity and effectiveness of the proposed method has been clarified fundamentally. However, this solver allows only small scale problems with small numbers of variables. Therefore, a revision is necessary to implement the method into a commercial MILP solver and conduct practical case studies.

In this paper, the aforementioned MILP method utilizing the hierarchical relationship between design and operation variables is revised to conduct the optimization calculation more efficiently. First, lower bounds for the optimal value of the objective function are evaluated by solving critical design and operation problems defined using the hierarchical relationship. Then, bounding operations with these lower bounds are conducted in the branch and bound methods used at both the levels. This revised method is implemented into an open MILP solver, and is applied to an illustrative case study on the optimal design of a gas engine cogeneration system. In addition, the method is implemented into a commercial MILP solver, and is applied to a practical case study on the optimal design of a gas turbine cogeneration system. The validity and effectiveness of the method are investigated through these case studies.

2. Formulation of optimal design problem

2.1. Basic concept

To consider seasonal and hourly variations in energy demands, a typical year is divided into multiple periods, and energy demands are estimated at each period. As shown in Fig. 1, a super structure for an energy supply system is created to match energy demand requirements. The super structure is composed of all the units of equipment considered as candidates for selection, and a real structure is created by selecting some units of equipment from the candidates. Furthermore, some units of equipment are operated to satisfy energy demands at each period. The selection, capacities, and numbers of equipment are considered as design variables, and the on/off status of operation and load allocation of equipment as operation ones. The hierarchical relationship between the design and operation variables is shown in Fig. 2. In this paper, the selection and capacities are expressed by binary variables, the numbers and on/off status of operation by integer ones, and the load allocation by continuous ones.

As fundamental constraints, performance characteristics of equipment and energy balance relationships are considered. If necessary, other constraints such as relationships between maximum demands and consumptions of purchased energy, and operational restrictions are considered. As the objective function to be minimized, the annual total cost is adopted typically, and is evaluated as the sum of annual capital cost of equipment and annual operational cost of purchased energy. These constraints and objective function are expressed as functions with respect to the design and operation variables.

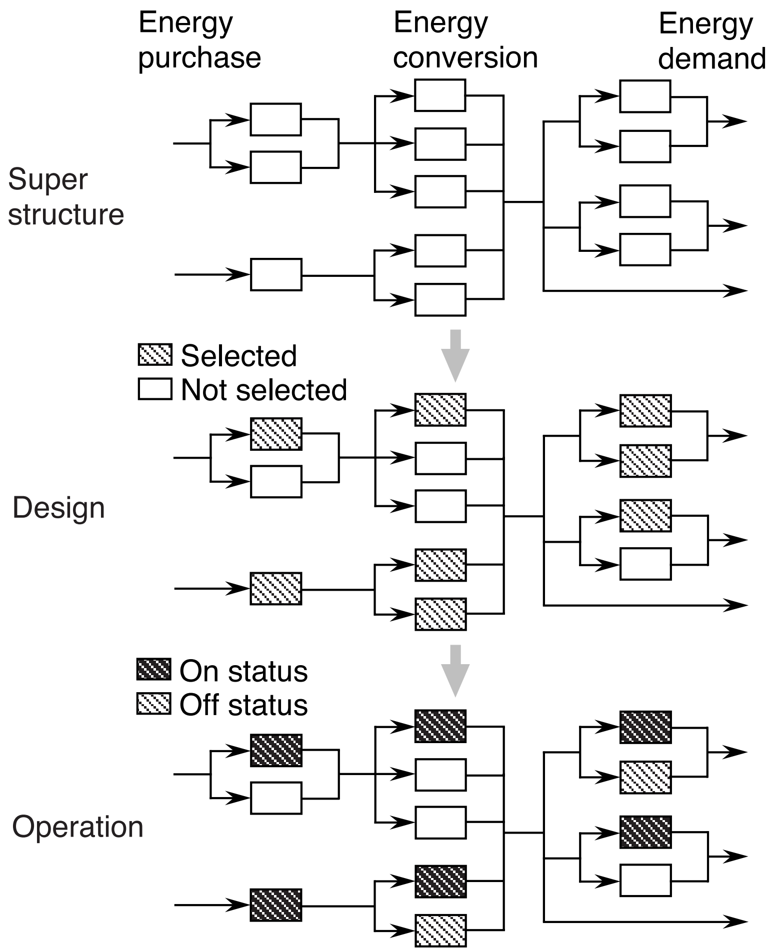


Fig. 1 Concept of super structure

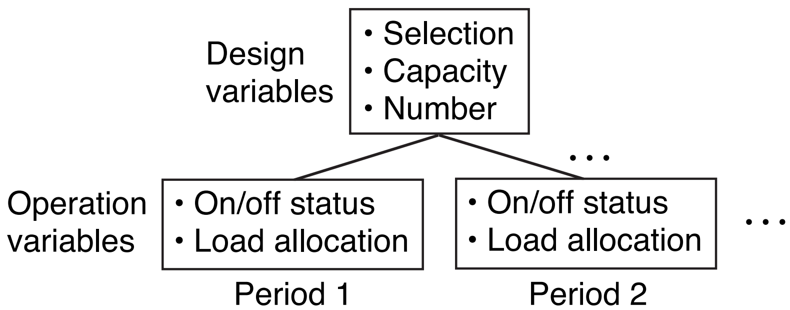


Fig. 2 Hierarchical relationship between design and operation variables

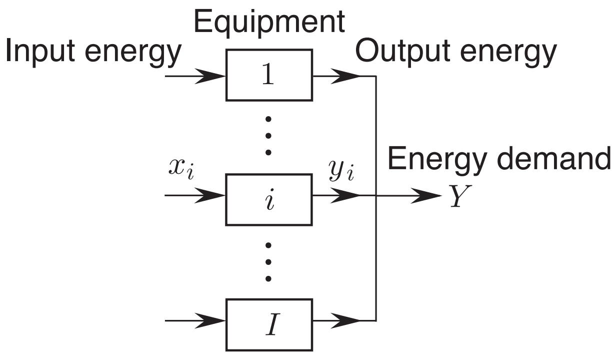


Fig. 3 Energy supply system with simple super structure

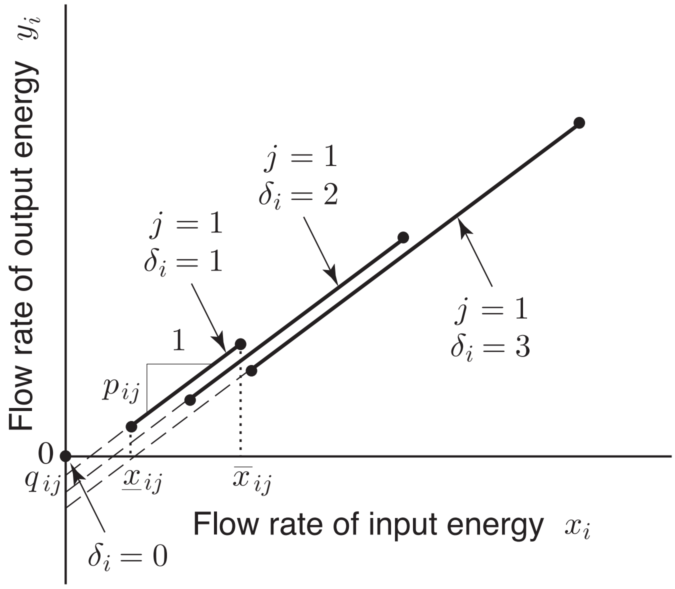
In the following, an optimal design problem is formulated for the energy supply system with a simple super structure shown in Fig. 3. The formulation can easily be extended to energy supply systems with complex super structures.

