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Integration of Duty Scheduling and Rostering to Increase Driver Satisfaction

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Keywords Benders decomposition · driver satisfaction · duty scheduling · duty templates · rostering

Abstract Integrated treatment of hitherto individual steps in the planning process of public transit companies discloses opportunities to reduce costs and to improve the quality of service. The arising integrated planning problems are complex and their solution requires the development of novel mathematical methods. This article proposes a mathematical optimization approach to integrate duty scheduling and rostering in public transit, which allows to significantly increase driver satisfaction at almost zero cost. This is important in order to to increase the attractiveness of the driver profession. The integration is based on coupling the subproblems by duty templates, which, compared to a coupling by duties, drastically reduces the problem complexity.

Mathematics Subject Classification (2000) 90B06 · 90B20 · 90C06 · 90C59

1 Introduction

The *planning process in public transit* is by default organized in a sequence of individual steps [Fengler and Kolonko(1997)]. At first *strategic planning*, including network, timetable, connection and price planning, has to be done, followed by *operational planning*, which consists of vehicle routing, duty scheduling, rostering, and personnel deployment planning. At the end of the planning process there is operations control consisting of vehicle and personnel disposition. [Borndörfer et al(2008)Borndörfer, Grötschel, and Jaeger]. Duty scheduling and rostering are the planning steps which assign the workload to the per-

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sonnel. The *duty scheduling* problem consists of finding duties for the drivers in order to cover a set of tasks (for a given day). These tasks are computed in the vehicle scheduling step. The *rostering* problem consists of assigning duties of a specific planning horizon to individual drivers. Such an assignment is called a *roster*. Duties and rosters have to fulfill many rules stemming from laws, regulations, and agreements with trade unions.

Due to the high complexity of duty scheduling and rostering these problems are traditionally solved sequentially. However, the so-computed duties are in general not an optimal starting point for rostering. In other words, the solutions of the sequential approach are in general not optimal for the integrated duty scheduling and rostering problem. Here is a huge potential for savings, because the average public transport company in Germany spends about 50% of its operational costs for personnel [Leuthardt(1998),Leuthardt(2000)]. Another important aspect of personnel planning is *driver satisfaction*, because this is of utmost concern for trade unions and workers councils, which in Germany are involved in the planning process. Cost minimization is usually the main point of duty scheduling, while rostering deals primarily with driver satisfaction. In our days, many public transport companies have problems to recruit new drivers. E.g., the average age of a bus driver working at Berlin's public transport company BVG is almost 50 years [Kurpjuweit(2011)]. Because of this problem it has become increasingly important to offer good working conditions for drivers. As the potential for salary increases is limited, driver satisfaction is becoming a main goal. Ideally one would like to create rosters which are maximizing driver satisfaction without generating relevant additional costs. To reach this goal, solving the duty scheduling and rostering problem in an integrated way seems to be very promising.

2 Duty scheduling & rostering in public transit

Personnel deployment in public transit consists of scheduling duties and assigning them to drivers. A *duty* is sequence of tasks and breaks, which can be done by a single driver on a working day. A duty has to fulfill legal requirements, in Germany the most important are determined in the Arbeitszeitgesetz (working time act) and the Lenkzeitverordnung (driving time regulation). There are also many rules and requirements which stem from individual negotations between trade unions and public transit companies.

The *tasks* of a duty are either continuous parts of timetabled or empty trips of a rotation or so called supplementary tasks, that is tasks, which have to be performed if a driver changes a vehicle or begins or ends a duty or a rotation. Typical supplementary tasks are checking the vehicle or preparing the cash register.

An *operating day* is a collection of days with the same tasks and rules. There are typically different operating days for working days, weekends and holidays, because the timetable differs significantly on these days.

