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Template-based Re-optimization of Rolling Stock Rotations

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Abstract Rolling stock, i.e., the set of railway vehicles, is among the most expensive and limited assets of a railway company and must be used efficiently. We consider in this paper the *re-optimization* problem to recover from unforeseen disruptions. We propose a *template concept* that allows to recover cost minimal rolling stock rotations from reference rotations under a large variety of operational requirements. To this end, *connection templates* as well as *rotation templates* are introduced and their application within a rolling stock rotation planning model is discussed. We present an implementation within the rolling stock rotation optimization framework ROTOR and computational results for scenarios provided by DB Fernverkehr AG, one of the leading railway operators in Europe.

Keywords Rolling Stock Rotation Problem, Re-optimization, Hypergraph-based Integer Programming, Rotation Patterns

1 Introduction

Rolling stock is among the most expensive and limited assets of a railway company and must therefore be used efficiently. The *Rolling Stock Rotation Problem* (RSRP) addresses this task. It deals with the cost minimal construction of *rolling stock rotations* to operate a given timetable of passenger trips by rail vehicles, including a large number of operational requirements like vehicle composition rules, maintenance constraints, infrastructure capacity constraints, and regularity requirements. The fundamental task is to construct

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cycles for the different types of vehicles in order to define the chronological sequence of activities which should be performed by those vehicles. These activities include passenger trips, empty movements, so called *deadhead trips*, and maintenance activities. A rolling stock rotation plan is then a set of rotations which cover all trips of the given timetable. Note, that trips which are operated by more than one vehicle are part of several rotations. A detailed problem description, a mixed integer programming formulation, and an algorithm to solve this problem in an integrated manner is described in detail in Borndörfer et al (2012).

The current paper is a further development of the paper Borndörfer et al (2014) and focuses on the *re-optimization* task.

The re-optimization setting for the RSRP arises in the following situation. At some point in time a railway undertaking has to tackle an instance of the RSRP and constructs a rolling stock rotation plan. We call this planning step *greenfield planning* or *greenfield optimization*. At a later point in time conditions of and assumptions about the original instance change. It is then inefficient and often even not feasible to operate the originally constructed rolling stock rotations that we call *reference rotations*.

The circumstances that lead to re-optimization scenarios are manifold. Examples are construction sites, technical failures, accidents, and strikes. In such situations, a new RSRP′ has to be solved. In the RSRP′ all requirements of the original RSRP have to be taken into account. The main difference to greenfield planning is that the reference rotations were already completely or partially implemented in operation. Crew was scheduled for vehicle operations and maintenance services, capacity consumption of parking areas was reserved, and most important railway tracks were already allocated for the deadhead trips of the reference rolling stock rotations. Hence, a major goal in constructing a solution to the RSRP′ is to change as little as possible in comparison to the reference rotations. This is called *re-optimization*. Re-optimization problems have received a huge amount of attention in the literature, e.g., see Haahr et al (2014); Budai et al (2010) for rolling stock applications, Secomandi and Margot (2009) for vehicle routing with stochastic demands, and Huisman (2007) for crew scheduling applications.

Recent literature overviews on the re-optimization in terms for rolling stock applications can be found in Nielsen (2011) and Wagenaar et al (2016). In comparison to most of all existing re-optimization approaches for rolling stock rotations our approach is especially tailored to reproduce as many details of the reference rotations as possible. Of course, this objective competes with the optimization of operational cost (where almost all other approaches are tailored to) but turns out to be of essential interest in the railway industry.

When it comes to re-optimization, almost everything about the original scenario can change. Rather typical examples are:

- the set of allowed vehicle configurations (see Section 2.2) of a timetabled trip can change,
- timetabled trips can be shortened, enlarged, or canceled,

– new timetabled trips can appear,
– fleet capacities can change, or
– new fleets may have to be introduced.

This large variety of possible changes makes it hard to insist on maintaining any particular properties of the reference solutions. A major question in re-optimization applications is therefore to identify appropriate structures in the reference solutions that can be recognized after re-optimization. An obvious candidate in rolling stock rotation planning is a dedicated connection between two timetabled trips in a reference rotation. To this end, we introduce *connection templates* that are used to veer the objective function for re-optimization towards the connections included in the reference rotations. In our application, however, it turned out that it is not enough to re-optimize on the basis of such "local" templates. To this end, we introduce additional *rotation templates* which we announce as the main contribution of the paper.

We use the term *template* to refer to a dedicated set of timetabled trips. This set is supposed "to be part of the reference rotations" and an optimization goal is to recreate its arrangement in the solution to the re-optimization scenario. In this way, *connection templates* refer to two succeeding timetabled trips and *rotation templates* refer to a (usually larger) set of timetabled trips that are all covered by a single rolling stock rotation of the reference solution.

We will show in this paper that templates can be integrated conveniently into a hypergraph-based multi-commodity flow framework by increasing the number of commodities.

The paper is organized as follows. In the next section we briefly review our "traditional" approach (see Borndörfer et al (2012, 2014); Borndörfer et al (2014)) to the RSRP including an integer programming formulation. The template-based re-optimization approach is discussed in Section 3. The final Section 4 elucidates the contribution of our approach with a computational study of scenarios provided by DB Fernverkehr AG, referring to individual computational aspects of rotation and connection templates.

