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Triangulated Manifolds with Few Vertices: Vertex-Transitive Triangulations I

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ZIB-Report 06-09 (März 2006)

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The computer enumeration of triangulated surfaces and combinatorial 3-manifolds was started by Altshuler and Altshuler & Steinberg in the early and mid seventies of the twentieth century: They explicitely enumerated all combinatorial 3-manifolds with up to 9 vertices [2], [7], [8], [9], all neighborly combinatorial 3-manifolds with 10 vertices [4], and all neighborly triangulated orientable surfaces with 12 vertices [5].

A conceptually slightly different enumeration algorithm for neighborly combinatorial 3-manifolds with a *vertex-transitive* cyclic or dihedral group action was presented by Kühnel and Lassmann [52] in 1985. With an implementation of their algorithm they enumerated all vertex-transitive neighborly 3-manifolds with a cyclic action for up to 15 vertices and with a dihedral action for up to 19 vertices.

Further enumerations results for 4-manifolds were obtained by Kühnel and Lassmann [50] (on the uniqueness of Kühnel's 9-vertex triangulation of the complex projective plane [49]), by Lassmann and Sparla [57] (on centrally symmetric triangulations of $S^2 \times S^2$ with 12 vertices), and by Casella and Kühnel [25] (on the existence of a 16-vertex triangulation of the K3-surface).

In general, however, there is no algorithm to enumerate combinatorial manifolds of dimension $d \geq 6$: By fundamental work of Novikov (cf. [82]), there is no algorithm to recognize (combinatorial) triangulations of spheres of dimension $d-1 \geq 5$, which would be needed to determine whether vertex-links are spheres (see [56] for a discussion).

In this paper, we describe an algorithm for the enumeration of (candidates of) vertex-transitive combinatorial d-manifolds. With a GAP implementation, MANIFOLD_VT [61], of our algorithm, we determine, up to combinatorial equivalence, all combinatorial manifolds with a vertex-transitive automorphism group on $n \leq 13$ vertices. With the exception of actions of groups of small order, the enumeration is extended to 14 and 15 vertices.

Our enumeration algorithm is, in part, based on the algorithm by Kühnel and Lassmann. Improvements and variants of our algorithm are used to enumerate all vertex-transitive triangulations of 3-manifolds with 16 and 17 vertices and all vertex-transitive neighborly surfaces with up to 22 vertices [69], centrally symmetric triangulations with a vertex-transitive cyclic action [65], vertex-transitive combinatorial pseudomanifolds [67], all triangulated surfaces with 9 and 10 vertices [64], and all combinatorial 3-manifolds with 10 vertices [58].

1 The Enumeration Algorithm

The aim of this section it to describe a basic algorithm for the enumeration of candidates for vertex-transitive combinatorial (respectively simplicial) manifolds as well as heuristical steps to further analyze these candidates.

The procedure consists of seven steps. In Step 1, the input parameters have to be fixed: the number of vertices n, the dimension d, and the vertex-transitive group action n^i on n vertices. In Step 2, all candidates for simplicial d-manifolds with n vertices are generated that are invariant under the transitive group action n^i. These candidates are tested heuristically whether they are manifolds in Step 3, then classified up to combinatorial equivalence in Step 4, and examined further in Steps 5–6. If they are combinatorial manifolds, then in Step 7 we heuristically try to determine their topological types.

Step 1: Fix the number of vertices $n \geq 4$ *and the dimension* $2 \leq d \leq n-2$.

Before starting with the enumeration of vertex-transitive triangulated d-manifolds with n vertices we need to know all vertex-transitive group actions on the given number n of vertices. The transitive permutation groups of small degree $n \leq 31$ were classified by Miller [72], [73], Butler and McKay [24], Royle [76], Butler [23], Conway, Hulpke, and McKay [26], and Hulpke [40], [41]; see Table 1 for the numbers of distinct actions that occur. In general, a finite group can have different group actions on n vertices. The respective permutation groups then are non-isomorphic as permutation groups but isomorphic as finite groups. If n is prime, then the corresponding transitive permutation groups are primitive. Generators for all the transitive permutation groups of degree $n \leq 31$ are available via the transitive permutation group library of the computer algebra package GAP [37].