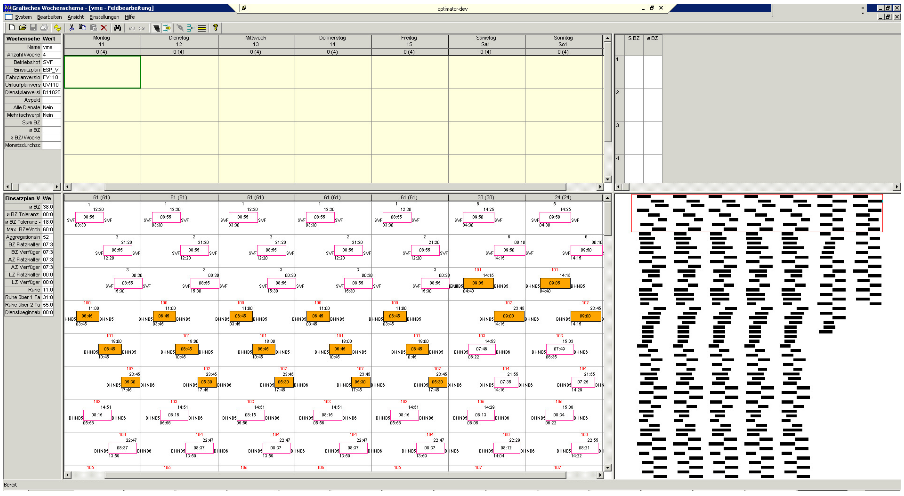

Fig. 1 Roster in tabular form.

Before assigning duties to specific drivers, many public transit companies generate an anonymous *roster* over a specific planning horizon (e.g. 6 or 8 weeks), in which duties are assigned to *rows*. Every row is constructed in such a way, that it could be legally assigned to a driver. A roster is typically displayed as a Gantt-Chart. Picture 1 shows a view of a roster in the software system `IVU.plan`. In the lower part, it displays the rows of a roster over the considered planning horizon. The lower right window shows a complete overview, while the left one shows a detailed zoom in. The upper part of this picture is used to edit unassigned duties and presents a statistic. The number of rows in a roster is equal to the number of drivers needed to perform all duties. The rosters here are anonymous, which means driver specific requirements such as working time accounts, qualification, and preplanned absences are not considered. The anonymous rosters have to be assigned to specific drivers in the next planning step; however, this planning step is not part of this article.

The rostering has to take *regulations* for daily and weekly resting periods into account. These resting periods are regulated by EU regulation 561/2006 [Europäisches Parlament und Rat(2006)] and the “Deutsche Fahrpersonalverordnung” (German driving personnel regulation) [Bundesministerium der Justiz(2005)]. Most important in these regulations is the daily resting time: between every two duties there has to be a resting period of at least 11 hours, and the weekly resting time: every week has to have at least one continuous resting period of 48 hours. This period can be shorten to 24 hours, but this has to be compensated by a longer weekly resting time within a specific time frame.

One of the most important objectives of rostering is to minimize the deviation of the *average working time* per time interval from a target value. This prevents (expensive) overtime or paid working time, without work to do. Most

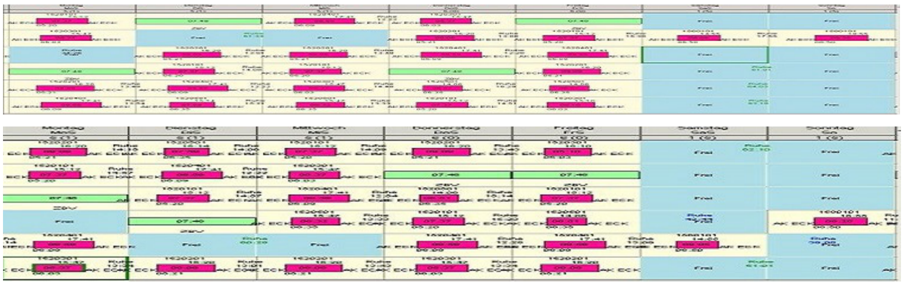

Fig. 2 Not connected weekend duties versus connected weekend duties.