2 Optimization of Rolling Stock Rotations

In this section we provide an abstract description of the rolling stock problem (RSRP). This description is based on a hypergraph model that constitutes the basis for an integer programming formulation of the RSRP which is described in Section 2.1. In the succeeding Section 2.2 we consider an example which we use in particular to explain the relation between *fleets* and *vehicle configurations* in rolling stock rotation planning. This relation is important for the subject of the paper as will become clear in Section 3.1.

We streamline the presentation of the RSRP to only those aspects which are needed for the discussion of the concept of templates. For further reading we refer to the papers Borndörfer et al (2012, 2014); Borndörfer et al (2014) that cover also, e.g., regularity, maintenance, and capacity requirements. We note that the template approach is compatible with all of these requirements.

2.1 The Rolling Stock Rotation Problem

The rolling stock rotation problem can be described as follows. We consider a cyclic planning horizon of one *standard week*. The set of timetabled passenger trips is denoted by T. Let V be a set of *nodes* representing departures and arrivals at origin respectively destination station of vehicles operating passenger trips of T, let $A \subseteq V \times V$ be a set of directed standard arcs, and $H \subseteq 2^A$ a set of *hyperarcs*. We consider an *arc-based* hypergraph definition in contrast to other variants in the literature, see Cambini et al (1997). The reason is simply that this allows a direct definition of which vehicles are coupled together and are jointly operating trips. Thus, a hyperarc $h \in H$ is a set of standard arcs representing the contained vehicles.

The RSRP *hypergraph* is denoted by $G = (V, A, H)$. The hyperarc $h \in H$ *covers* $t \in T$, if each standard arc $a \in h$ represents an arc between the departure and arrival of t. We define the set of all hyperarcs that cover $t \in T$ by $H(t) \subseteq H$. By defining hyperarcs many technical requirements such as vehicle configuration and regularity aspects can be handled directly by the hypergraph model, see Borndörfer et al (2012) and Section 2.2. The RSRP is to find a cost minimal set of hyperarcs $H^\star \subseteq H$ such that each timetabled trip $t \in T$ is covered by exactly one hyperarc $h \in H^\star$ and $\bigcup_{h \in H^\star} h$ is a set of *rotations*, i.e., a set packing of cycles (each node is covered at most one time). Hence, a rotation is a cycle in G that runs through the standard week at least once.

We define the sets of hyperarcs that go into and out of the node $v \in V$ as $H(v)^{\text{in}} := \{h \in H \,|\, \exists\, a \in h : a = (u, v)\}$ and $H(v)^{\text{out}} := \{h \in H \,|\, \exists\, a \in h : a = (v, w)\}$, respectively. Introducing a binary decision variable x_h, which equals to one if h is selected, and its cost c_h for each hyperarc $h \in H$, the RSRP can be stated as an integer program as follows:

$$\min \sum_{h \in H} c_h x_h, \tag{IP}$$

$$\sum_{h \in H(t)} x_h = 1 \qquad \forall t \in T, \tag{1}$$

$$\sum_{h \in H(v)^{\text{in}}} x_h = \sum_{h \in H(v)^{\text{out}}} x_h \qquad \forall\, v \in V, \tag{2}$$

$$x_h \in \{0, 1\} \qquad \forall \in H. \tag{3}$$

The objective function of model (IP) minimizes the total cost of operating a timetable. For each trip $t \in T$ the covering constraints (1) assign exactly one hyperarc of $H(t)$ to t. The equalities (2) are flow conservation constraints for each node $v \in V$ that imply the set of cycles in the arc set A. Finally, (3) state the integrality constraints for the decision variables.

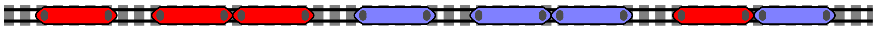

Fig. 1 Possible vehicle configurations composed of two fleets, i.e., a red fleet and a blue fleet.

2.2 Vehicle Configurations

In this section we describe the relation between fleets and vehicle configurations in the context of the hypergraph-based model presented in the previous section.

A *fleet* is a basic type of railway vehicles. For example, the slightly more than 220 Intercity-Express railway vehicles of DB Fernverkehr AG are partitioned into several structurally identical sets of vehicles such as ICE1, ICE2, ICE3, etc., which form fleets. A *vehicle configuration* is a multiset of fleets. A vehicle configuration models the requirement that rolling stock vehicles are coupled together to operate trips. A vehicle configuration is defined as a multiset such that vehicles of a dedicated fleet can appear multiple times.

Let F be the set of fleets. For example, if we consider the set $F = \{\text{Red}, \text{Blue}\}$ of fleets, one can create the vehicle configurations $\{\text{Red}\}$, $\{\text{Blue}\}$, $\{\text{Red}, \text{Red}\}$, $\{\text{Red}, \text{Blue}\}$, and $\{\text{Blue}, \text{Blue}\}$ of size of at most two. These vehicle configurations are illustrated in Figure 1. Fleets and vehicle configurations play important roles in intercity planning. This is because rolling stock vehicles can be coupled together *on the fly*, i.e., no technical equipment or crew is needed for coupling or decoupling activities, and they can happen frequently, even with the drawback of additional time that is required to perform such an event. Thus, determining the vehicle configuration for each timetabled trip is a basic question in rolling stock rotation planning.