2.2. Selection, capacities, and numbers of equipment

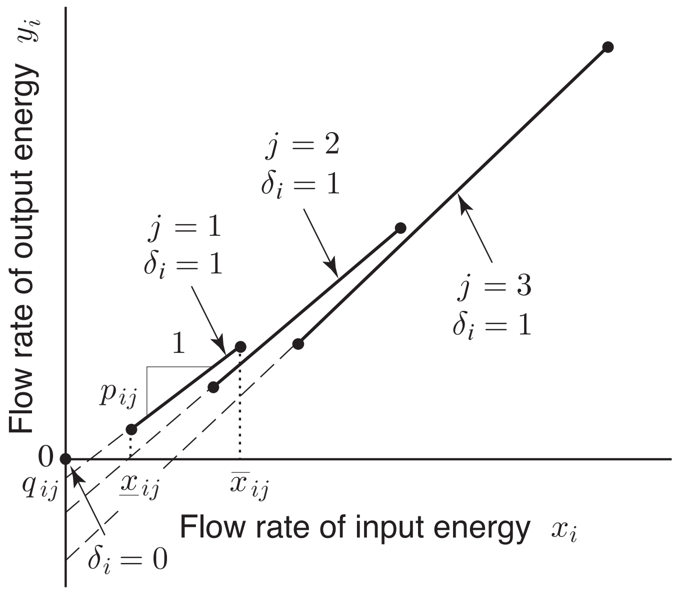
The energy supply system is composed of I blocks, each of which corresponds to a type of equipment. The capacity of the ith type of equipment is selected from its J_i candidates. In addition, the number of the ith type and the jth capacity of equipment is determined within its maximum N_{ij}. The selection and number of the ith type and the jth capacity of equipment are designated by the binary variable γ_{ij} and the integer variable η_{ij}, respectively. By these definitions, the following equations are obtained:

$$\left.\begin{array}{l} \eta_{ij}/N_{ij} \leq \gamma_{ij} \leq \eta_{ij} \quad (j = 1, 2, \cdots, J_i) \\ \sum\limits_{j=1}^{J_i} \gamma_{ij} \leq 1 \\ \gamma_{ij} \in \{0, 1\} \quad (j = 1, 2, \cdots, J_i) \\ \eta_{ij} \in \{0, 1, \cdots, N_{ij}\} \quad (j = 1, 2, \cdots, J_i) \end{array}\right\} (i = 1, 2, \cdots, I) \tag{1}$$

Here, it is assumed that multiple units with the same capacity can be selected for a type of equipment. To select multiple units with different capacities for a type of equipment, multiple blocks for the type of equipment should be included in the system.



(a) Dependence on number of equipment at on status of operation



(b) Dependence on capacity of equipment

Fig. 4 Modeling of performance characteristics of equipment

2.3. Performance characteristics of equipment

A relationship between the flow rates of input and output energy is shown in Fig. 4 as performance characteristics of the ith type and the jth capacity of equipment. Here, the discontinuity of the relationship due to the number of equipment at the on status of operation is expressed by an integer variable, and the relationship for multiple units of equipment is approximated by a linear equation as follows:

$$\left.\begin{array}{l} y_i(k) = \sum_{j=1}^{J_i} p_{ij}\gamma_{ij}x_i(k) + \sum_{j=1}^{J_i} q_{ij}\gamma_{ij}\delta_i(k) \\ \sum_{j=1}^{J_i} \underline{x}_{ij}\gamma_{ij}\delta_i(k) \leq x_i(k) \leq \sum_{j=1}^{J_i} \bar{x}_{ij}\gamma_{ij}\delta_i(k) \\ \delta_i(k) \leq \sum_{j=1}^{J_i} \eta_{ij} \\ \delta_i(k) \in \{0, 1, \cdots, \max_{1 \leq j \leq J_i} N_{ij}\} \end{array}\right\} (i = 1, 2, \cdots, I;\ k = 1, 2, \cdots, K) \tag{2}$$

where δ_i is the integer variable for the number of equipment at the on status of operation. Here, it is assumed that δ_i units of equipment are operated at the same load level, and the sums of the flow rates of input and output energy are expressed by the continuous variables x_i and y_i, respectively. This assumption is validated if the simple performance characteristics expressed by Eq. (2) are used. p_{ij}, q_{ij}, $\underline{x}_{ij}$, and $\bar{x}_{ij}$ are the performance characteristic values of the ith type and the jth capacity of equipment, i.e., p_{ij} and q_{ij} are the slope and intercept, respectively, of the linear relationship between the flow rates of input and output energy for a unit of equipment at the on status of operation, and $\underline{x}_{ij}$ and $\bar{x}_{ij}$ are the lower and upper limits, respectively, for the flow rate of input energy for a unit of equipment at the on status of operation. The argument k is the index for periods, and K is the number of periods. The first equation in Eq. (2) expresses the flow rate of output energy as a function with respect to that of input energy when δ_i units of equipment are at the on status of operation, and makes the flow rate of output energy zero when all the units of equipment are at the off status of operation. The second equation in Eq. (2) makes the flow rate of input energy within its lower and upper limits when δ_i units of equipment are at the on status of operation, and zero when all the units of equipment are at the off status of operation. The third equation in Eq. (2) means that the number of equipment at the on status of operation may not be larger than that selected. Since δ_i is common to all the capacities, $\max_{1 \leq j \leq J_i} N_{ij}$ is used as its maximum in the fourth equation in Eq. (2).

Since δ_i is common to all the capacities, this formulation keeps the number of integer variables as small as possible, which makes the computation time as short as possible.

2.4. Annual total cost and energy balance relationship

As mentioned previously, the annual total cost is adopted as the objective function z to be minimized, and is expressed by

$$z = \sum_{i=1}^{I} \left(R \sum_{j=1}^{J_i} c_{ij} \eta_{ij} + \varphi_i \sum_{k=1}^{K} T(k) x_i(k) \right) \tag{3}$$

where R is the capital recovery factor, c_{ij} is the capital cost of the ith type and the jth capacity of equipment, φ_i is the unit cost for energy charge of the input energy consumed by the ith type of equipment, and T is the duration per year of each period.

As the energy balance relationship, the following equation is considered:

$$\sum_{i=1}^{I} y_i(k) = Y(k) \quad (k = 1, 2, \cdots, K) \tag{4}$$

where Y is the energy demand at each period.

2.5. Linearization of nonlinear terms

To reformulate this optimal design problem as a MILP one, the nonlinear terms due to the products of the binary variable γ_{ij} and the continuous and integer variables x_i and δ_i in Eq. (2) are replaced with the continuous variables ξ_{ij} and ζ_{ij} as follows: $\xi_{ij}(k) = \gamma_{ij}x_i(k)$ and $\zeta_{ij}(k) = \gamma_{ij}\delta_i(k)$, respectively. As a result, Eq. (2) is reduced to

$$\left.\begin{array}{l} y_i(k) = \sum_{j=1}^{J_i} p_{ij}\xi_{ij}(k) + \sum_{j=1}^{J_i} q_{ij}\zeta_{ij}(k) \\ \sum_{j=1}^{J_i} \underline{x}_{ij}\zeta_{ij}(k) \le x_i(k) \le \sum_{j=1}^{J_i} \bar{x}_{ij}\zeta_{ij}(k) \\ \delta_i(k) \le \sum_{j=1}^{J_i} \eta_{ij} \\ \delta_i(k) \in \{0, 1, \cdots, \max\limits_{1 \le j \le J_i} N_{ij}\} \end{array}\right\} (i = 1, 2, \cdots, I;\ k = 1, 2, \cdots, K) \tag{5}$$

