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Designing Inspector Rosters with Optimal Strategies

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Abstract

We consider the problem of enforcing a toll on a transportation network with limited inspection resources. We formulate a game theoretic model to optimize the allocation of the inspectors, taking the reaction of the network users into account. The model includes several important aspects for practical operation of the control strategy, such as duty types for the inspectors. In contrast to a formulation in [1] using flows to describe the users' strategies we choose a path formulation and identify dominated user strategies to significantly reduce the problem size. Computational results suggest that our approach is better suited for practical instances.

1 Introduction

In the past years, a lot of work has been done in the application of game theoretic models to real-world security problems. These applications range from airport security [4] over protection of wildlife reserves [4] to toll (or fare) control in transportation networks [1, 3]. In [4], the authors give an overview on projects with security games where mostly no network structures are considered. Models for fare evasion in public transport are studied in [3], but the work is focussed more on theoretical results than on practical issues. For practical operation it is important to include the notion of duties for inspection units and the concept of control areas as subparts of the network where controls can be conducted. Both of these extensions are taken into account in [1] where a game theoretic formulation for the enforcement of a toll on a transportation network is studied.

In this paper we reformulate the toll enforcement problem of [1]. By identifying dominated user strategies, we significantly reduce the size of the presented MIP and LP formulations to compute the control strategy in a Stackelberg and Nash equilibrium, respectively. Computational results for real-world instances show that the new approach outperforms the existing formulation.

2 The Toll Enforcement Game

We consider a user network $G_0 = (V_0, E_0)$ with nodes V_0 and directed arcs E_0 with costs $c_e \geq 0$. For a given time interval, typically one day or one week, we consider an equidistant time discretization $\mathcal{T} = \{0, \ldots, T-1\}$. A time-expanded graph $G = (V, E)$ is constructed by adding a copy of G_0 for every time window $t \in \mathcal{T}$. In addition, we are given k commodities $(s_i, t_i, d_i) \in V \times V \times \mathbb{N}$ describing the number of users d_i travelling from s_i to t_i. We make the simplifying assumption that every network user starts and ends his trip within the same time window. Every user going from s_i to t_i is supposed to pay a toll (or fare) of τ_i. In contrast to the driving costs c_e on arc e, the users can decide not to pay the toll and risk a fine $f \gg \tau_i$ if caught evading. In order to enforce the toll, a number of κ inspection units can be allocated throughout the network. However, the possible distributions of the inspectors are subject to a number of spatial, temporal and legal constraints which will be specified later. In the following, we describe a game between the network users and the inspectors concerning the users' payment of the toll. While there is one player for every origin-destination pair (s_i, t_i), the inspectors are aggregated as one player choosing a joint control strategy.

Users' strategies: The set Σ_i of pure strategies of player i can be divided into toll paying strategies Σ_i^{pay} and toll evading strategies Σ_i^{ev}. If we consider the user network as the toll evading network where no toll is paid, we have

$$\Sigma_i^{ev} = \{P \mid P \text{ is an } s_i\text{-}t_i\text{-path in } G\}.$$

If player i decides to pay the toll τ_i she will take a shortest s_i-t_i-path with respect to the travel costs c. Considering the payoff functions we can assume that there is a single toll paying strategy for player i and we write $\Sigma_i^{pay} = \{\sigma_i^{pay}\}$.

With the mixed strategy $x^i = (x_0^i, x_1^i, \ldots, x_{k_i}^i)$ we say that player i commits to σ_i^{pay} with probability x_0^i and to $P_j^i \in \Sigma_i^{ev}$ with probability x_j^i. The joint strategy of the users is denoted by $x = (x^1, \ldots, x^k)$.

We would like to point out that [1] uses an equivalent formulation which describes the toll paying strategy of player i as an s_i-t_i-path in an adopted user network. Consequently, every mixed strategy of player i can be seen as an s_i-t_i-flow of unit value in this network.