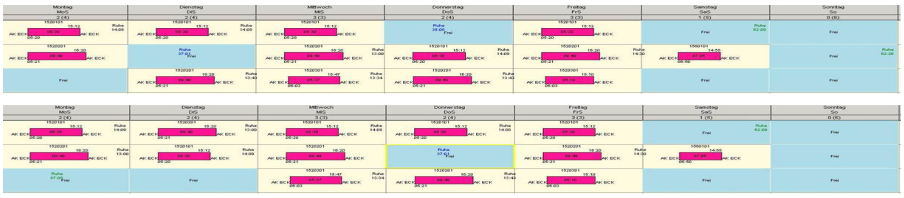

Fig. 3 Isolated duty versus connected duties.

of the other objectives can be subsumed as maximizing driver satisfaction. This includes arranging *free weekends* favorably (at best on Saturdays and Sundays), avoiding erratic starting times of duties, and avoiding too many stressful duties per given time interval, e.g., per week.

Picture 2 shows examplarily the Gantt-Chart of a roster over 7 days (one column for every day of the week). The upper window shows a solution with two continuous weekend duties, while the lower one shows a solution with two weekend duties, which are spread over two weekends, i.e., one weekend has a duty on Sunday but not on Saturday and the next weekend has a duty on Saturday but not on Sunday. The upper solution is usually favored by the drivers, whereas occupational physicians prefer the lower one. Every company has to find the right way for themselves for this conflicting goals. There are more examples of goals in rostering, such as the avoidance of isolated duties, shown in Picture 3. The upper part shows a Grantt-Chart of a roster that has an isolated duty on Friday within the first week, i.e. there is a duty on Friday but not on the Thursday before and Saturday after. Such duties are very disliked by the driving personnel. In the lower part of the picture the drivers have at least two consecutive duties. All these goals and rules have to be considered in detail by public transit companies.

In practice there exist even more *variants of rostering* than the ones described above. In some companies, for example, at first the placement of free days is planned and then the duties are inserted on the remaining days. Often a cyclic roster is built, in which the drivers perform at first the duties from the first row, the the second, and so on "rotating through the scheme", such that every driver has done the same duties in the same sequence after completing the whole schedule. This kind of roster has the advantage, that it is perceived as fair by the drivers. Furthermore, it is possible to get a compact and clear presentation of the roster in this way. Sometimes the rostering step is entirely omitted; the duties are then assigned to specific drivers directly.

All personnel planning processes have in common that one reaches better results if the need of rostering is anticipated in the duty scheduling step. In practice one tries to get a good initial situation for rostering through duty mix constraints in duty scheduling or by penalizing types of duties, which give problems in the rostering step. For example, if a planner observes that there are too much late duties on Fridays to find a good roster, he is able to give them higher objective values in the duty scheduling problem or limit their number by a constraint and rerun the duty scheduling optimizer. So, one of the most important questions using optimization systems, is how to adjust the parameters in a general setting to anticipate the latter planning steps and to improve the overall solution quality. The integrated duty scheduling and rostering is a step in the direction of automating and simplifying such an iterative planning process.

3 Integrated duty scheduling & rostering

Duty scheduling and rostering are both complicated by itself, and integrating them is naturally even more complicated. This is probably the reason why the literature on integrated duty scheduling and rostering is so scarce. In fact, the only article we are aware of was published in 2013 by Mesquita, Moz, Paias, and Pato [Mesquita et al(2012)Mesquita, Moz, Paias, and Pato]. This article presents an integrated vehicle, duty scheduling, and duty rostering model which is formulated as an integer linear program. Using Benders decomposition, the integrated problem is subdivided into two smaller problems. The rostering problem is treated as the subproblem while the integrated vehicle and duty scheduling problem is the master problem. These problems are then iteratively solved, relaxing the master and subproblem and using a branch and bound method. The solutions of the linear relaxations of the master and subproblem and of the branch and bound schemes are computed using the optimization software CPLEX 11.0. This process is carried out for 10 iterations. The authors report solutions of real world scenarios. One set of instances considers four depots and has up to 238 timetabled trips per week. The other set of instances has up to 280 timetabled trips per week and there is a single depot for vehicles. The planning horizon was seven weeks in both cases. The authors used a cyclic days-off pattern which already guarantees some rules for rostering, e.g., the minimal number of free days per week. Note that the rosters generated in this way are not cyclic.