Note that a vehicle configuration does not have an ordering in itself. In particular, the individual positions of the railway vehicles within a vehicle configuration is not defined. We call an ordered set of railway vehicles where the positions and orientations of the individual vehicles are determined a *vehicle composition*. The details on the extension of the model from vehicle configurations to vehicle compositions can be found in Borndörfer et al (2014). We do not discuss vehicle compositions here because the presentation in terms of less complex vehicle configurations is sufficient to motivate the re-optimization approach of this paper. However, the implementation and the presented computational results are for RSRP models on the composition level.

For each timetabled trip of T the set of possible vehicle configurations is given as an input data to the RSRP. The concrete choice of one of these vehicle configurations is to be made in rolling stock rotation planning. For that purpose, the nodes and hyperarcs are arranged as follows. A node of the RSRP hypergraph is associated with the departure or arrival of a vehicle of a dedicated fleet of a timetabled trip. Each hyperarc of the RSRP hypergraph represents the connection of the involved nodes with a dedicated vehicle configuration.

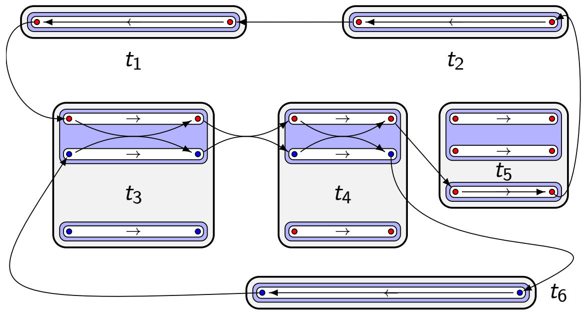

Fig. 2 Two rotations consisting of 4 standard arcs and 5 hyperarcs (of size 2) in a hypergraph model $G = (V, A, H)$ with $|T| = 6$ and $|V| = 24$.

Figure 2 shows a small example to illustrate the relations between trips, nodes, and fleets as well as between hyperarcs and vehicle configurations. All red and blue circles are nodes of the RSRP hypergraph. In particular the departure node at the origin station of a trip is on the left hand side within the box and the arrival node at the destination is on the right hand side, respectively. Arcs define the consecutive operation of activities, e.g., the arc from the arrival node of trip t_6 to one of the departure nodes of trip t_3 models that this blue vehicle operates t_3 in double-traction after t_6. Note that couple cost of the blue vehicle can be directly associated with this arc.

Hyperarcs are depicted as a collection of connected and curved arcs. Hyperarcs within a box represent rolling stock vehicles operating the corresponding timetabled trip together, e.g., in box t_3. Thus, hyperarcs connecting boxes represent rolling stock vehicles turning together from the first trip to the second trip, e.g., from t_3 to t_4.

The modeling benefits of hyperarcs can directly be seen in Figure 2 in which jointly activities of vehicles, e.g., coupling or decoupling, can be distinguished from activities related to a single vehicle. Note that in case of using formulations based on standard graphs this relation has to be modeled using additional constraints which handle the cases if arcs are chosen together or not.

The colors of the circles indicate the two fleets – red and blue. The hyperarcs correspond to a solution. The hyperarcs that connect the departures and arrivals of timetabled trips are elements of the sets $H(t)$ for trips $t \in T$, i.e., the set of hyperarcs which can be used to cover the timetabled trips. For $H(t_3)$, $H(t_4)$, and $H(t_5)$ only the hyperarcs that were chosen in the solution are shown. The vehicle configuration that is used to cover a dedicated timetabled trip is implied by the fleets of the tail and head nodes of a hyperarc.

3 Re-optimization by Templates

Our approach for the re-optimization of rolling stock rotations is designed to make use of the same algorithmic framework as greenfield optimization, i.e.,

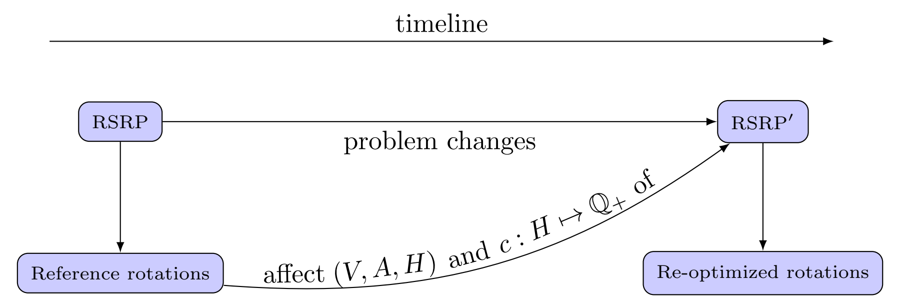

Fig. 3 Re-optimization setting

we treat the RSRP′ as a special variant of the RSRP. The specialized parts are the shape of the hypergraph (V, A, H) and the objective function $c : H \mapsto \mathbb{Q}_+$, see Figure 3. These parts are obviously dependent from a reference solution that is part of the input data of the RSRP′ and is the solution of an closely related RSRP. As previously mentioned, this solution is a set of rotations where each rotation is a cycle through the hypergraph. The total number of rolling stock vehicles required to operate each single rotation is the number times the rotation passes the underlying cyclic standard week.

The general concept of templates is a soft modeling approach which modifies the objective function in such a way that preserving characteristics of the reference solution will be rewarded in the objective. Section 3.1 explains how rotation templates affect the shape of the hypergraph, i.e., by introducing commodities for the combination of the fleets and the vehicle types used in the reference solution. The connection templates presented in Section 3.2 only influence the objective function. This is described in Section 3.3.

