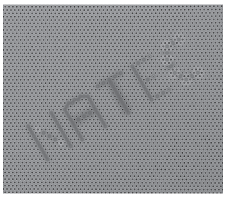
Comparison of five computational methods for computing Q factors and resonance wavelengths in photonic crystal membrane cavities



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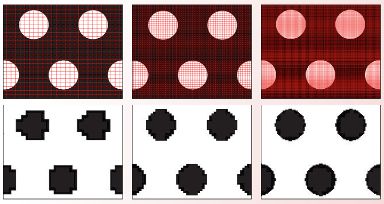
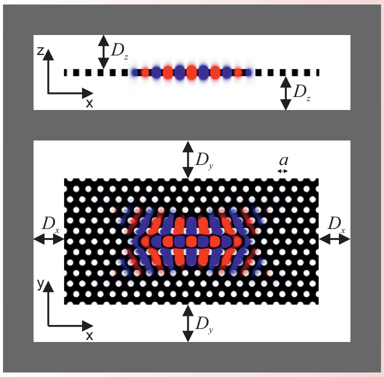
The focus of this work is the fundamental cavity mode of photonic crystal membrane L*n* cavities, for which we determine the resonance wavelength λ and the cavity quality (Q) factor for two cavity lengths using five well-established numerical simulation techniques.

- **FDTD** : Finite-Difference Time-Domain technique (Lumerical)
- **FDFD** : Finite-Difference Frequency-Domain technique (in-house code)
- **FEM1** : Finite Elements Method (JCMWave, eigenvalue analysis)
- **FEM2** : Finite Elements Method (JCMWave, scattering analysis)
- **FEM3** : Finite Elements Method (COMSOL, eigenvalue analysis)
- **FMM** : Fourier Modal Method (in-house code)
- **SIE** : Surface Integral Equation approach (in-house code)



Discretisation scheme employed in the finite-difference models.

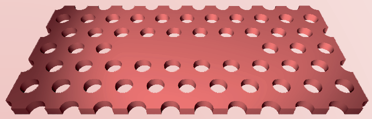
For each method we vary relevant resolution parameters described in the tables below and study the convergence of the resonance wavelength λ and the quality factor Q.



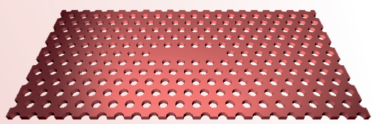
Geometry of the photonic crystal L9 cavity and optical field profile of the M1 mode.

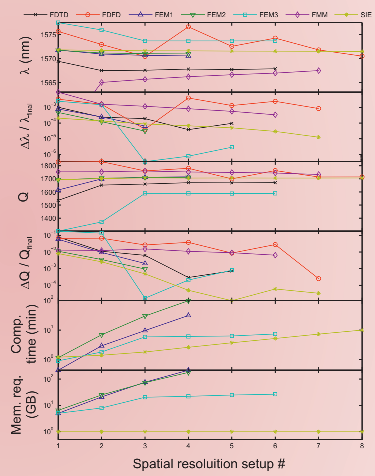
All the methods except the SIE technique employ a limited computational domain. It is thus necessary to implement perfectly matched layers (PMLs) to mimic an open geometry. We also study the convergence of λ and Q with respect to domain size setups to investigate the performance of the PMLs.

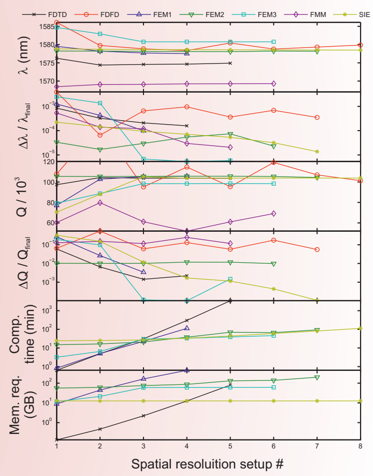
L5 cavity

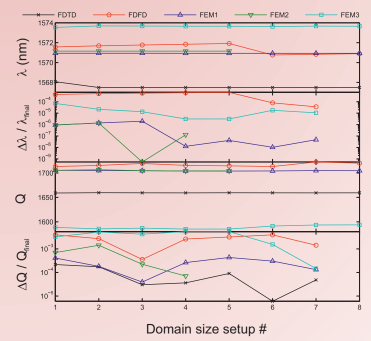


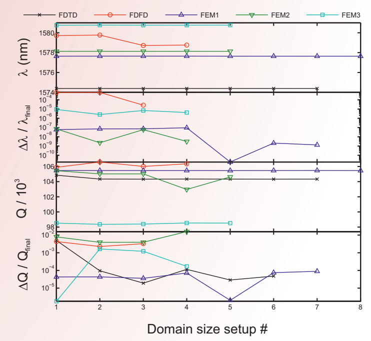
L9 cavity











The simulation methods are all fully rigourous state-of-the-art 3D simulation tools, and ideally, they should compute the same λ and Q. However, significant variations in the results are observed even for the smaller L5 cavity. Additionally to absolute values, we also plot the relative errors $\Delta\lambda / \lambda_{final}$ and $\Delta Q / Q_{final}$. Again the relative errors of the methods differ greatly with respect to variations in both the resolution and the domain size.