In addition, the following linear constraints are introduced to make the aforementioned replacement valid:

$$\left.\begin{array}{l} \underset{\sim}{x}_i(k)\gamma_{ij} \le \xi_{ij}(k) \le \tilde{x}_i(k)\gamma_{ij} \\ x_i(k) - \tilde{x}_i(k)(1 - \gamma_{ij}) \le \xi_{ij}(k) \le x_i(k) \end{array}\right\}$$
$$(i = 1, 2, \cdots, I;\ j = 1, 2, \cdots, J_i;\ k = 1, 2, \cdots, K) \tag{6}$$

$$\left.\begin{array}{l} \underset{\sim}{\delta}_i(k)\gamma_{ij} \le \zeta_{ij}(k) \le \tilde{\delta}_i(k)\gamma_{ij} \\ \delta_i(k) - \tilde{\delta}_i(k)(1 - \gamma_{ij}) \le \zeta_{ij}(k) \le \delta_i(k) \end{array}\right\}$$
$$(i = 1, 2, \cdots, I;\ j = 1, 2, \cdots, J_i;\ k = 1, 2, \cdots, K) \tag{7}$$

with lower and upper bounds for the continuous and integer variables x_i and δ_i as follows: $\underset{\sim}{x}_i(k) = 0$ and $\tilde{x}_i(k) = \max\limits_{1 \le j \le J_i} N_{ij}\bar{x}_{ij}$, and $\underset{\sim}{\delta}_i(k) = 0$ and $\tilde{\delta}_i(k) = \max\limits_{1 \le j \le J_i} N_{ij}$, respectively.

2.6. Vector representation of formulation

To show the solution process below, the reformulated MILP problem is expressed using vectors for variables and constraints. The binary and integer design variables γ_{ij} and η_{ij} are expressed as

$$\boldsymbol{\gamma} = (\gamma_{11}, \cdots, \gamma_{IJ_I}, \eta_{11}, \cdots, \eta_{IJ_I})^{\mathrm{T}} \tag{8}$$

The integer and continuous operation variables δ_i and x_i, y_i, ξ_{ij}, and ζ_{ij} are expressed as

$$\left.\begin{aligned} &\boldsymbol{\delta}(k) = (\delta_1(k), \cdots, \delta_I(k))^{\mathrm{T}} \\ &\boldsymbol{x}(k) = (x_1(k), \cdots, x_I(k), y_1(k), \cdots, y_I(k), \xi_{11}(k), \cdots, \xi_{IJ_I}(k), \zeta_{11}(k), \cdots, \zeta_{IJ_I}(k))^{\mathrm{T}} \end{aligned}\right\} \quad (k = 1, 2, \cdots, K) \tag{9}$$

respectively. Then, Eq. (1) and Eqs. (3) through (7) are reduced to

$$\left.\begin{aligned} \text{min.} \quad & z = f_0(\boldsymbol{\gamma}) + \sum_{k=1}^{K} f_k(\boldsymbol{\delta}(k), \boldsymbol{x}(k)) \\ \text{sub. to} \quad & \boldsymbol{g}_0(\boldsymbol{\gamma}) \leq \mathbf{0} \\ & \boldsymbol{g}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) \leq \mathbf{0} \ \ (k = 1, 2, \cdots, K) \\ & \boldsymbol{h}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) = \mathbf{0} \ \ (k = 1, 2, \cdots, K) \end{aligned}\right\} \tag{10}$$

where f_0 and f_k denote the terms composed of the design and operation variables, respectively, in the objective function of Eq. (3). $\boldsymbol{g}_0$ denotes the inequality constraints of Eq. (1) which relate design variables. $\boldsymbol{g}_k$ denotes the inequality constraints of the second and third equations in Eq. (5), and Eqs. (6) and (7) which relate design and operation variables. $\boldsymbol{h}_k$ denotes the equality constraints of Eq. (4) and the first equation in Eq. (5) which relate design and operation variables.

If the values of the design variables $\boldsymbol{\gamma}$ are assumed, the constraints $\boldsymbol{g}_k$ and $\boldsymbol{h}_k$ become independent at each period. Namely, $\boldsymbol{\gamma}$ acts as coupling constraints for all the operation variables.

3. Solution in consideration of hierarchical relationship

3.1. Basic concept

Some commercial MILP solvers which are recently available can solve large scale problems in practical computation times. However, the MILP problem under consideration has the feature that it becomes extremely large scale with increases in the numbers of types and capacities of equipment, and periods, I, J, and K, respectively. In such cases, even commercial MILP solvers may not derive the optimal solutions in practical computation times. In this paper, a special solution method is proposed in consideration of the hierarchical relationship between the design and operation variables. A flow chart for the solution method is shown in Fig. 5.

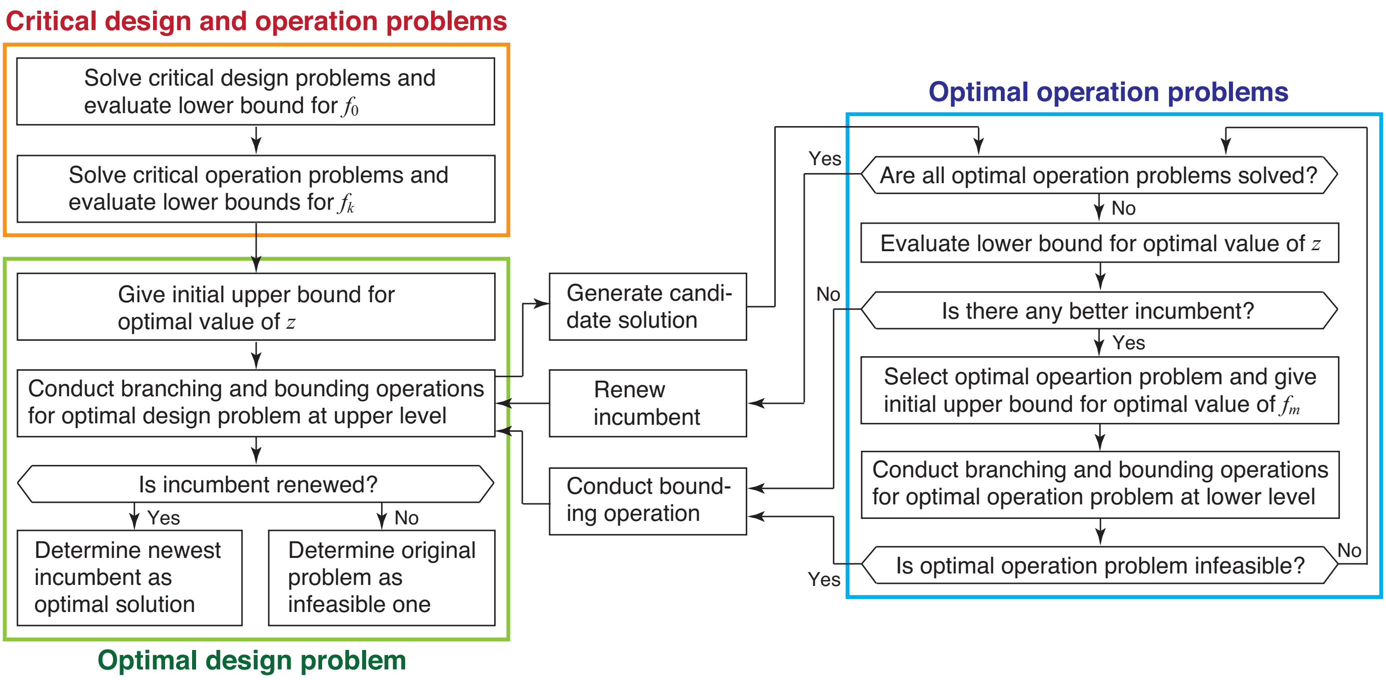


Fig. 5 Flow chart for solution method in consideration of hierarchical relationship between design and operation variables

The optimal design problem of Eqs. (1) through (4) has the hierarchical relationship between the design and operation variables. The reformulated MILP problem also has a similar relationship. The design variables at the upper level are the binary and integer variables $\boldsymbol{\gamma}$, while the operation variables at the lower level are the integer variables $\boldsymbol{\delta}(k)$ and the continuous variables $\boldsymbol{x}(k)$ at each period k. The values of these design and operation variables at all the periods should be optimized simultaneously. However, if the values of the design variables are given tentatively at the upper level, the values of the operation variables can be optimized independently at each period at the lower level. This feature leads to the following hierarchical solution process.