We present in this article a similar idea, which is also based on a Benders decomposition approach of the integrated duty scheduling and rostering problem. In contrast to the approach of Mesquita, Moz, Paias, and Pato [Mesquita et al(2012)Mesquita, Moz, Paias, and Pato], we do not schedule vehicles. And we examine a new approach to solve the integrated problem, we introduce *duty templates* to couple the problems not only on specific duties, but on classes of duties. to reduce the number of coupling constraints significantly.

Duty templates are sets of duties, which are characterized by properties that are important for rostering. A more detailed description can be seen in [Borndörfer et al(2013)Borndörfer, Langenhan, Löbel, Schulz, and Weider]. Such properties can be starting and ending times or durations of duties. Possible duty templates would be, for example, the set of early duties or the set of night duties.

Our idea is to define a set of disjoint duty templates containing all possible duties and then to compute rosters based on the numbers of duties per duty template in a given duty schedule instead of rosters based on specific duties. That is, we compute rosters that are sequences of duty templates, ore more exactly rosters that are sequences of typical representants of duty templates. The idea is that the aggregation of duties to duty templates allows to concentrate on information that is relevant for solving the rostering problem. In doing so, we achieve a considerable reduction of the problem size, because, on the one hand, the amount of coupling conditions is reduced from the number of duties to the number of duty templates and, on the other hand, the amount of possible rosters in the rostering subproblem is significantly reduced. We will show that this technique reduces the computation time and allows to solve larger and more complex scenarios. However, introducing duty templates for coupling leads to a less exact coupling of the problems which can lead to slightly worse solutions than the exact coupling. Our computations show, that there is a trade off between computation time and solution quality depending on the selected duty templates.

The actual solving of the integrated duty scheduling and rostering model is done by *Benders decomposition*. The integrated problem is split into its components that are solved several times:

- the duty scheduling problem, with additional Benders cuts to anticipate the requirements of the rostering problem is solved as the so called master problem,
- the (dual) rostering problem, that stems from fixing the solution of the master problem as the subproblem. The dual solution of the subproblem is used to generate constraints for the master problem, so called Benders cuts, for the next iteration.

This process is iterated for a fixed number of iterations. The Benders cuts carry information of the rostering problem to duty scheduling, that in the best case synchronizes the solutions of this two planning problems to an optimal over-all solution. Using duty templates for coupling Benders cuts can be interpreted as systematically determined duty mix conditions, analogously to the ones used in a manual planning process.

For our computations we used two optimization components integrated in the commercial ivu.plan-system [Scholz(2011)] to solve the individual problems: The rostering problem is solved with the optimization tool WS-OPT, which also computes feasible dual variables for the master problem. The master problem, in our case the duty scheduling problem, is solved with the optimization tool DS-OPT [Borndörfer et al(2003)Borndörfer, Grötschel, and Löbel]. Our approach handles the Benders cuts using a Lagrangian relaxation, so one has to expect a duality gap. We remark that the duty scheduling problem decomposes into problems for each operating day, which can be solved in parallel.

4 Mathematical model

The integrated duty scheduling and rostering problem with templates can be formulated as an integer program as follows:

$$
\begin{array}{lllr}
(\mathsf{ISP}) & \min \sum_{d\in\mathcal{D}} c_d x_d + z & & \\
\text{s.t.} & \sum_{d\ni t} x_d = 1, & \forall t\in\mathcal{T}, & (1)\\
 & \sum_{d\in\mathcal{D}} M_{bd} x_d \le r_b, & \forall b\in\mathcal{M}, & (2)\\
 & \sum_{r\in\mathcal{R}} f_r y_r = z & & (3)\\
 & \sum_{r\ni d(s)} y_r = \sum_{d\in s} x_d, & \forall s\in\mathcal{S}, & (4)\\
 & \sum_{r\in\mathcal{R}} N_{br} y_r \le q_b & \forall b\in\mathcal{N}, & (5)\\
 & y_r \in \mathbb{N}, & \forall r\in\mathcal{R} & (6)\\
 & x_d \in \{0,1\}, & \forall d\in\mathcal{D}. & (7)
\end{array}
\qquad (1)
$$