Inspector's strategies: The spatio-temporal allocation of the inspectors is done by assigning duties to control areas. A duty can start at the beginning of every time window $t \in \mathcal{T}$ and is scheduled for a fixed number L of consecutive time windows. The control takes place on given *control areas* $\mathcal{A} = \{a_1, \ldots, a_m\}$ with a predefined adjacency of control areas $A' \subseteq \mathcal{A}^2$. For every part $l = 1, \ldots, L$ of the duty the inspector can switch from a_i to a_j iff $(a_i, a_j) \in A'$.

We define the set $\mathcal{D}$ of *control duties* to be

$$\mathcal{D} := \left\{ \left(t, (a^1, a^2, \ldots, a^L)\right) \in \mathcal{T} \times \mathcal{A}^L \;\middle|\; (a^i, a^{i+1}) \in A' \right\}.$$

The set of the inspector's pure strategies can then be described as $\{C \subseteq \mathcal{D} \mid |C| \leq \kappa\}$.

In the following, we construct a duty graph $D = (W,\ A)$ to obtain a more elegant representation of the inspector's strategy set Σ_{insp}. For every time window t, every duty part l and every control area s_i we have a *control node* $(t, l, a_i) \in W$. If $l < L$ we introduce an arc $\big((t, l, a_i), (t, l+1, a_j)\big)$ iff $(a_i, a_j) \in A'$. Now we add additional nodes t^s and t^t for every time window t and insert arcs $\big(t^s, (t, 1, a_i)\big)$ and $\big((t, L, a_i), t^t\big)$ for every $a_i \in \mathcal{A}$. Finally, we introduce a super source d^s and a super sink d^t and arcs (d^s, t^s) and (d^t, t_d) for all $t \in \mathcal{T}$.

We can observe that there is a one-to-one correspondence between control duties and d^s-d^t-paths in D. The set of strategies for the inspection player can thus be formulated as

$$\Sigma_{insp} := \{p \mid p \text{ is a } d^s\text{-}d^t\text{-flow of value} \leq \kappa \text{ in } D\}.$$

For a given strategy $p \in \Sigma_{insp}$, the control intensities $q = (q_e)$ on arcs E of the user network G can be obtained by a given linear transformation, i.e. $q = Tp$. The induced control intensity q_e can be interpreted as the expected number of controls on arc $e \in E$. We follow the notation of [1] and define the set $\mathcal{Q}$ of induced control intensities q on G to be

$$\mathcal{Q} := \{Tp \mid p \in \Sigma_{insp}\}.$$

Payoffs: While the inspection player wants to maximize his total income, the users aim to minimize their total costs consisting of travel costs and toll costs or expected fine. The travel costs of player i choosing strategy $\sigma \in \Sigma_i$ are denoted by c_i^σ. If $\sigma = \sigma_i^{pay}$ then c_i^σ is the length of a shortest s_i-t_i-path with respect to c. For $\sigma = P \in \Sigma_i^{ev}$ we have $c_i^\sigma = \sum_{e \in P} c_e$.

If player i chooses strategy σ_i^{pay}, the player's and inspector's payoffs are independent of the chosen control strategy $p \in \Sigma_{insp}$. Then, the total costs of player i are

$$-\pi_i(p, \sigma_i^{pay}) := c_i^{\sigma_i^{pay}} + \tau_i,$$

while the inspector's profit from player i in this case is $\pi_{insp}^i(p, \sigma_i^{pay}) := \tau_i$.

Let us now assume, that player i chooses the evading strategy $P \in \Sigma_i^{ev}$ while the inpector plays $p \in \Sigma_{insp}$. With the induced control intensities $q = Tp$ on G we have $-\pi_i(p, P) := c_i^P + \sum_{e \in P} f q_e$ where the first term accounts for travel costs while the second term is the expected fine. Accordingly, the inspector's gain from player i is $\pi_{insp}^i(p, P) := \sum_{e \in P} f q_e$. Note that we use a simplified formula for the expected fine where we assume that evaders can be fined several times. However, our results show that the probability of being controlled more than once is very small for a reasonable number of controllers. With the above formula we also assume that the payoff for player i does not depend on the actions of the other users as we take no congestion effects into account. Given the control strategy p and the joint users' strategy $x = (x^1, \ldots, x^k)$, we have