In place of the original problem of Eq. (10), the optimal design and operation problems at the upper and lower levels are defined as follows:

Optimal design problem at upper level

$$\left.\begin{array}{ll} \text{min.} & z = f_0(\boldsymbol{\gamma}) + \sum_{k=1}^{K} f_k(\boldsymbol{\delta}(k), \boldsymbol{x}(k)) \\ \text{sub. to} & \boldsymbol{g}_0(\boldsymbol{\gamma}) \leq \mathbf{0} \\ & \boldsymbol{g}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) \leq \mathbf{0} \quad (k = 1, 2, \cdots, K) \\ & \boldsymbol{h}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) = \mathbf{0} \quad (k = 1, 2, \cdots, K) \\ & \text{with } \boldsymbol{\delta}(k) \text{ relaxed to continuous variables} \end{array}\right\} \tag{11}$$

Optimal operation problems at lower level

$$\left.\begin{array}{ll} \text{min.} & f_k(\boldsymbol{\delta}(k), \boldsymbol{x}(k)) \\ \text{sub. to} & \boldsymbol{g}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) \leq \mathbf{0} \\ & \boldsymbol{h}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) = \mathbf{0} \\ & \text{with } \boldsymbol{\gamma} \text{ given tentatively at upper level} \end{array}\right\} (k = 1, 2, \cdots, K) \tag{12}$$

respectively. The optimal design problem at the upper level is defined by relaxing

$\boldsymbol{\delta}(k)$ to continuous variables in the original problem of Eq. (10), while the optimal operation problem at the lower level is defined at each period by adopting f_k as the objective function and giving the values of the design variables $\boldsymbol{\gamma}$.

The optimal values of $\boldsymbol{\gamma}$ are searched at the upper level through the branching and bounding operations used in the branch and bound method. During this search, when the values of the design variables $\boldsymbol{\gamma}$ are given tentatively, they are transferred to the optimal operation problems, and the optimal operation problem is solved independently at each period at the lower level by the branch and bound method under the values of $\boldsymbol{\gamma}$ given tentatively, and its result is returned to the optimal design problem. If an optimal operation problem at a period is infeasible, for example, because the deficit in energy supply arises, the tentative values of $\boldsymbol{\gamma}$ cannot become the optimal ones and are discarded, and thus the bounding operation is conducted between the two levels. Otherwise, i.e., if the optimal operation problems at all the periods are feasible, the optimal values of the operation variables $\boldsymbol{\delta}(k)$ and $\boldsymbol{x}(k)$ are determined, the part of the objective function f_k is evaluated correspondingly, and the value of the objective function z is evaluated by adding f_0 using the tentative values of $\boldsymbol{\gamma}$. If the value of z is larger than or equal to an upper bound for the optimal value of z, or the value of z for the incumbent obtained previously, the tentative values of $\boldsymbol{\gamma}$ cannot become the optimal ones and are discarded, and thus the bounding operation is conducted between the two levels. Otherwise, the solution corresponding to the tentative values of $\boldsymbol{\gamma}$ becomes a new incumbent, and the previous incumbent is replaced with this new one. When all the branches are searched in the optimal design problem, the incumbent results in the optimal solution of the original problem of Eq. (10).

The number of all the variables in the optimal design problem is the same as that in the original problem of Eq. (10). However, the number of the binary and integer variables in the optimal design problem is much smaller than that in the original problem. Therefore, the optimal design problem needs a smaller memory size as well as a shorter computation time to conduct the branching and bounding operations in the branch and bound method. In addition, the number of the variables in the optimal operation problem at each period is quite small, and the optimal operation problem can be solved easily. As a result, the proposed method has better features in memory size and computation time as compared with the direct solution of the original problem.

3.2. Evaluation of lower bounds

It is possible to conduct the optimization calculations for the optimal design problem at the upper level and the optimal operation problems at the lower level using the branch and bound method. However, it is suitable to introduce bounding operations to conduct the optimization calculations efficiently. Here, lower bounds for parts of the objective function are derived in consideration of the hierarchical relationship. In addition, bounding operations using the lower bounds are proposed in consideration of the hierarchical relationship.

Since the objective function is divided into the terms composed of the design and operation variables, lower bounds are also evaluated for the corresponding terms independently. First, the following critical design problem at each period is considered:

$$\left.\begin{array}{ll} \text{min.} & f_0(\boldsymbol{\gamma}) \\ \text{sub. to} & \boldsymbol{g}_0(\boldsymbol{\gamma}) \leq \mathbf{0} \\ & \boldsymbol{g}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) \leq \mathbf{0} \\ & \boldsymbol{h}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) = \mathbf{0} \end{array}\right\} (k = 1, 2, \cdots, K) \tag{13}$$

This problem is obtained by considering the design variables $\boldsymbol{\gamma}$ and only the operation variables at the kth period $\boldsymbol{\delta}(k)$ and $\boldsymbol{x}(k)$ as well as the term composed of only the design variables in the objective function f_0. This means that the coupling constraints by $\boldsymbol{\gamma}$ for the operation variables other than those at the kth period are removed, and that the objective function is adopted only for the design variables. The optimal solution of this problem gives the minimum of f_0 subject to the constraints for the operation variables at the kth period. Thus, the maximum among the minimums of f_0 for all the K critical design problems becomes a lower bound for f_0 in the original problem of Eq. (10). This lower bound is designated by $\underset{\sim}{f_0}^{\mathrm{D}}$ to show the process of conducting bounding operations below.

Second, the following critical operation problem at each period is considered:

$$\left.\begin{array}{ll} \text{min.} & f_k(\boldsymbol{\delta}(k), \boldsymbol{x}(k)) \\ \text{sub. to} & \boldsymbol{g}_0(\boldsymbol{\gamma}) \leq \mathbf{0} \\ & \boldsymbol{g}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) \leq \mathbf{0} \\ & \boldsymbol{h}_k(\boldsymbol{\gamma}, \boldsymbol{\delta}(k), \boldsymbol{x}(k)) = \mathbf{0} \end{array}\right\} (k = 1, 2, \cdots, K) \tag{14}$$

This problem is obtained by replacing f_0 with f_k in the aforementioned critical design problem. This means that the coupling constraints by $\boldsymbol{\gamma}$ for the operation variables other than those at the kth period are removed, and that the objective function is adopted only for the operation variables at the kth period. The optimal solution of this problem gives the minimum f_k subject to the constraints for the operation variables at the kth period. Thus, this minimum becomes a lower bound for f_k in the original problem of Eq. (10). This lower bound is designated by $\underset{\sim}{f_k}^{\mathrm{O}}$ to show the process of conducting bounding operations below.

The number of binary and integer variables in each of these critical design and operation problems is much smaller than that in the original problem of Eq. (10), and each problem can be solved easily. In addition, if the optimal values of the design variables $\boldsymbol{\gamma}$ for all the critical design and operation problems coincide with one another, the obtained solutions give the optimal solution of the original problem, although such a case rarely arises. This is because the coupling constraints by $\boldsymbol{\gamma}$ removed in the critical design and operation problems are satisfied, and the lower bounds $\underset{\sim}{f_0}^{\mathrm{D}}$ and $\underset{\sim}{f_k}^{\mathrm{O}}$ become the optimal values of f_0 and f_k, respectively.