Here $\mathcal{T}$ is the set of tasks that has to be scheduled, $\mathcal{D}$ is the set of duties, and $\mathcal{M}$ is the set of base constraints. The binary variables x_d state whether duty $d \in \mathcal{D}$ is used in a solution or not. The binary variables x_d with the associated costs c_d and with constraints (ISP)(1,2,7) is the standard set-partitioning-model (DSP) for duty scheduling. Condition (ISP)(1) guarantees that every task is performed exactly once. Condition (ISP)(2) is used to control the distribution of duty types, average paid time of duties, and similiar constraints.

The set of duty templates is denoted by $\mathcal{S}$, the feasible set of rosters by $\mathcal{R}$ and the set of mix conditions for rosters by $\mathcal{N}$. A typical duty mix constraint for rosters could for example restrict the working time per week. In this case N_{br} would be the working time of roster r in a specific week and q_b the associated limit. The binary variables y_r state whether roster $r \in \mathcal{R}$ is used in a solution. The binary variables y_r with the associated costs f_r and with constraints (ISP)(3-6) is a set-partition-model (RSP) for rostering, which is also pretty standard besides the duty templates.

The rosters consist in this model of *duty-representants* of templates denoted by $d(s) \in ssubseteq\mathcal{D}$. A representant can possibly be scheduled several times. The coupling conditions (ISP)(4) ensure that all duties used in a duty schedule correspond to the right number of template representants scheduled in rosters. To achieve this the set of the templates has to be a partition of the set of duties, i.e., every duty has to be in exactly one template. A typical template contains, for example, all duties, which start between 6 and 8 o'clock; a representant of this template could be a duty that starts "in the middle" of this interval, that is at 7 o'clock; the rosters are generated as sequences of such representants.

If we define a template per duty, that is $\mathcal{S} = \{\{d\} : d \in \mathcal{D}\}$, then we have an exact model for the integrated duty scheduling and rostering. This is basically the model presented in [Mesquita et al(2012)Mesquita, Moz, Paias, and Pato] without vehicle routing. The main problem with this approach is the large number of coupling constraints $(\mathsf{ISP})(4)$: one constraint for every possible duty. The number of possible duties grows exponentially with the number of tasks that have to be scheduled. For practical problem instances we quickly reach a size of millions or even billions of coupling constraints. The number of variables is also growing exponentially, which means that the exact models are only solvable for rather small instances.

The use of templates drastically reduces the number of coupling constraints. Instead of one constraint per duty just one constraint per template is required. However, we do not have a guarantee, that coupling with templates generates good solutions, but our computations show, that it works in practice.

5 Benders decomposition

For solving the model (ISP) a Benders decomposition approach is used. Using this approach the integrated optimization problem is solved by splitting it into smaller problems and solving the slightly modified subproblems iteratively. Within the iteration process solutions of the subproblems are synchronized by exchanging information. At first a duty schedule is computed, then a roster based on the duties of this schedule. The resulting roster is used to generate Benders cuts that are added to the duty scheduling problem. The Benders cuts can be interpreted as numerical duty mix conditions that direct the duty schedule towards more "roster friendliness". Iteratively duty schedules and rosters are computed, until the model converges or the iteration limit is reached.