		FDTD	FDFD	FEM1	FEM2	FEM3	FMM	SIE
L5	λ_{final}	1567.9	1570.5	1570.5	1571.0	1573.7	1567.5	1571.5
	Q_{final}	1671.2	1714.9	1715.9	1711.5	1589.4	1733.4	1706.6
L9	λ_{final}	1574.9	1579.9	1577.5	1578.2	1580.8	1569.3	1578.5
	Q_{final}	103613	101289	105595	104387	98376	69128	103901

Table 1: Spatial resolution setup parameters for L5 cavity

Setup #	FDTD $a/\Delta x$	FDTD $\frac{\sqrt{3}}{2}a/\Delta y$	FDTD $L_z/\Delta z$	FDFD Δ/a [a]	FEM1 p	FEM1 h (nm)	FEM2 p	FEM2 h (nm)
1	13	13	10	0.094	2	150	2	130
2	26	26	20	0.087	3	150	3	130
3	52	52	40	0.080	4	150	4	130
4	104	104	80	0.073	5	150	5	130
5	208	208	160	0.066				
6	438	438	250	0.059				
7				0.052				
8				0.045				

Setup #	FEM3 Min. h (nm)	FEM3 Max. h (nm)	FMM l_y	FMM l_z	SIE IPI	SIE p	SIE g
1	400	650	10	10	0	3-1	4
2	100	550	30	10	1	3-1	4
3	25	450	35	10	2	3-1	4
4	7.5	350	40	10	3	3-1	4
5	5	300	45	10	4	3-1	4
6	2.2	220	50	10	5	3-1	4
7			60	10	6	3-1	4
8					7	3-1	4

[a] $\Delta x = \Delta y = \Delta z = \Delta$.

Table 2: Domain size setup parameters for L5 cavity

Setup #	FDTD D_x (a)	FDTD D_y $(a\sqrt{3}/2)$	FDTD D_z (a)	FDFD ΔD (a) [a]	FEM1 D (nm) [b]	FEM2 D (nm) [b]
1	6	6	0.5	0.053	50	0
2	6	6	2	0.106	100	10
3	6	6	4	0.159	200	100
4	6	6	8	0.212	300	200
5	6	6	12	0.265	400	500
6	6	6	16	0.318	500	
7			20	0.371	750	
8				0.424	1000	

Setup #	FEM3 D_x (a)	FEM3 D_y (a)	FEM3 D_z (a)
1	2	2	4
2	4	2	4
3	8	2	4
4	10	2	4
5	15	2	4
6	20	2	4
7	25	2	4
8	30	2	4

[a] $D_x = D_y = a/8 + \Delta D$. $D_z = a/2 + \Delta D$. [b] $D_x = D_y = D_z = D$.

Table 3: Spatial resolution setup parameters for L9 cavity

Setup #	FDTD $a/\Delta x$	FDTD $\frac{\sqrt{3}}{2}a/\Delta y$	FDTD $L_z/\Delta z$	FDFD Δ/a [a]	FEM1 p	FEM1 h (nm)	FEM2 p	FEM2 h (nm)
1	13	13	10	0.097	2	250	3	200
2	26	26	20	0.091	3	250	3	160
3	52	52	40	0.085	4	250	3	130
4	104	104	80	0.079	5	250	3	115
5	208	208	160	0.073			3	100
6				0.067			3	90
7				0.061			3	80
8				0.055				

Setup #	FEM3 Min. h (nm)	FEM3 Max. h (nm)	FMM l_y	FMM l_z	SIE IPI	SIE p	SIE g
1	400	650	30	10	0	3-1	4
2	100	550	30	14	1	3-1	4
3	25	450	30	16	2	3-1	4
4	7.5	350	30	18	3	3-1	4
5	5	300	30	20	4	3-1	4
6	2.2	220	30	24	5	3-1	4
7					6	3-1	4
8					7	3-1	4

[a] $\Delta x = \Delta y = \Delta z = \Delta$.

Table 4: Domain size setup parameters for L9 cavity

Setup #	FDTD D_x (a)	FDTD D_y $(a\sqrt{3}/2)$	FDTD D_z (a)	FDFD ΔD (a) [a]	FEM1 D (nm) [b]	FEM2 D (nm) [b]
1	3	3	1	0	50	0
2	3	3	3	0.06	100	10
3	3	3	5	0.12	200	100
4	3	3	7	0.18	300	200
5	3	3	10		400	500
6	3	3	14		500	
7			18		750	
8					1000	

Setup #	FEM3 D_x (a)	FEM3 D_y (a)	FEM3 D_z (a)
1	2	2	4
2	2	2	6
3	2	2	8
4	2	2	10
5	2	2	12

[a] $D_x = D_y = a/8 + \Delta D$. $D_z = a/2 + \Delta D$. [b] $D_x = D_y = D_z = D$.

Jakob Rosenkrantz de Lasson et al, Optics Express, in preparation.