3.3. Bounding operations

3.3.1 Optimal design problem

As aforementioned, the optimal design problem at the upper level of Eq. (11) is considered in place of the original problem of Eq. (10) by relaxing the binary operation variables $\boldsymbol{\delta}(k)$ to continuous ones. The optimal values of the design variables $\boldsymbol{\gamma}$ are searched by the branch and bound method. During this search, the bounding operation is conducted at each branching node at the upper level using a conventional method. The continuous relaxation problem corresponding to each branching node is usually solved, and the value of the objective function z for its optimal solution is used as a lower bound for the bounding operation. In the problem under consideration, this value can be divided into those for the design and operation variables. Thus, these values are also used as lower bounds for the optimal values of f_0 and f_k. These lower bounds are designated by $\underset{\sim}{f_0}^{\mathrm{C}}$ and $\underset{\sim}{f_k}^{\mathrm{C}}$. On the other hand, the aforementioned lower bounds $\underset{\sim}{f_0}^{\mathrm{D}}$ and $\underset{\sim}{f_k}^{\mathrm{O}}$ are used at all the branching nodes. Therefore, more effective lower bounds for the optimal values of f_0 and f_k can be selected from the two ones. As a result, a lower bound for the optimal value of the objective function z is evaluated as follows:

$$\underset{\sim}{z} = \max(\underset{\sim}{f_0}^{\mathrm{C}}, \underset{\sim}{f_0}^{\mathrm{D}}) + \sum_{k=1}^{K} \max(\underset{\sim}{f_k}^{\mathrm{C}}, \underset{\sim}{f_k}^{\mathrm{O}}) \tag{15}$$

If this value is larger than or equal to an upper bound for the optimal value of z, or the value of z for the incumbent $\tilde{z}$, the bounding operation is conducted at the corresponding branching node at the upper level. Otherwise, the branching operation is further continued.

3.3.2. Optimal operation problems

The optimal operation problem at each period at the lower level of Eq. (12) is considered by giving the values of the design variables $\boldsymbol{\gamma}$ tentatively. Each optimal operation problem can be solved independently from the other ones. This is because the values of the design variables $\boldsymbol{\gamma}$ are given tentatively, and the coupling constraints by $\boldsymbol{\gamma}$ are removed. Thus, the optimal operation problems are solved sequentially. Here, the following bounding operation between the two levels can be applied. Before solving each optimal operation problem in the specified order, a lower bound for the optimal value of the objective function z is evaluated as follows:

$$\underset{\sim}{z} = f_0 + \sum_{k \in \mathrm{S}} f_k + \sum_{k \in \mathrm{U}} \max(\underset{\sim}{f_k}^{\mathrm{C}}, \underset{\sim}{f_k}^{\mathrm{O}}) \tag{16}$$

where S and U are the sets for the solved and unsolved optimal operation problems, respectively. This equation means that the first term in the right hand side is calculated from the values of the design variables $\boldsymbol{\gamma}$, that the second term is calculated from the optimal values of the objective functions f_k for the solved optimal operation problems, and that the third term is calculated from the lower bounds for the unsolved optimal operation problems, where $\underset{\sim}{f_k}^{\mathrm{C}}$ is a lower bound for the optimal value of f_k evaluated by the continuous relaxation problem after the values of all the design variables are given tentatively. If this value is larger than or equal to an upper bound for the optimal value of z, or the value of z for the incumbent $\tilde{z}$, it is judged that the values of the design variables cannot give the optimal solution of the original problem. Thus, the bounding operation is conducted between the two levels without solving the unsolved optimal operation problems, and this information is transferred to the optimal design problem. Else if the optimal operation problem is infeasible, the bounding operation is conducted between the two levels without solving the unsolved optimal operation problems, and this information is also transferred to the optimal design problem. Otherwise, the next optimal operation problem is solved. This same procedure is repeated until the last optimal operation problem is solved. Since the value of $\max(\underset{\sim}{f}_k^{\mathrm{C}}, \underset{\sim}{f}_k^{\mathrm{O}})$ is replaced with the optimal value of f_k after the kth optimal operation problem is solved, the lower bound $\underset{\sim}{z}$ increases as the optimal operation problems are solved sequentially, which heightens the possibility of conducting the bounding operation before solving the next optimal operation problem.

When each optimal operation problem is solved by the branch and bound method, the bounding operation is conducted at each branching node at the lower level using the conventional method. The continuous relaxation problem corresponding to each branching node is usually solved as aforementioned. To conduct this bounding operation more efficiently during the solution of an optimal operation problem, say, the mth one, an initial upper bound for the optimal value of the objective function f_m is evaluated as follows:

$$\tilde{f}_m = \tilde{z} - \left(f_0 + \sum_{k \in \mathrm{S}} f_k + \sum_{k \in \mathrm{U} \setminus \{m\}} \max(\underset{\sim}{f}_k^{\mathrm{C}}, \underset{\sim}{f}_k^{\mathrm{O}}) \right) \tag{17}$$

If a lower bound for the optimal value of f_m is larger than this initial upper bound, the *m*th optimal operation problem is infeasible, and the bounding operation is conducted at the lower level. In addition, the bounding operation is conducted between the two levels without solving the unsolved optimal operation problems, and this information is transferred to the optimal design problem.

3.4. Implementation into solvers

3.4.1 Open solver

The proposed method is implemented into an open MILP solver published in [22]. This MILP solver is a simple one using the branch and bound method based on the depth first rule for selecting the branching node, and is applicable to small scale problems with small numbers of variables. Here, this MILP solver is used to solve all the optimization problems necessary for the proposed method, or not only the optimal design and operation problems at the upper and lower levels, respectively, but also the critical design and operation problems. Two sets of the solver are prepared to consider the optimal design and operation problems interactively. In addition, a program is prepared to control the flow of the optimization calculations for the optimal design and operation problems. It is called by the solver for the optimal design problem and exchanges information between the optimal design and operation problems.

The evaluation of the lower bound by Eq. (15) and the bounding operation based on it are incorporated into the solver for the optimal design problem. The evaluation of the lower bound by Eq. (16) and the bounding operation based on it are incorporated into the program for controlling the flow of all the optimization calculations. The evaluation of the upper bound by Eq. (17) is incorporated into the solver for the optimal operation problems.

3.4.2 Commercial solver

The proposed method is also implemented into a commercial MILP solver IBM ILOG CPLEX Optimization Studio V12.5.1 [23]. Since this MILP solver is one of the powerful commercial solvers using the linear programming based branch and cut method, and is applicable to large scale problems with large numbers of variables. Here, this MILP solver can be used to solve the optimal design problem at the upper level as well as the critical design and optimal operation problems. Since the optimal operation problems at the lower level are small scale, the aforementioned open MILP solver is used to solve them. In addition, some functions for CPLEX are utilized to control the flow of the optimization calculations for the optimal design and operation problems. They are called by the solver for the optimal design problem and exchanges information between the optimal design and operation problems.

The evaluation of the lower bound by Eq. (15) and the bounding operation based on it cannot be incorporated into the commercial solver for the optimal design problem. The evaluation of the lower bound by Eq. (16) and the bounding operation based on it are incorporated into the program for controlling the flow of all the optimization calculations. The evaluation of the lower bound by Eq. (16) and the bounding operation based on it with $U = \emptyset$, or without solving any optimal operation problems are used in place of the evaluation of the lower bound by Eq. (15) and the bounding operation based on it. The evaluation of the upper bound by Eq. (17) is incorporated into the open solver for the optimal operation problems.