3.1 Rotation Templates

In large railway companies the rolling stock rotations of a single vehicle type are often split into different sub rotations, i.e., due to different depots of the vehicles or different on board equipment. This has to be considered during re-optimization and in the best case completely recovered after the re-optimization process. To do so we define rotation templates. A rotation template is a set of timetabled trips that are all contained in a reference rotation. In order to motivate the idea of rotation templates we consider a single reference rolling stock rotation as a cycle that covers a subset of timetabled trips $T_{\text{ref}} \subseteq T$ (and possibly more in its associated original RSRP). We assume that the set T_{ref} is contained in (the timetable of) the RSRP′, i.e., re-optimization scenario. A seemingly "small" change (a classical 2-opt move) to this reference rotation is to interchange two connections of timetabled trips, i.e., if $t_1 \in T_{\text{ref}}$ is connected to $t_2 \in T_{\text{ref}}$ and $t_3 \in T_{\text{ref}}$ is connected to $t_4 \in T_{\text{ref}}$ in the reference rotation. We might consider to connect t_1 to t_4 and t_3 to t_2 in a solution for the re-optimization scenario. Note that this splits up the given rotation of the reference solution into two disjoint cycles. Hence, only two of the four trips are contained in the same rotation. The other two trips are covered by a different rolling stock rotation. This is not desired in industrial practice and therefore penalized in our template approach. In fact, an important "non-local" requirement refers to the whole set T_{ref} of timetabled trips of a dedicated reference rotation. Namely, it is generally desired that most of the timetabled trips of T_{ref} remain together in a rolling stock rotation *after* re-optimization. The purpose of our *rotation templates* is to provide control about the distribution of the timetabled trips among the rotations produced by re-optimization.

The implementation of this purpose works as follows. As already introduced, we denote by F the set of fleets that is available for the operation of the timetable of the re-optimization scenario. Let R be the set of rolling stock rotations that appear in the reference rotations. The idea is to "refine" the set of fleets F into a larger set of fleets such that its elements can be distinguished by the reference rotations R. To this end, we are given a non-empty set of *refined fleets* $F_R(f)$ for each (original) fleet $f \in F$ in the input data for the re-optimization scenario. The set $F_R(f)$ is created in an appropriate way, e.g., if three reference rotations are operated by railway vehicles of the fleet $f \in F$ we would create at least three refined fleets in $F_R(f)$ that correspond to the reference rotations operated by f. Note that it might also be of practical interest to create refined fleets for $f \in F$ that correspond to reference rotations that were not operated by f before re-optimization. This is an exceptional case, which is made, e.g., to allow to only change the fleet of a reference rotation. In this way, it is possible to consider all elements of the Cartesian product $F \times R$ as refined fleets. But the typical case is to refine $f \in F$ to as many refined fleets as reference rotations are operated by f. Therefore, we assume $\left|\bigcup_{f\in F}(F_R(f))\right| = |R|$ in the following.

The (refined) re-optimization hypergraph G_R is built in a (more or less) straightforward way on the basis of the refined fleets. All rules and requirements for the original case directly carry over to the refined hypergraph that can be denoted as

$$G_R = \left(\bigcup_{r\in R} (V_r \cup S_r), \bigcup_{r\in R} A_r, H_R \right).$$

The nodes and *standard arcs* of G_R decompose into independent *standard graphs* $(V_r \cup S_r, A_r)$ for each reference rotation $r \in R$. By construction, each such graph $(V_r \cup S_r, A_r)$ is an exact one-to-one copy of nodes and arcs that we consider in the case without reference rotations.

Thus, the nodes as well as the standard arcs of a hyperarc can be distinguished in terms of reference rotations. E.g., traversing $v \in V_r$ has the meaning of traversing a node in a rolling stock rotation that corresponds to the reference rotation $r \in R$. By choosing appropriate objective function coefficients we gain full control over the distribution of the timetabled trips among the reference rotations in a solution to the re-optimization scenario, see Section 3.3.

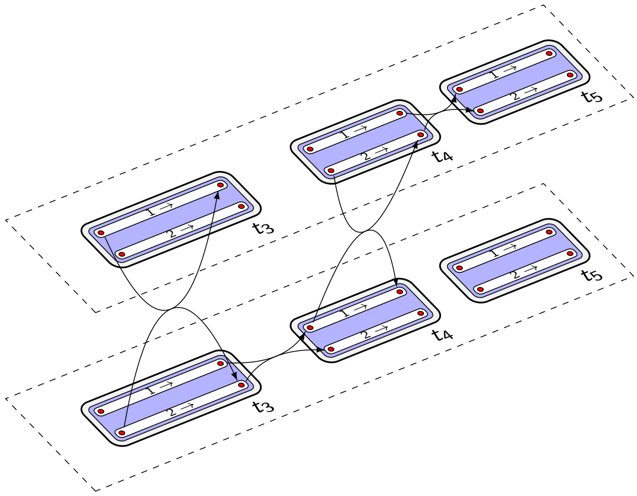

Fig. 4 Refined hyperarcs of the re-optimization hypergraph.