To derive Benders cuts we use the LP-relaxation of the rostering subproblem (RSP):

$$\begin{aligned}
(\mathsf{RSP}) \quad & \min \sum_{r \in \mathcal{R}} f_r y_r = z \\
\text{s.t.} \quad & \sum_{r \ni d(s)} y_r = \sum_{d \in s} x_d, \ \forall s \in \mathcal{S}, \\
& \sum_{r \in \mathcal{R}} N_{br} y_r \leq q_b \qquad \forall b \in \mathcal{N}, \\
& y_r \geq 0, \qquad \forall r \in \mathcal{R}.
\end{aligned} \tag{2}$$

Dualization leads to the *Benders subproblem*:

$$
\begin{array}{llr}
(\mathsf{DRSP}) & \max \sum_{s\in\mathcal{S}} \lambda_s \sum_{d\in s} x_d + \sum_{b\in\mathcal{N}} \mu_b q_b = \quad z & \\
\text{s.t.} & \sum_{r\ni d(s)} \lambda_s \quad + \sum_{b\in\mathcal{N}} \mu_b N_{br} \leq f_r, \ \forall r \in \mathcal{R}, & (3) \\
 & \lambda_s \in \mathbb{R} \qquad\qquad\qquad\qquad \forall s \in \mathcal{S}, & \\
 & \mu_q \leq 0, \qquad\qquad\qquad\qquad \forall q \in \mathcal{N}. &
\end{array}
$$

For simplicity we assume that the feasible domain of (RSP) is non-empty and bounded, i.e., for every duty schedule it is possible to generate feasible but maybe very expensive rosters. In practice this can be achieved by introducing slack variables with suitable penalty costs. Then (DRSP) is bounded and its solution space is a polytope. We denote the vertices of this polytope by J. (DRSP) is equivalent to the following optimization problem:

$$
(\mathsf{DRSP}) \quad \max_{(\lambda,\mu)\in J} \sum_{s\in\mathcal{S}} \lambda_s \sum_{d\in s} x_d + \sum_{b\in\mathcal{N}} \mu_b q_b = z. \tag{4}
$$

Insertion of z in (ISP) gives rise to the *Benders masterproblem* (BDSP):

$$
\begin{array}{llllr}
(\mathsf{BDSP}) & \min \sum_{d\in\mathcal{D}} c_d x_d + z & & & \\
\text{s.t.} & \sum_{d\ni t} x_d & = 1, & \forall t \in T, \quad (1) & \\
 & \sum_{d\in\mathcal{D}} M_{bd} x_d & \leq r_b, & \forall b \in \mathcal{M}, \quad (2) & (5) \\
 & \sum_{s\in\mathcal{S}} \lambda_s \sum_{d\in s} x_d + \sum_{b\in\mathcal{N}} \mu_b q_b & \leq z, & \forall (\lambda,\mu) \in J, \ (3) & \\
 & x_d \in \{0,1\}, & & \forall d \in \mathcal{D}, &
\end{array}
$$

with the Benders cuts (BDSP)(3). If one would compute all this cuts at once, one would just need one iteration for a duty schedule that is optimally adapted to the roster (or rather its LP-relaxation).

It is more efficient to iteratively identify the important Benders cuts using a cutting plane procedure than to enumerate all vertices. For this approach we use a subset of the benders cuts and start with the empty set. The solution of subproblem (DRSP) for a subset $J' \subset J$ is a lower bound on the optimal objective value and (BDSP) with respect to J' is a relaxation of the masterproblem. Using the solution of the subproblem one decides in every iteration, whether there are improving Benders cuts; if so, they are added to the masterproblem.

Benders cuts have almost the same structure as duty mix conditions. However, due to technical reasons DS-OPT, the duty scheduling optimizer used here, is right now not able to cope with arbitrary Benders cuts. Thus, we are using Lagrange relaxation (see, e.g., [Lemaréchal(2001)]) to relax the Benders

Algorithm 1 Lagrange-relaxed Benders decomposition with templates.

Input: The sets T, $\mathcal{M}$, $\mathcal{N}$, $\mathcal{D}$, $\mathcal{S}$ and $\mathcal{R}$. A function $r : \mathcal{S} \to \mathcal{D}$.
Output: A feasible solution x^*, y^* of (ISP).
1: $i \leftarrow 1$, $J \leftarrow \emptyset$
2: Let γ be the zero vector and $k = 0$.
3: **repeat**
4: Solve (LDSP) and let x^i be the solution.
5: Solve (DRSP) with $\mathcal{R}^* = \{r(s) : d \in s \wedge x^i_d = 1\}$.
6: Let (λ^i, μ^i) be a solution of (DRSP).
7: **if** $(\lambda^i, \mu^i) \in J$ **then**
8: **go to** 15
9: **else**
10: $J \leftarrow J \cup (\lambda^i, \mu^i)$
11: Guess κ^*.
12: Compute k and γ.
13: $i \leftarrow i + 1$
14: **until** end
15: Solve (RSP) with the best set of duties $\mathcal{D}^*$. Let y^* be the solution of (RSP).
16: $x^* \leftarrow x^i$.