In order to use CPELX as a solver for the optimal design problem at the upper level, the following three features have to be implemented.

1) Reject all the solutions found in CPLEX not to renew the incumbent at the upper

level, although the corresponding values of the design variables are used to solve the optimal operation problems at the lower level.

2) Set a cutup value to remove branching nodes with their lower bounds larger than the value of the objective function for a new incumbent, if it is found by solving the optimal operation problems at the lower level.
3) Prohibit generating the same solutions more than once to find a new incumbent efficiently.

Features 1) and 2) can be implemented using the incumbent callback of CPLEX. The incumbent callback is called when an integer solution is found but before this solution replaces the incumbent during the CPLEX solution procedure. It interrupts the solution procedure to reject the solution, invokes a solver for the optimal operation problems at the lower level, sets a cutup value if a new incumbent is found, and continues the solution procedure. Feature 3) can be implemented using the cut and lazy constraint callbacks of CPLEX. When a solution is found during the CPLEX solution procedure, the corresponding constraint which prohibits generating the same solutions is added for the continued solution procedure. This constraint is obtained by expressing all the integer variables with binary ones and using the values of the binary variables corresponding to the solution as shown in [24, 25]. However, it is not allowed to add this constraint in the incumbent callback because of the restriction of the library. Thus, the incumbent is stored in the incumbent callback, while the corresponding constraint is added in the cut or lazy constraint callback.

4. Case studies

4.1. Illustrative case study using open solver

4.1.1 Summary

As an illustrative case study, the proposed method is applied to the optimal design of a gas engine cogeneration system for electricity and hot water supply [17]. Since the super structure is very simple, and the number of periods is relatively small, only the open solver is used in this study. The optimal design problem is solved by not only the proposed method but also the conventional method, and the results obtained by these methods are compared with each other. The following three cases are investigated by the proposed method to show the effect of the bounding operations based on the lower bounds on the computation efficiency: In case I, Eqs. (15) through (17) are not considered; In case II, Eqs. (16) and (17) are considered; In case III, Eqs. (15) through (17) are considered. The optimal design problem has already been solved in case I and the case that the maximum demands of electricity and city gas are considered as continuous variables [17]. Both the design and operation variables are simultaneously optimized by the conventional method using the same open solver. All the optimization calculations are conducted on a MacBook Air with Mac OS X 10.6.7.

4.1.2. Conditions

Figure 6 shows the super structure for the cogeneration system, which has two gas engine cogeneration units with a same capacity and two gas-fired auxiliary boilers with a same capacity. Table 1 shows the capacities and performance characteristic values of candidates of equipment for selection. Although the performance characteristic values are shown only at the rated load level, their changes at part load levels are also taken into account. In addition to the equipment, the maximum demands of electricity and city gas purchased from outside utility companies are also determined. However, the proposed method allows only binary and integer variables in the optimal design problem at the upper level. Thus, the maximum demands of electricity and city gas are treated using integer variables, and are selected among discrete values by 10.0 kW and 1.0 Nm^3/h, respectively.

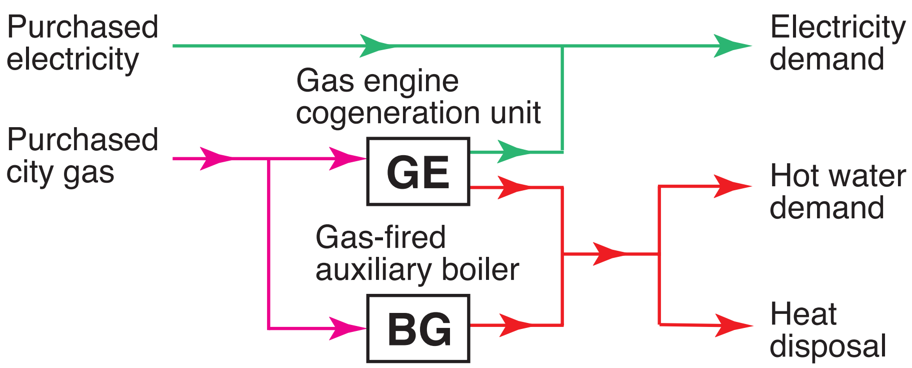


Fig. 6 Configuration of gas engine cogeneration system in illustrative case study

Table 1 Capacities and performance characteristic values of candidates of equipment for selection in illustrative case study

Equipment	Capacity/performance*	Candidate	
	#	1	2
Gas engine cogeneration unit	Max. power output kW	25.0	35.0
	Max. hot water output kW	38.4	52.7
	Power generating efficiency	0.335	0.340
	Heat recovery efficiency	0.515	0.511
Gas-fired auxiliary boiler	#	1	2
	Max. hot water output kW	99.0	198.0
	Thermal efficiency	0.886	0.900

*At rated load level

Table 2 Capital unit costs of equipment, and unit costs for demand and energy charges of utilities in illustrative case study

Equipment/utility		Unit cost
Gas engine cogeneration unit		225.0×10^3 yen/kW
Gas-fired auxiliary boiler		9.0×10^3 yen/kW
Electricity	Demand charge	1685 yen/(kW·month)
	Energy charge	12.08 yen/kWh
City gas	Demand charge	630 yen/(Nm^3/h·month)
	Energy charge	60.0 yen/Nm^3

The annual total cost is adopted as the objective function. Table 2 shows the capital unit costs of equipment as well as the unit costs for demand and energy charges of electricity and city gas. In evaluating the annual total cost, the capital recovery factor is set at 0.778 with the interest rate 0.02 and the life of equipment 15 y.

A hotel with the total floor area of 3 000 m^2 is selected as the building which is supplied with electricity and hot water by the cogeneration system. To take account of seasonal and hourly variations in energy demands, a typical year is divided into three representative days in summer, mid-season, and winter whose numbers of days per year are set at 122, 122, and 121 d/y, respectively, and each day is further divided into 3, 6, and 12 sampling time intervals of 8, 4, and 2 h, respectively. Thus, the year is divided into 9, 18, and 36 periods correspondingly.

4.1.3. Results and discussion

Table 3 shows the optimal values of the design variables obtained by the proposed method. In the case with the number of periods 9, two units of gas engine cogeneration unit #1 are installed. In the cases with the number of periods 18 and 36, a unit of gas engine cogeneration unit #1 and a unit of gas-fired auxiliary boiler #1 are installed. These are because in the former case the electricity and hot water demands are averaged and resultantly balanced, which is advantageous to cogeneration, and in the latter cases the electricity and hot water demands are not balanced in some periods.