The crux of G_R is that its hyperarcs H_R do not refine as easy as the nodes and standard arcs do. The hyperarcs of the original hypergraph particularly model movements of vehicle configurations, which are multisets of original fleets. If we refine the original fleets, we, consequently, have to also refine the set of original vehicle configurations, which movements are modeled by hyperarcs H_R of G_R. This is much more complicated.

For example, consider a hyperarc $h = \{a_1, a_2\} \in H$ of the original hypergraph and assume that $F(a_1) \in F$ and $F(a_2) \in F$ denote the original fleets that the standard arcs $a_1, a_2 \in h$ model, i.e., $\{F(a_1), F(a_2)\}$ is the vehicle configuration of h. In the re-optimization scenario we have $|F_R(F(a_1))|$ alternative refined fleets for a_1 as well as $|F_R(F(a_2))|$ for a_2. The crucial point is that it is necessary to consider all multisets of size two of elements (i.e., fleets) of $F_R(F(a_1)) \cup F_R(F(a_2))$ because an optimal distribution of the railway vehicles that are coupled together at h among the refined fleets is not known a priori. In fact, in a solution to the re-optimization scenario it could be efficient to perform the movement of h by two coupled railway vehicles with *equal* refined fleets. This already gives $|F_R(F(a_1)) \cup F_R(F(a_2))|$ alternatives. Moreover, it could also be even optimal to (arbitrarily) take two *different* refined fleets from $F_R(F(a_1)) \cup F_R(F(a_2))$ in order to refine h.

By Figure 4, which is based on Figure 2, we illustrate the situation of the refined hyperarcs in the re-optimization hypergraph G_R. Suppose that the original hypergraph $(V \cup S, A, H)$ is illustrated by one of the two layers. Further, assume that two reference rotations for the red fleet are given. As explained, the nodes of $(V \cup S, A, H)$ become refined, i.e., just copied. Obviously, these copies are represented by the two layers that lie on top of each other in Figure 4. We also assume that we want to operate the timetabled trips t_3, t_4, and t_5 by two coupled railway vehicles of the red fleet. In the re-optimization scenario, we have to decide from which refined fleet (which corresponds to a reference rotation for the original red fleet) the two individual railway vehicles originate. This translates to hyperarcs as is exemplary shown in Figure 4. It is possible to operate the individual railway vehicles by vehicles of the same refined fleets. This situation is implied if one chooses the hyperarc that connects t_3 to t_4 (see the layer below) or t_4 to t_5 (see the layer above) in a solution. But, as can be seen by the hypearcs that operate t_3 and t_4 in Figure 4, it is naturally also possible to take vehicles from different refined fleets.

Thus, an original hyperarc may be refined to many hyperarcs of the refined hypergraph. The refinement of the original hyperarcs H to H_R of the re-optimization hypergraph G_R directly derives from the refinement of the vehicle configurations. But, it turns out that the number of vehicle configurations increases–dramatically.

In order to illustrate this blow-up, we consider a vehicle configuration that has not been refined yet, i.e., a multiset of elements of F, by using the following notation:

$$\{f_1^1, f_1^2, \ldots, f_1^{m_1}, f_2^1, f_2^2, \ldots, f_2^{m_2}, \ldots, f_n^1, f_n^2, \ldots, f_n^{m_n}\}. \tag{4}$$

The multiset (4) denotes a single vehicle configuration of size $\sum_{i=1}^{n} m_i$ that is allowed for the operation of a dedicated timetabled trip of T. In this notation we assume $n \in \mathbb{Z}_+$ and that two fleets $f_i, f_j \in F$ have different subscripts $i, j \in \{1, \ldots, n\}$ if and only if they identify different fleets of F. Therefore, the vehicle configuration (4) is composed of n different fleets where multiple appearances of a dedicated fleet are distinguished by the superscripts. In this way, fleet f_i appears exactly $m_i \in \mathbb{Z}_+$ times in the vehicle configuration, $i \in \{1, \ldots, n\}$.

In the re-optimization model we consider $F_R(f_i)$ as the new (i.e., refined) fleets for the original fleet $f_i \in F$ that appeared m_i times in the original vehicle configuration (4). Already for this m_i appearances we have

$$\binom{|F_R(f_i)| + m_i - 1}{m_i} = \frac{(|F_R(f_i)| + m_i - 1)!}{m_i!(|F_R(f_i)| - 1)!} \tag{5}$$

possibilities to form a (refined) multiset from the elements of the set $F_R(f_i)$. The Formula (5) denotes the number of possibilities to choose m_i elements from a base set of size $|F_R(f_i)| + m_i - 1$. The base set can be interpreted as the refined fleets $F_R(f_i)$ plus $m_i - 1$ artificial elements that indicate the multiple choice of an already taken element. As already mentioned, all of this (refined) multisets have to be considered because an optimal distribution of the railway vehicles that are coupled together while operating a timetabled trip among the refined fleets is not known a priori.