cuts:

$$
\begin{aligned}
&(\mathsf{LDSP}) \quad \min \sum_{d\in\mathcal{D}} c_d x_d + \max_{\substack{\kappa_j \geq 0,\\ \sum_{j\in J}\kappa_j = 1}} \sum_{j\in J} \kappa_j \left(\sum_{s\in\mathcal{S}} \lambda^j_s \sum_{d\in s} x_d + \sum_{b\in\mathcal{N}} \mu^j_b q_b \right) \\
&\text{s.t.} \quad \sum_{d\in\mathcal{D}(t)} x_d = 1, && \forall t \in T, \\
&\qquad \sum_{d\in\mathcal{D}} M_{bd} x_d \leq r_b, && \forall b \in \mathcal{M}, \\
&\qquad x_d \in \{0,1\}, && \forall d \in \mathcal{D}.
\end{aligned}
\tag{6}
$$

Model (LDSP) is a "pure" duty scheduling problem. Only the costs of the duties are modified w.r.t. λ. All duties of the same duty template, i.e., all $d \in s$ are modified by the same value $\sum_{j\in J} \kappa_j \lambda^j_s$.

Let now κ^*_j be optimal Lagrange multipliers, then we can write:

$$
\begin{aligned}
&(\mathsf{LDSP}) \quad \min \sum_{d\in\mathcal{D}} (c_d + \gamma_d) x_d + k \\
&\text{s.t.} \quad \sum_{d\in\mathcal{D}(t)} x_d = 1, && \forall t \in T, \\
&\qquad \sum_{d\in\mathcal{D}} M_{bd} x_d \leq r_b, && \forall b \in \mathcal{M}, \\
&\qquad x_d \in \{0,1\}, && \forall d \in \mathcal{D}.
\end{aligned}
\tag{7}
$$

With

$$
k = \sum_{j\in J} \kappa^*_j \sum_{b\in\mathcal{N}} \mu^j_b q_b \qquad \text{and} \qquad \gamma_d = \sum_{j\in J} \kappa^*_j \sum_{s\ni d} \lambda^j_s. \tag{8}
$$

Lagrange multipliers κ^* can be computed with subgradient and bundle methods. This requires some computational effort, but our computations show, that it is sufficient to guess some multipliers. In this article we weight all Benders cuts with the same weight, that is, $\kappa_j^* = 1/|J|$ is used for all $j \in J$; see also section 6.

After the last Benders iterations, an exact roster without templates has to be computed. We simply use the duties of the benders master problem, that gave us the best bound.

Algorithm 1 shows the complete algorithm as pseudo-code.

6 Computational results

We have tested our algorithm on two real world scenarios of a large urban public transit company. These scenarios consist of three different days of operation: The first day of operation includes duty elements which are valid from Monday to Friday, the second one contains all elements of Saturday, and the third one corresponds to Sunday. Table 1 and 2 contains the key figures of the scenarios: The number of tasks per operating day, the number of duties of the sequential solution and the running time of the duty scheduling problem (without Benders cuts) in minutes:seconds.

In our tests, we perform 30 Benders iterations, i.e., we solve 30 duty scheduling and rostering problems. The duty scheduling problems are solved independent from one another. The planning horizon for the rostering problem is three weeks, so each duty has to be scheduled three times and each roster has the length of three weeks. The Lagrange multipliers κ for the Benders cuts in the masterproblem are chosen so that every Benders cuts gets the same weight. This achieved the best results in our tests.