Table 3 Optimal values of capacities and numbers of equipment, and maximum demands of utilities in illustrative case study

Number of periods	Equipment/utility	Candidate	Number	Capacity
9 (3×3)	Gas engine cogeneration unit	#1	2	50.0 kW
	Gas-fired auxiliary boiler	–	–	–
	Electricity maximum demand	–	–	50.0 kW
	City gas maximum demand	–	–	13.0 Nm^3/h
18 (3×6)	Gas engine cogeneration unit	#1	1	25.0 kW
	Gas-fired auxiliary boiler	#1	1	99.0 kW
	Electricity maximum demand	–	–	80.0 kW
	City gas maximum demand	–	–	10.0 Nm^3/h
36 (3×12)	Gas engine cogeneration unit	#1	1	25.0 kW
	Gas-fired auxiliary boiler	#1	1	99.0 kW
	Electricity maximum demand	–	–	80.0 kW
	City gas maximum demand	–	–	13.0 Nm^3/h

Table 4 Comparison of conventional and proposed methods in terms of solution and computation time in illustrative case study

Number of periods	Conventional method		Proposed method		
	Solution and objective $\times 10^6$ yen/y	Compu-tation time s	Solution and objective $\times 10^6$ yen/y	Case	Compu-tation time s
9 (3×3)	Optimal 10.45	3072.4	Optimal 10.45	I	1.9
				II	1.3
				III	0.7
18 (3×6)	Feasible 10.54	3600.0*	Optimal 10.44	I	5.3
				II	4.5
				III	2.0
36 (3×12)	Feasible 10.90	3600.0*	Optimal 10.49	I	9.2
				II	7.4
				III	4.6

*Attains limit for computation time

Table 5 Numbers of candidate solutions and optimal operation problems in illustrative case study

Number of periods	Number of candidate solutions				Number of optimal operation problems	
	All	Removed at upper level	Removed at lower level	Renewed incumbents	All	Solved
9 (3×3)	246	171	73	2	2214	347
18 (3×6)	203	136	61	6	4428	663
36 (3×12)	190	133	54	3	8856	1052

Table 4 shows the results obtained by both the methods to compare with each other in terms of solutions and computation times. In the case with the number of periods 9, both the methods derive the optimal solution. However, the conventional method needs a much longer computation time. In the cases with the number of periods 18 and 36, the proposed method derives the optimal solutions in practical computation times. However, the conventional method does not derive the optimal solutions but only feasible ones within the limit for the computation time. Although the computation times in cases I by the proposed method are short, the addition of the bounding operations by the lower bounds shortens the computation times in cases II and III.

Table 5 shows the numbers of all the candidate solutions which can be generated at the upper level, the candidate solutions removed by the bounding operations at the upper and lower levels, the renewed incumbents, and all the optimal operation problems and the solved ones. The first and fifth numbers are obtained in case I. The second through fourth and sixth numbers are obtained in case III. This result shows that the bounding operations at the upper and lower levels remove 67 to 70 and 28 to 30 %, respectively, of the candidate solutions. As a result, the number of the renewed incumbents is only less than 3 % of that of the candidate solutions. In addition, the number of the solved optimal operation problems is only 11 to 16 % of that of all the optimal operation problems.

4.2. Practical case study using commercial solver

4.2.1 Summary

As a practical case study, the proposed method is applied to the optimal design of a gas turbine cogeneration system for electricity, cold water, and steam supply [16]. Since the super structure is relatively complex, and the number of periods is relatively large, both the commercial and open solvers are used in this study. The optimal design problem is solved by not only the proposed method but also the conventional method, and the results obtained by these methods are compared with each other. Here, only cases I and II are investigated by the proposed method to show the effect of the bounding operation based on the lower bound on the computation efficiency, because Eq. (15) cannot be incorporated into the commercial solver as aforementioned. Both the design and operation variables are simultaneously optimized by the conventional method using the same commercial solver. The optimal design problem has already been solved by the conventional method in the case that the number of periods is only 20, and the capacity of the receiving device for purchasing electricity as well as the maximum demands of electricity and city gas are considered as continuous variables [16]. All the optimization calculations are conducted on a MacBook Air with Mac OS X 10.6.7.

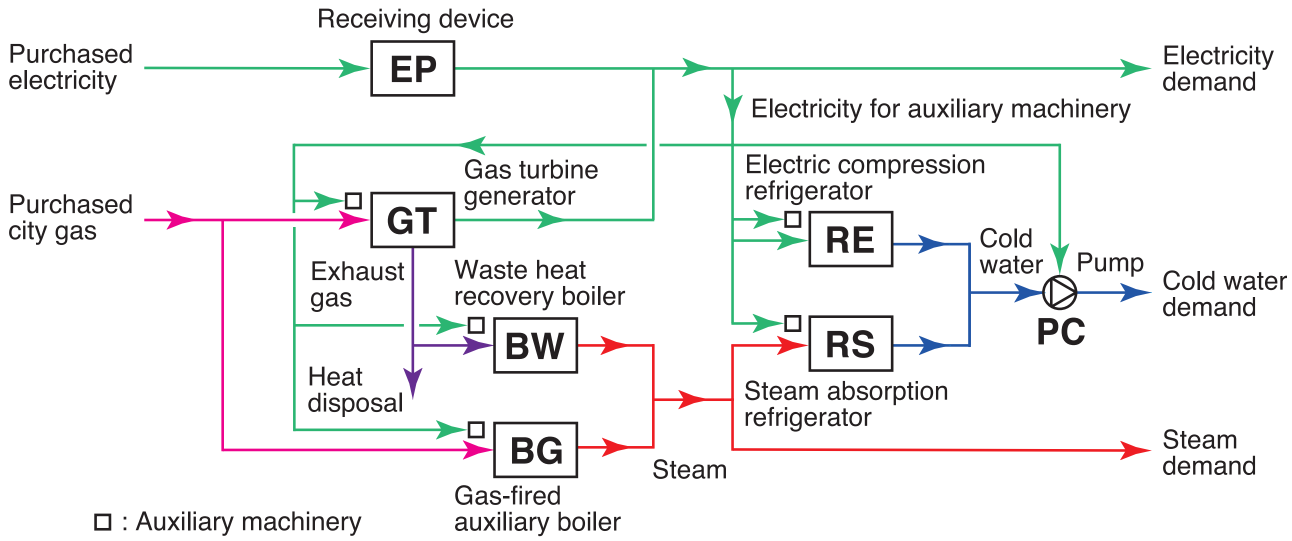


Fig. 7 Configuration of gas turbine cogeneration system in practical case study

4.2.2. Conditions

Figure 7 shows the super structure for the cogeneration system, which has four gas turbine generators with a same capacity, four waste heat recovery boilers with a same capacity, four gas-fired auxiliary boilers with a same capacity, four electric compression refrigerators with a same capacity, four steam absorption refrigerators with a same capacity, a receiving device for purchasing electricity, and pumps for supplying cold water. It is assumed that the gas turbine generators and waste heat recovery boilers are selected together as cogeneration units. Table 6 shows the capacities and performance characteristic values of candidates of equipment for selection. Although the performance characteristic values are shown only at the rated load level, their changes at part load levels are also taken into account. In addition to the equipment, the maximum demands of electricity and city gas purchased from outside utility companies are also determined, and are selected among discrete values by 1.0 MW and 0.5×10^3

Nm^3/h, respectively. The capacity of the receiving device for purchasing electricity is also selected among discrete values by 1.0 MW correspondingly. The pumps are common to all the possible structures, and only their power consumption is considered.

Table 6 Capacities and performance characteristic values of candidates of equipment for selection in practical case study

Equipment	Capacity/performance*	Candidate			
Gas turbine cogeneration unit	#	1	2	3	4
	Max. power output MW	1.29	1.60	2.00	2.40
	Max. steam output MW	5.69	3.34	4.10	4.57
	Power generating efficiency	0.140	0.173	0.169	0.179
	Heat recovery efficiency	0.617	0.362	0.347	0.341
	#	5	6	7	8
	Max. power output MW	2.93	3.50	3.54	4.36
	Max. steam output MW	6.44	6.97	6.89	8.92
	Power generating efficiency	0.256	0.271	0.273	0.273
	Heat recovery efficiency	0.563	0.540	0.531	0.559
	#	9	10		
	Max. power output MW	5.23	5.32		
	Max. steam output MW	8.91	9.05		
	Power generating efficiency	0.301	0.306		
	Heat recovery efficiency	0.513	0.521		
Gas-fired auxiliary boiler	#	1	2	3	4
	Max. steam output MW	5.24	6.55	7.86	9.82
	Thermal efficiency	0.92	0.92	0.92	0.92
Electric compression refrigerator	#	1	2	3	4
	Max. cooling output MW	2.82	3.52	4.22	5.28
	Coefficient of performance	4.57	4.73	4.76	5.04
Steam absorption refrigerator	#	1	2	3	4
	Max. cooling output MW	3.46	5.18	6.91	8.64
	Coefficient of performance	1.20	1.20	1.20	1.20