Formula (5) only denotes the blow-up for a single original fleet (contained in an original vehicle configuration). The final number of refined vehicle configurations for the original vehicle configuration (4) is

$$\prod_{i=1}^{n} \binom{|F_R(f_i)| + m_i - 1}{m_i}. \tag{6}$$

In the RSRP applications at DB Fernverkehr AG, a typical re-optimization case involves $|F_R(f)| = 7$ refined fleets for a dedicated original fleet $f \in F$. Let $f = \text{Red}$ be the original fleet and let $F_R(\text{Red})=\{\text{Red}_1, \text{Red}_2, \text{Red}_3, \text{Red}_4, \text{Red}_5, \text{Red}_6, \text{Red}_7\}$. Then, the number of possible (refined) vehicle configurations for the original vehicle configuration $\{\text{Red}, \text{Red}\}$ increases to not less than 28. In this situation, the refined vehicle configurations for $\{\text{Red}, \text{Red}\}$ read:

$$
\begin{array}{llll}
\{\text{Red}_1, \text{Red}_1\}, & \{\text{Red}_2, \text{Red}_2\}, & \{\text{Red}_3, \text{Red}_4\}, & \{\text{Red}_4, \text{Red}_7\}, \\
\{\text{Red}_1, \text{Red}_2\}, & \{\text{Red}_2, \text{Red}_3\}, & \{\text{Red}_3, \text{Red}_5\}, & \{\text{Red}_5, \text{Red}_5\}, \\
\{\text{Red}_1, \text{Red}_3\}, & \{\text{Red}_2, \text{Red}_4\}, & \{\text{Red}_3, \text{Red}_6\}, & \{\text{Red}_5, \text{Red}_6\}, \\
\{\text{Red}_1, \text{Red}_4\}, & \{\text{Red}_2, \text{Red}_5\}, & \{\text{Red}_3, \text{Red}_7\}, & \{\text{Red}_5, \text{Red}_7\}, \\
\{\text{Red}_1, \text{Red}_5\}, & \{\text{Red}_2, \text{Red}_6\}, & \{\text{Red}_4, \text{Red}_4\}, & \{\text{Red}_6, \text{Red}_6\}, \\
\{\text{Red}_1, \text{Red}_6\}, & \{\text{Red}_2, \text{Red}_7\}, & \{\text{Red}_4, \text{Red}_5\}, & \{\text{Red}_6, \text{Red}_7\}, \\
\{\text{Red}_1, \text{Red}_7\}, & \{\text{Red}_3, \text{Red}_3\}, & \{\text{Red}_4, \text{Red}_6\}, & \{\text{Red}_7, \text{Red}_7\}.
\end{array}
$$

All of these refined vehicle configurations are, indeed, considered explicitly in our computations. Therefore, our modeling trick has a dramatic impact on the size of the arising hypergraphs as well as on the fractionality of corresponding solutions of LP relaxations.

However, it turns out that these disadvantage are often mitigated substantially by the information gain that the reference solution provides, in particular when large parts do not have to be changed, which is the usual case in industry. Indeed, the resulting increase in integrality is completely paying for the increase in size, see Section 4. Of course, one must be able to deal with such very large scenarios in the first place. The algorithmic key technology that allows this is the coarse-to-fine column generation method (see Borndörfer et al (2014)) which we use in our computations.

Note that Borndörfer et al (2013) propose a similar template concept for the re-optimization of duty schedules. The main idea is to define *duty scheduling templates*, which are able to model "non-local" requirements such as the (more or less detailed) distribution of breaks in duties. To this end, a pricing problem with a dedicated graph is solved for each duty type template individually. The individual graphs allow to model re-scheduling requirements and can be seen as copies of some original graph. Then, these copies are modified in order to provide a template for some "non-local" structures. In this sense, rotation templates and duty scheduling templates are related.

3.2 Connection Templates

The connection templates modify the objective function in such a way as to recover a connection between two timetabled trips, called tail and head, in a reference rotation. They are based on a configuration routine that extracts all needed information from the reference rotations. This configuration routine iterates all connections between timetabled trips of the reference rotations. Figure 5 illustrates such a connection for a trip with train number 709 that is connected to a trip with train number 408. The configuration routine associates the following data with such a connection:

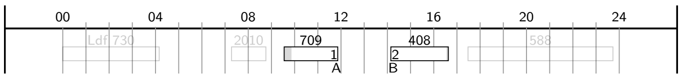

Fig. 5 Connection of a reference rotation performed by a rolling stock vehicle that is contained in a vehicle configuration. The arrival is performed with position one in orientation Tick at location A. The vehicle departs with position two and orientation Tick at location B.

– arrival and departure times,
– arrival and departure locations,
– train identifies at arrival and departure,
– fleet of the vehicle performing the connection,
– reference rotation in that the connection is contained,
– vehicle configurations at arrival, taken for the connection, and at departure,
– orientations at arrival and departure,
– positions at arrival and departure, and
– maintenance services performed during the connection.

We consider a *connection template* as a data record that is composed of all data associated with a single connection of a reference rotation.