$$\pi_i(p, x^i) = x_0^i\, \pi_i(p, \sigma_i^{pay}) + \sum_{j=1}^{k_i} x_j^i\, \pi_i(p, P_j^i)$$

$$\text{and} \quad \pi_{insp}(p, x) = \sum_{i=1}^{k} \left(x_0^i\, \pi_{insp}^i(p, \sigma_i^{pay}) + \sum_{j=1}^{k_i} x_j^i\, \pi_{insp}^i(p, P_j^i) \right).$$

We denote by $BR_i(p)$ the set of best responses of player i to the control strategy p, i.e. $BR_i(p) := \arg\max_{x^i} \pi_i(p, x^i)$.

3 Computing Equilibria

Stackelberg Equilibrium: In most security games and fare evasion models the classical concept of Stackelberg equilibria is applied. A Stackelberg game is a bilevel game where the players are divided into leaders and followers. First, each leader (in our case the inspection player) commits to a strategy, then the followers choose a strategy after observing the leaders' strategy. Let p be a control strategy and x be a joint strategy of the users, then

$$(p,x) \text{ is a strong Stackelberg equilibrium } :\iff (p,x) \in \operatorname*{arg\,max}_{(\tilde{p},\tilde{x})\,:\,\tilde{x}^i \in BR_i(\tilde{p})} \pi_{insp}(\tilde{p},\tilde{x}).$$

Note that the notion of strong Stackelberg equilibria implies that the followers break ties in favor of the leader. As a consequence, we only need to consider pure strategies of the followers [2]. While the existence of a strong Stackelberg equilibrium is always guaranteed, the respective optimization problem is NP-hard in general [2].

In the following we present a mixed integer program (MIP) to compute a leader strategy of a Stackelberg equilibrium for the toll enforcement game.

$$\max_{q,y,\mu} \quad \sum_i d_i \left(y_i - \sum_{\sigma \in \Sigma_i} \mu_i^\sigma \, c_i^\sigma \right) \tag{1a}$$

$$\text{s.t.} \quad 0 \le c_i^{\sigma_i^{pay}} + \tau_i - y_i \le M \left(1 - \mu_i^{\sigma_i^{pay}}\right) \qquad \forall i \tag{1b}$$

$$0 \le c_i^P + \sum_{e \in P} f q_e - y_i \le M \left(1 - \mu_i^P\right) \qquad \forall P \in \Sigma_i^{ev} \; \forall i \tag{1c}$$

$$\sum_{\sigma \in \Sigma_i} \mu_i^\sigma = 1 \qquad \forall i \tag{1d}$$

$$\mu_i^\sigma \in \{0,1\} \qquad \forall \sigma \in \Sigma_i \; \forall i \tag{1e}$$

$$q \in \mathcal{Q} \tag{1f}$$

The objective (1a) is to maximize the inspector's income. This can be done by considering the total costs y_i of an optimal strategy of player i subtracted by her travel costs. The costs y_i are bounded from above by the costs of the toll paying strategy (1b) and the costs of any evasion strategy (1c). The binary variable μ_i^σ indicates if $\sigma \in \Sigma_i$ is a best response to the control q. Constraints (1b) and (1c) also guarantee that $\mu_i^\sigma = 0$ if σ is not a best response for player i. Equation (1d) and the second term in the objective function make sure that each follower breaks ties in favor of the leader. Finally, in (1f) we force q to be induced by a control flow $p \in \Sigma_{insp}$.