For every fixed pair of n and d, we treat the corresponding transitive group actions in decreasing group order. The two transitive group actions on n vertices of largest group order are the actions of the symmetric group S_n and of the alternating group A_n. Both groups are transitive on the unordered $(d+1)$-subsets of the set $\{1,\dots,n\}$ for all $1 \leq d \leq n-2$. Since the only d-manifold on n vertices, invariant under one of these two actions, is the $(n-2)$-sphere S^{n-2} triangulated as the boundary of the $(n-1)$-simplex

Table 1: The number of transitive group actions on $n \leq 31$ vertices.

n	4	5	6	7	8	9	10	11	12	13	14	15	16	17
#	5	5	16	7	50	34	45	8	301	9	63	104	1954	10

n	18	19	20	21	22	23	24	25	26	27	28	29	30	31
#	983	8	1117	164	59	7	25000	211	96	2392	1854	8	5712	12

with $n = d + 2$ vertices, we can start for $n > d + 2$ with the next smaller permutation group from the list of group actions for the respective n.

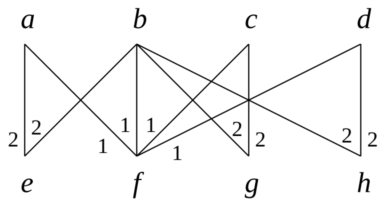
Let n^i be the i-th transitive permutation group on n vertices from the GAP library and let ${}^d n^i$ be its induced action on the $(d + 1)$-subsets of the set $\{1, \ldots, n\}$.

Every time that our enumeration algorithm produces a new candidate for a combinatorial manifold in Step 2, we test in Step 4 whether we have found a combinatorially equivalent candidate before. If not, then the combinatorial automorphism group of our candidat is the current permutation group n^i. (Since we proceed with decreasing group order, the order of the automorphism group of our candidate cannot be larger: All examples with a larger vertex-transitive group action have been enumerated before.)

Step 2 (Enumeration): Determine all pure d-dimensional simplicial complexes on n vertices that have the pseudomanifold property and that are invariant unter the vertex-transitive group action n^i.

The 'candidates' for vertex-transitive combinatorial manifolds with n vertices that we are going to build are pure d-dimensional simplicial complexes M that are invariant under the group action n^i. The collection of facets of every such M is a union of orbits of $(d+1)$-tuples with respect to the induced action ${}^d n^i$ of the permutation group n^i on the set of $(d+1)$-subsets of the ground set $\{1, \ldots, n\}$.

In addition, we require that M has the *pseudomanifold property*, that is, every $(d-1)$-dimensional face of M must be contained in *precisely two* d-dimensional facets. By transitivity, we say that an orbit of $(d-1)$-dimensional faces is *included t times* in an orbit of d-dimensional facets if each $(d-1)$-dimensional member of the first orbit is included in t sets of the latter orbit. If there is a $(d-1)$-dimensional orbit that is included three or more times in a d-dimensional orbit, then this d-dimensional orbit cannot be used for composing M, since this would violate the pseudomanifold property. In a preprocessing step we sort out all these d-orbits. It then can



happen that there are some $(d-1)$-orbits that are not included (or included only once) in any (in one) of the remaining d-orbits. We sort out these $(d-1)$-orbits (and the d-orbits containing these $(d-1)$-orbits) as well and iterate this procedure as long as possible.

We next associate a weighted bipartite graph with the remaining d- and $(d-1)$-orbits as nodes and an edge of weight t between two nodes whenever a $(d-1)$-orbit is included t-times in a d-orbit. Let us, for example, consider the action 7^2 of the dihedral group D_7 on $n = 7$ vertices and let $d = 3$. There are four 3-dimensional orbits

$$\begin{array}{rl}
a: & 1234, 2345, 3456, 4567, 1567, 1237, 1267 \\
b: & 1235, 2346, 2456, 3457, 1345, 3567, 1456, \\
 & 2347, 1467, 1247, 2567, 1236, 1257, 1367 \\
c: & 1245, 2356, 3467, 1457, 1347, 1256, 2367 \\
d: & 1246, 2357, 1356, 1346, 2457, 2467, 1357
\end{array}$$