3.3 Objective Function Configuration

This section is about the objective function $c : H \mapsto \mathbb{Q}_+$ of the re-optimization problem RSRP$'$. We configure the objective function in such a way that it reflects all aspects for greenfield and for re-optimization. The objective function for $h \in H$ consists of 9 different *sub cost functions* $c_i : H \mapsto \mathbb{Q}_+$ for $i = 1, \ldots, 9$ that correspond to 9 different objective features.

$$\begin{pmatrix} c_1(h) \\ c_2(h) \\ c_3(h) \\ c_4(h) \\ c_5(h) \\ c_6(h) \\ c_7(h) \\ c_8(h) \\ c_9(h) \end{pmatrix} \begin{array}{l} \left.\begin{array}{l} \ldots \text{ vehicles} \\ \ldots \text{ trip distance} \\ \ldots \text{ deadhead distance} \\ \ldots \text{ maintenance services} \end{array}\right\} \text{greenfield optimization} \\ \left.\begin{array}{l} \ldots \text{ deviating configurations} \\ \ldots \text{ deviating fleets} \\ \ldots \text{ deviating rotations} \\ \ldots \text{ deviating connections} \\ \ldots \text{ deviating maintenance services} \end{array}\right\} \text{rotation dependent} \end{array} \tag{7}$$

The individual value that c assigns to the hyperarc $h \in H$ is defined as:

$$c(h) := \sum_{i=1}^{9} c_i(h).$$

All objective features for greenfield optimization are also present in re-optimization scenarios. We reduced the different cost criteria to the ones that fit to the scope of the paper, nevertheless our implementation deals with additional cost criteria. For the objective features for greenfield optimization we refer to previous paper Borndörfer et al (2012).

In re-optimization scenarios it might not be beneficial to reproduce all details of a dedicated connection template in the solution to the re-optimization scenario. In fact, this can even be impossible. Our idea to reasonably carry over the data of the connection templates to the re-optimization scenario is as follows. We penalize deviations to the reference rotations for each detail and for each hyperarc individually by the objective features 5 to 9.

The connection templates are evaluated as follows. Let $h \in H$ be a hyperarc of the re-optimization hypergraph. In a first step we reinterpret h in the reference rotations, i.e., we search through all connection templates and check if h matches some details of a template. A cost value $c_i(h)$ for $i = 5, \ldots, 9$ is then set to an appropriate value if h models a deviation w.r.t. the reference rotation plan. This is possible because the hyperarcs of our hypergraph are distinguished by the same details. In particular, the hyperarcs can be distinguished by reference rotations as described in Section 3.1. If a deviation is unavoidable, e.g., if a succeeding timetabled trip is cancelled w.r.t. c_8, the cost values are clearly not affected.

We also consider departures and arrivals of the re-optimization scenario to match the connection templates of the reference rotation plan if the corresponding times and locations vary slightly. Thereby, small timetable changes, e.g., a shift of an arrival time by five minutes or a shortened timetabled trip (by, e.g., one stop located ten kilometres before the stop location in the reference rotations) does not prevent the configuration routine from recognizing desired (details of) connections of the reference rotations for re-optimization.

Examples for the adjustment of the objective function values for deviations are:

- If $h \in H$ implies that one vehicle will operate $t \in T$ in a rolling stock rotation that differs from its reference rotation (and it is possible to operate within the reference rotation) we set $c_7(h) = 1 \cdot P_7$ ($c_6(h) = 1 \cdot P_6$) where P_7, $P_6 \in \mathbb{Q}$ equals the appropriate penalty costs, otherwise $c_7(h) = 0$ ($c_6(h) = 0$).
- Let $h \in H$ a hyperarc connecting the timetabled trips $t_1 \in T$ and $t_2 \in T$. If the arrival of t_1 and the departure of t_2 exist in the reference rotations and are not connected, we set $c_8(h) = |h| \cdot P_8$ where $P_8 \in \mathbb{Q}$ denotes appropriate penalty costs.
- If $h \in H$ implies a different maintenance service before or after a timetabled trip $c_9(h) = 1 \cdot P_9$, otherwise $c_9(h) = 0$ with respective penalty costs $P_9 \in \mathbb{Q}$.

By this construction, we translate all deviations into the objective function c that is composed of $c_5, \ldots, c_9$. In this way we are able to handle many technical re-optimization details simply by changing objective function coefficients and by the refinement of the hypergraph described in Section 3.1.

4 Computational Study: The Benefit of Templates

In this section, we report on a set of computational experiments carried out to assess the modeling power of the introduced template approach implemented within the rolling stock optimizer ROTOR, see Borndörfer et al (2012). We evaluate the impact of the different templates on the model's computational tractability and on the practical quality of the produced solutions, i.e., on the modeling accuracy. The main idea of the computational study is to run model versions that do not apply connection or rotation templates for a reasonable set of real-wold instances and to compare the results of these experiments with the extended approach that uses both templates.

Our implementation makes use of the commercial mixed integer programming solver `CPLEX 12.6` as internal LP solver. All computations were performed on computers with an Intel(R) Xeon(R) CPU E31280 with 3.50 GHz, 8 MB cache, and 16 GB of RAM in multi thread mode with four cores. The algorithm presented in Borndörfer et al (2012) is used in every case.

Table 1 Key numbers of the instances

instance	trips	trip distance	compositions	fleets	maintenances	unrecognized trips	recognizable trips	$\|V\|$	$\|H\|$
RSRP_1	788	555686	2	2	8	187	22	5702	10026804
RSRP_2	788	555686	2	2	8	189	22	5702	10027556
RSRP_3	665	265657	2	1	0	198	184	7680	16465044
RSRP_4	793	430770	11	5	32	50	7	8350	33113628
RSRP_5	785	426459	11	5	32	169	120	8330	33069736
RSRP_6	53	37501	5	3	20	40	40	498	138558
RSRP_7	670	263602	3	2	0	27	27	7650	16645161
RSRP_8	670	263602	3	2	0	27	25	7642	16596517

The set of instances that have been chosen for the experiments emerge from reasonable real-world applications that are to be solved at our cooperation partner DB Fernverkehr AG. Except instance RSRP_6, which is a rather small test instance, all instances cover a big part of the German high speed railway network for different weeks and deal with infrastructure construction periods. For example instances RSRP_3, RSRP_4, and RSRP_5 are scenarios for infrastructure constructions in Frankenwald, Cologne, and Aachen. In these three cases several trips have

– increased running times up to one hour,
– additional orientation changes during the trip,
– changed origin or destination stations,
– or were cancelled.