*At rated load level

Table 7 Capital unit costs of equipment, and unit costs for demand and energy charges of utilities in practical case study

Equipment/utility		Unit cost
Gas turbine generator		230.0×10^3 yen/kW
Waste heat recovery boiler		9.6×10^3 yen/kW
Gas-fired auxiliary boiler		6.6×10^3 yen/kW
Electric compression refrigerator		34.4×10^3 yen/kW
Steam absorption refrigerator		30.1×10^3 yen/kW
Receiving device		56.3×10^3 yen/kW
Electricity	Demand charge	1740 yen/(kW·month)
	Energy charge	10.77 yen/kWh (Summer) 9.79 yen/kWh (Others)
City gas	Demand charge	2033 yen/(Nm^3/h·month)
	Energy charge	30.88 yen/Nm^3

The annual total cost is adopted as the objective function. Table 7 shows the capital unit costs of equipment as well as the unit costs for demand and energy charges of electricity and city gas. In evaluating the annual total cost, the capital recovery factor is set at 0.964 with the interest rate 0.05 and the life of equipment 15 y.

Two hotels and four office buildings with the total floor area of 383.7×10^3 m^2 are selected as the buildings which are supplied with electricity, cold water, and steam by the cogeneration system. To take account of seasonal and hourly variations in energy demands, a typical year is divided into three representative days in summer, mid-season, and winter whose numbers of days per year are set at 122, 122, and 121 d/y, respectively, and each day is further divided into 24 sampling time intervals of 1 h, respectively. Thus, the year is divided into 72 periods correspondingly.

4.2.3. Results and discussion

Table 8 shows the optimal values of the design variables obtained by the proposed method. Three units of gas turbine cogeneration unit #8 are selected. This is because the power generating and waste heat recovery efficiencies at the rated load level are the second and first highest among the ten candidates, and additionally these efficiencies at part load levels are relatively high among the candidates. A unit of electric compression refrigerator #4 is selected. This is because the coefficient of performance at the rated load level is highest among the four candidates. A unit of gas-fired auxiliary boiler #3 and four units of steam absorption refrigerator #2 are selected to supplement steam and cold water supply, respectively.

Table 8 Optimal values of capacities and numbers of equipment, and maximum demands of utilities in practical case study

Equipment/utility	Candidate	Number	Capacity
Gas turbine cogeneration unit	#8	3	13.08 MW
Gas-fired auxiliary boiler	#3	1	7.86 MW
Electric compression refrigerator	#4	1	5.28 MW
Steam absorption refrigerator	#2	4	20.72 MW
Receiving device	–	–	4.00 MW
Electricity maximum demand	–	–	4.00 MW
City gas maximum demand	–	–	4.50×10^3 Nm^3/h

Table 9 Comparison of conventional and proposed methods in terms of solution and computation time in practical case study

Conventional method		Proposed method		
Solution and objective $\times 10^9$ yen/y	Computation time s	Solution and objective $\times 10^9$ yen/y	Case	Computation time s
Feasible 1.451	10599.5*	Optimal 1.451	I	945.1
			II	529.4

*Attains limit for computation memory

Table 9 shows the results obtained by both the methods to compare with each other in terms of solution and computation time. The proposed method derives the optimal solution in a practical computation time even in case I. The addition of the bounding operation by the lower bound shortens the computation time in cases II. On the other hand, the conventional method also derives the optimal solution, but it cannot be ascertained whether the derived solution is optimal or not because the limit for computation memory is attained. In addition, it takes an extremely long computation time.

Table 10 Numbers of candidate solutions and optimal operation problems in practical case study

Number of candidate solutions				Number of optimal operation problems	
All	Removed before lower level	Removed at lower level	Renewed incumbents	All	Solved
636	171	463	14	45792	15708

Table 10 shows the numbers of all the candidate solutions which can be generated at the upper level, the candidate solutions removed by the bounding operations before and while solving the optimal operation problems at the lower level, the renewed incumbents, and all the optimal operation problems and the solved ones. The first and fifth numbers are obtained in case I. The second through fourth and sixth numbers are obtained in case II. This result shows that the bounding operations before and while solving the optimal operation problems at the lower level remove more than 26 and 72 %, respectively, of the candidate solutions. As a result, the number of the renewed incumbents is only less than 3 % of that of the candidate solutions. In addition, the number of the solved optimal operation problems is only less than 35 % of that of all the optimal operation problems.

5. Conclusions

A MILP method utilizing the hierarchical relationship between design and operation variables has been proposed to solve the optimal design problem of energy supply systems efficiently. At the upper level, the optimal values of design variables have been searched by the branch and bound method. At the lower level, the values of operation variables have been optimized independently at each period by the branch and bound method under the values of design variables given tentatively during the search at the upper level. Especially, lower bounds for the optimal value of the objective function have been evaluated by solving critical design and operation problems defined using the hierarchical relationship. Then, bounding operations with these lower bounds have been conducted in the branch and bound methods used at both the levels. This method has been implemented into open and commercial MILP solvers, and has been applied to illustrative and practical case studies on the optimal design of cogeneration systems. The validity and effectiveness of the method have been investigated through these case studies. The following main results have been obtained:

1) Through the illustrative case study using the open solver, it has turned out that the proposed method can derive the optimal solutions in much shorter computation times as compared with the conventional method.
2) Through the practical case study using the commercial solver, it has turned out that the proposed method can derive the optimal solutions in practical computation

times, although the conventional method may not do it because of the limit for computation memory.

3) Through the illustrative and practical case studies, it has turned out that the bounding operations based on the lower bounds are very effective to reduce the computation times in the proposed method.

These results will enable one to use the proposed method as an effective tool for providing rational design and operation solutions of energy supply systems. In addition, although the proposed method has been intended for the optimal design of energy supply systems, it will be applicable to that of other systems with the hierarchical relationship between design and operation variables.

Acknowledgements

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Nomenclature

c : capital cost of equipment

f : part of objective function

$\boldsymbol{g}$: vector for inequality constraints

$\boldsymbol{h}$: vector for equality constraints

I : number of types of equipment

J : number of capacities of equipment

K : number of periods

k : index for periods

N : maximum number of selected equipment

p : slope of linear relationship between flow rates of input and output energy of equipment

q : intercept of linear relationship between flow rates of input and output energy of equipment

R : capital recovery factor

S : set of indices for solved optimal operation problems

T : duration per year of period

U : set of indices for unsolved optimal operation problems

x : flow rate of input energy of equipment

$\boldsymbol{x}$: vector for continuous operation variables

$\underline{x}$: lower limit for flow rate of input energy of equipment

$\bar{x}$: upper limit for flow rate of input energy of equipment

Y : energy demand

y : flow rate of output energy of equipment

z : annual total cost (objective function)

γ : binary variable for selection of equipment

$\boldsymbol{\gamma}$: vector for binary and integer design variables

δ : number of equipment at on status of operation

$\boldsymbol{\delta}$: vector for integer operation variables

ζ : product of γ and δ

η : number of selected equipment

ξ : product of γ and x

φ : unit cost for energy charge of input energy

$\underset{\sim}{(\ )}$: lower bound

$\tilde{(\ )}$: upper bound

Subscripts

i : index for types of equipment

j : index for capacities of equipment

k, m : part of objective function composed of operation variables for kth or mth optimal operation problem

0 : part of objective function composed of design variables

Superscripts

C : continuous relaxation problem

D : critical design problem

O : critical operation problem

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