(where 1234 denotes the tetrahedron with vertices 1, 2, 3, and 4, etc.) and four 2-dimensional orbits

$$\begin{array}{rl}
e: & 123, 234, 456, 345, 567, 167, 127 \\
f: & 124, 235, 356, 346, 245, 467, 457, \\
 & 134, 157, 137, 156, 237, 126, 267 \\
g: & 125, 236, 256, 347, 145, 367, 147 \\
h: & 135, 246, 357, 146, 247, 257, 136
\end{array}$$

with associated weighted graph

For composing a vertex-transitive pure d-dimensional simplicial complex with the pseudomanifold property we have to form combinations of d-orbits, such that the resulting total weight of (the incident edges of) every contained (d–1)-orbit is exactly two. To find such combinations fast, we have to choose appropriate data structures for the enumeration. If the acting group G has small group order, i.e., $|G| = m \cdot n$ with m small, then the corresponding weighted bipartite graph will be sparse. Thus we best use adjacency lists to represent the graphs. Since the graphs are bipartite, the resulting lists can be displayed for every graph in form of a matrix (with missing entries) that has a row for every d-orbit and a column for every $(d-1)$-orbit. For the above graph the matrix is

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} e & f & g & h \end{array} \\ \left(\begin{array}{cccc} 2 & 1 & & \\ 2 & 1 & 2 & 2 \\ & 1 & 2 & \\ & 1 & & 2 \end{array} \right) \end{array}$$

In terms of this associated matrix, the problem of finding pure vertex-transitive simplicial complexes with the pseudomanifold property translates to finding all combinations of row vectors such that their vector sum has entries 0 or 2 only. Missing entries 'contribute' 0 in the summation and therefore can be neglected in the computation. In our implementation we determine the valid combinations via *backtracking*.

We group the rows of the matrix in *blocks*, such that the rows of every block have their first non-zero entry at the same position. If we assume that the corresponding orbits of facets were ordered lexicographically, then the rows of the first block have their first non-zero entry at the first position, and from block to block the position of the first non-zero entry of the respective rows increases. The above matrix has two blocks.

We start the backtracking with the zero row-vector as *current sum vector* and introduce a *pointer* that points to the next row-vector that is to be added to the current sum vector. Initially, the pointer is set to the first row. When the first row has been added to the zero vector the pointer is set to the second row as the next row to be added, if possible: As soon as the current sum vector has 2 as its first entry, then no further row from the first block can be added without violating the pseudo-manifold property. Thus we can set the pointer to the first row of the next block of the matrix, etc.

As soon as the current sum vector is *closed*, i.e., has entries 2 or 0 only, the corresponding combination of d-orbits gives a vertex-transitive pure simplicial complex with the pseudomanifold-property and therefore a candidate for a vertex-transitive simplicial manifold. If we would add further rows of the matrix to a closed vector, then we might eventually obtain another closed vector that is the sum of two closed vectors. However, the corresponding simplicial complex then is *not strongly connected* (i.e., there is a pair of facets that cannot be joined by a path which moves from facet to facet only across $(d-1)$-faces), and therefore cannot be a connected manifold. To avoid this, we set the pointer to END.

Whenever the pointer points to END, then in the following step we go one level up in the backtracking tree (by subtracting the last row of the combination from the current sum vector) and set the pointer to the next row after the deleted row. If the deleted row was the last row of the matrix, then the pointer is set to END another time and we go up one level further. We also set the pointer to END when after a summation at least one entry of the sum vector is larger than two: such sum vectors are *invalid*.

One further case to set the pointer to END is when the current sum vector has an entry 1 at its, say, k-th position and the current pointer points to a row that has missing entries at its positions 1 to k: such a sum vector can never be completed to a closed vector by adding rows that come after the current row. (We use an auxiliary variable to keep track of the position of the first non-zero entry of the row to which the pointer currently points to.)