Nash Equilibrium: We also study the Nash equilibria of the toll enforcement game which can be derived for the present case as follows:

$$(p,x) \text{ is a Nash equilibrium } :\iff p \in \operatorname*{arg\,max}_{\tilde{p} \in \Sigma_{insp}} \pi_{insp}(\tilde{p},x) \text{ and } x^i \in BR_i(p).$$

The existence of a Nash equilibrium in the toll enforcement game is guaranteed and an optimal strategy for the inspection player can be computed by linear programming due to the following important result from [1]: Let x be a joint mixed strategy for the users, then

$$p \in \underset{\tilde{p}\in\Sigma_{insp}}{\arg\max}\ \pi_{insp}(\tilde{p}, x^i) \iff p \in \underset{\tilde{p}\in\Sigma_{insp}}{\arg\max} \sum_{i=1}^{k} -\pi_i(\tilde{p}, x^i).$$

Therefore, the inspection player aims to maximize the costs of the users in a Nash equilibrium and his optimal strategy can be computed with the following linear program (LP):

$$\begin{aligned}
&\max_{q,r} && \sum_i d_i\, r_i && && \text{(2a)}\\
&\text{s.t.} && r_i \le c_i^{\sigma_i^{pay}} + \tau_i && \forall i && \text{(2b)}\\
& && r_i \le \sum_{e\in P} c_e + f q_e && \forall P \in \Sigma_i^{ev}\ \forall i && \text{(2c)}\\
& && q \in \mathcal{Q} && && \text{(2d)}
\end{aligned}$$

Due to (2a) the inspection player aims to maximize the total costs of the users. The costs for player i described by r_i are bounded from above by the costs of the toll paying strategy (2b) and also by the costs of her evading strategies (2c). Again, we force q to be induced by a control flow $p \in \Sigma_{insp}$ (2d).

Dominated strategies: The number Σ_i^{ev} of toll evading strategies for player i is potentially huge compared to the size of the network. It is well known that the number of paths in a graph can be exponential in the number of edges. To avoid a potentially great number of constraints (1c) and (2c) the authors of [1] use a flow formulation to describe the users' strategies.

In practice however, user networks are normally sparse and there are not a huge number of possible user paths, especially if we exclude dominated strategies. In those networks, the travel costs represent the largest share of the user's total costs while toll costs or expected fines are secondary. As a result, the travel costs of most s_i-t_i-paths exceed the toll paying costs of $c_i^{\sigma_i^{pay}} + \tau_i$. A great number of strategies $P_j^i \in \Sigma_i^{ev}$ are thus dominated by the honest strategy σ_i^{pay}.

We use a preprocessing algorithm to compute the honest costs for every player i and apply a modified version of Yen's k-shortest path algorithm [5] to find the s_i-t_i-paths in G with length $\le c_i^{\sigma_i^{pay}} + \tau_i$ and thereby build the set Σ_i^{ev}.

4 Computational Results

We applied the presented approaches to three real-world instances of the German motorway network. The instances were provided by the federal office for goods transport who is responsible for the truck toll enforcement on German motorways. The commodities are based on historical data and we schedule the duties for an exemplary

Table 1: Computation of the inspector's strategy in a Nash equilibrium for three real-world instances with $|\mathcal{T}| = 168$ and $L = 2$. We compare the flow formulation of (2) taken from [1] to the presented path formulation with non-dominated strategies. Computation time includes preprocessing, building and solving time, RAM shows the maximum memory usage during the computation.

instance	$\lvert V_0\rvert$	$\lvert E_0\rvert$	k	# rows in reduced LP	# columns in reduced LP	computation time in s	RAM
I1_flow	112	220	118 917	929 445	510 453	482	4.6 GB
I1_paths				67 299	75 194	29	0.4 GB
I2_flow	196	394	220 204	2 997 920	1 569 627	17 095	23.0 GB
I2_paths				157 870	167 317	214	3.7 GB
I3_flow	319	672	365 603	7 593 778	3 718 269	–	killed
I3_paths				270 799	235 759	338	7.7 GB

week with 4-hour time windows and duties with two parts. The optimization was run on a Linux PC (3.6 GHz, 8 cores, 32 GB RAM) and we used CPLEX as an LP and MIP solver.

We also computed Stackelberg equilibria for the above instances using the path formulation in the MIP (1). The computation time and RAM usage were similar to the respective Nash equilibria. Noting that the computation of a Stackelberg equilibrium is at least as hard as computing a Nash equilibrium, we expect the results from Table 1 to carry over.

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