The scenarios contain the following main cost factor of the rostering subproblem:

- **Costs per row:** Every row (i.e., every driver) has a cost facor of 2 units.
- **Below weekly working time:** Every member of staff should work 39 hours a week. Exceeded working time is panelized by 0.7 units per hour and undercutting by 1.5 units per hour.
- **Isolated duties:** There should be at least two consecutive days with duties. Every isolated duty ,i.e., a day with duty but without duty the day before and after, is panelized with 1 unit.
- **consecutive weekends:** A weekend should be free either, or have a duty on both days. A violation of this rule costs 2.5 units.

The number of rows in a roster is basically determined by the duties and the given weekly working time. It can happen that the weekly working time can not be achieved for planning reasons, then more rows are needed. A decrease of the duty scheduling's objective value is mainly accomplished by reduction for other cost factors.

Table 1 Small scenario.

operating day	Mo-Fr	Sa	Su	$\sum$
#tasks	297	174	113	1,772
#duties	27	15	11	161
run time duty scheduling [mm:ss]	0:21	0:04	0:09	–

Table 2 Large scenario.

operating day	Mo-Fr	Sa	Su	$\sum$
#tasks	711	424	367	4.346
#duties	64	37	33	390
run time duty scheduling [mm:ss]	16:58	5:49	7:14	–

Table 3 Results

Scenario	small			large	
	sequential	exact	templates	sequential	templates
total costs	491.64	465.63	474.12	1198.32	1155.28
duty costs	296.74	296.93	297.22	721.00	720.87
rostering costs	194.9	168.70	176.90	477.32	434.41
#rows	34	33	35	81	81
below opt paid time	26,5	25,7	26,5	62,7	63,2
#isolated duties	4	–	–	5	6
#non consecutive weekends	26	18	20	68	50
running time [hh:mm]	01:50	61:19	09:00	22:00	128:41

Table 3 shows our computational results for the small and the large scenario. The columns 2 to 4 list the results for the small scenario and the last two columns the results for the large scenario. The column "sequential" displays the results of the classical sequential approach: First solve the duty scheduling and then the rostering problem. The column "exact" displays the results for the integrated Benders approach using exact coupling, i.e., the coupling is based on proper duties and not on duty templates. The column "templates" displays the results of our Benders decomposition algorithm 1 using a coupling on duty templates. As templates we used sets of duties that were characterized by the same start- and end-time window. For this purpose the operating days were subdivided into time windows of two hours length. A template is then defined by a start-time window i_1 and a end-time window i_2, i.e., the template (i_1, i_2) consists of all duties that start during the interval i_1 and end during the interval i_2. So there are at most 66 different templates a day, 462 templates a week and 1386 templates for the planning horizon of 3 weeks. As representative $d(s)$ for a template s we chose a duty that starts in the middle of the interval $i_1(s)$ and ends in the middle of interval $i_2(s)$. Templates that only considered the starting time of the duty did not work out, because their structure was too rough.

The row “total costs” shows the value of the overall costs of the best solution of the integrated duty scheduling and rostering problem. The following two rows display the cost values for the duty scheduling and rostering problem. The next four rows contain the main cost components of the rosters. The last row “running time” shows the total running time in hours:minutes.

It can be clearly seen that the integrated approach significantly improves the rostering results at almost no extra costs for the duty scheduling problem. Compared to the coupling on templates the results of an exact coupling are slightly better on the one hand, but, on the other hand, the running time increases drastically. The results of the small and large scenario show roughly the same effect. Computations for an exact coupling for the large scenario have not been carried out since the expected running time (approx. 30 times the running time of the sequential problem) of about 25 days was too large.

7 Conclusion

It is possible to coordinate current optimization components for duty scheduling and rostering, without any manual coordination, such that a better overall result is achieved. This is done by an automatic approach that intelligently iterates using the appropriate computing resources. In the considered case of integrated duty scheduling and rostering the driver satisfaction could be significantly increased without coursing additional costs in duty scheduling. These results are transferable to other (integrated) problems of a planning chain. Thus it is again possible to identify and exploit considerable potentials in public transit.

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