To prepare the rolling stock rotations for these maintenance periods re-optimized solutions had to be computed.

The main characteristics, i. e., distance and number of trips, number of compositions, number of fleets, and number of vehicle maintenance types are shown in Table 1. The Columns marked with *unregognized trips* give the number of trips in the reference solution that can not be associated with a trip of the new timetable. In more detail these trips are somehow disturbed, i.e., a deviating arrival or departure time or location. *Recognizable trips* are the trips of the reference solution that deviate from the ones in the new timetable, but could be identified via a connection template. Note that each trip can only belong to two connection templates either as tail or head trip of a connection template.

For each instance we ran four experiments for each combination of with or without rotation or connection templates. Table 2 lists the number of trips that deviate from their reference rotations, i.e., trips of the optimized solution where its rotation differs from the rotation corresponding to the rotation template of the trip, in column two and seven, respectively. The next columns show the total, the rotation dependent, and the greenfield costs of the solution, and the computation time needed for the computations without using the rotation templates. Table 3 gives the numbers for the experiment with rotation templates. All computations finish with an optimality gap of less than 1%. In case of computations without the rotation template the solutions were evaluated with the objective function of the rotation template computations.

Table 2 Key numbers of the computational results with `ROTOR 2.4` and `CPLEX 12.6` without using rotation templates

	no connection templates					connection templates				
instance	rotation deviations	total costs($\cdot 10^3$)	rotation dep. costs($\cdot 10^3$)	greenfield costs($\cdot 10^3$)	computation time(in s)	rotation deviations	total costs($\cdot 10^3$)	rotation dep. costs($\cdot 10^3$)	greenfield costs($\cdot 10^3$)	computation time(in s)
RSRP_1	66	2526	205	2321	14786	66	2451	132	2319	9451
RSRP_2	40	2516	189	2327	15653	40	2442	109	2333	13396
RSRP_3	28	3939	74	3865	5661	28	3939	74	3865	4012
RSRP_4	303	6232	573	5659	16299	303	6232	573	5659	22105
RSRP_5	299	6412	650	5762	37543	299	6356	589	5767	19878
RSRP_6	28	542	79	463	10	28	520	56	464	51
RSRP_7	536	7922	1210	6712	5338	536	7916	1204	6712	4508
RSRP_8	538	7951	1225	6726	3511	538	7940	1214	6726	4091

Analyzing Table 2 reveals that using connection templates is a first step towards producing similar rotations in comparison with the given reference solution. It reduces the number of unrecognized trips which leads to an increased number of trips that are penalized in the model for deviations from the associated trips of the reference rotations. This penalization has an significant influence on the run time of the computations which can be seen in both tables in the decreasing numbers for rotation dependent costs and the algorithms runtime.

The more effective tool to preserve the reference rotations is indeed the rotation template. Its usage decreases the deviations of trips from their reference rotation significantly. Another positive or at least not negative result is that if one compares the greenfield costs, which are of coarse much closer to the real operative costs, there is no big increase or drawback of including templates or not. Surprisingly, some solutions for the runs including both types of templates had even slightly better greenfield costs than the respective solutions without considering the templates. Additionally, using the rotation templates has a positive effect on the computation time as it makes deviating rotation much more unattractive which can be seen in Table 3.

Table 3 Key numbers of the computational results of `ROTOR 2.4` and `CPLEX 12.6` with using rotation templates

	no connection templates					connection templates				
instance	rotation deviations	total costs($\cdot 10^3$)	rotation dep. costs($\cdot 10^3$)	greenfield costs($\cdot 10^3$)	computation time(in s)	rotation deviations	total costs($\cdot 10^3$)	rotation dep. costs($\cdot 10^3$)	greenfield costs($\cdot 10^3$)	computation time(in s)
RSRP_1	0	2397	80	2317	6034	0	2319	0	2319	1698
RSRP_2	0	2411	83	2328	5265	0	2336	3	2333	1775
RSRP_3	0	3884	19	3865	2052	0	3883	18	3865	1806
RSRP_4	0	5684	15	5669	6557	0	5684	15	5669	6785
RSRP_5	0	5868	71	5797	17172	0	5827	39	5788	8650
RSRP_6	0	482	19	463	46	0	464	0	464	34
RSRP_7	7	6963	231	6732	4492	7	6955	223	6732	2306
RSRP_8	6	6988	233	6755	4076	6	6974	219	6755	2087

5 Conclusion

We have shown that a hypergraph model for the optimization of rolling stock rotations can be extended to the task of re-optimization using a template concept. Using both types of templates results in a decrease of the computation time to solve instances of our test set of 77.7% and a cost reduction of 8.67% in geometric mean compared to the results without using one of the templates.

Though, it turns out that templates are a powerful concept that allow to compute cost minimal rolling stock rotations under a large variety of requirements for re-optimization scenarios appearing at DB Fernverkehr AG.

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