For the above matrix we get the following sequence of combinations:

$-:$	$(\ 0\ \ 0\ \ 0\ \ 0\ )$	Set pointer to a.
$a:$	$(\ 2\ \ 1\ \ 0\ \ 0\ )$	First entry is 2: set pointer to c.
$a+c:$	$(\ 2\ \ 2\ \ 2\ \ 0\ )$	*Candidate!* Set pointer to END.
$a:$	$(\ 2\ \ 1\ \ 0\ \ 0\ )$	Set pointer to d
$a+d:$	$(\ 2\ \ 2\ \ 0\ \ 2\ )$	*Candidate!* Set pointer to END.
$a:$	$(\ 2\ \ 1\ \ 0\ \ 0\ )$	Set pointer to END.
$-:$	$(\ 0\ \ 0\ \ 0\ \ 0\ )$	Set pointer to b.
$b:$	$(\ 2\ \ 1\ \ 2\ \ 2\ )$	Set pointer to c.
$b+c:$	$(\ 2\ \ 2\ \ 4\ \ 2\ )$	Invalid combination! Set pointer to END.
$b:$	$(\ 2\ \ 1\ \ 2\ \ 2\ )$	Set pointer to d.
$b+d:$	$(\ 2\ \ 2\ \ 2\ \ 4\ )$	Invalid combination! Set pointer to END.
$b:$	$(\ 2\ \ 1\ \ 2\ \ 2\ )$	Set pointer to END.
$-:$	$(\ 0\ \ 0\ \ 0\ \ 0\ )$	Set pointer to c.
$c:$	$(\ 0\ \ 1\ \ 2\ \ 0\ )$	Set pointer to d.
$c+d:$	$(\ 0\ \ 2\ \ 2\ \ 2\ )$	*Candidate!* Set pointer to END.
$c:$	$(\ 0\ \ 1\ \ 2\ \ 0\ )$	Set pointer to END.
$-:$	$(\ 0\ \ 0\ \ 0\ \ 0\ )$	Set pointer to d.
$d:$	$(\ 0\ \ 1\ \ 0\ \ 2\ )$	Set pointer to END.
$-:$	$(\ 0\ \ 0\ \ 0\ \ 0\ )$	Set pointer to END.

Thus, for the above example there are three valid combinations, $a+c$, $a+d$, and $c+d$, which are further examined in Step 3 and Step 4.

Step 3 (Combinatorial Tests): Remove complexes from the list of candidates that cannot be manifolds.

Pure simplicial complexes with the pseudomanifold property can be regarded as the most general form of pseudomanifolds, as they comprise proper *combinatorial manifolds* (on which we will concentrate in the following) as well as *combinatorial pseudomanifolds*, in particular, *Eulerian* *manifolds* (see [59, Ch. 3]). For every simplicial complex that we found in Step 2, we perform simple tests to exclude complexes that cannot be connected manifolds.

We first test whether the candidate complex is connected: For example, the 1-dimensional complex consisting of the edges 13, 15, 24, 26, 35, and 46 is invariant under the cyclic shift $(1, 2, 3, 4, 5, 6)$ and thus is a vertex-transitive pure simplicial complex with the pseudomanifold property. However, it is not connected: it is the union of two disjoint (empty) triangles. In order that a connected simplicial complex is a combinatorial manifold the link of any proper face has to be a combinatorial sphere. Two necessary conditions for this are that the links are connected and have the Euler characteristic of a sphere. In our implementation, we test these conditions for the link of one vertex v_0 (transitivity), for the link of every edge containing v_0 if $d \geq 3$, and for the link of every triangle containing v_0 if $d \geq 4$. (If the number of vertices is restricted to $n \leq 15$ it is, in most cases, not necessary, but expensive to test the links of higher-dimensional faces.)

These tests have to be altered only slightly if we want to enumerate all vertex-transitive Eulerian manifolds or all vertex-transitive combinatorial pseudomanifolds for a given vertex-transitive group action; see [59, Ch. 3].

Step 4 (Combinatorial Equivalence): Remove complexes from the list of candidates that, up to combinatorial equivalence, have appeared before.

We next classify, up to *combinatorial equivalence* (i.e., up to relabeling the vertices), the candidates which survived Step 3. Two basic combinatorial invariants are particularly helpful for this: the f-vector and the *Altshuler-Steinberg determinant* [7] of a candidate, i.e., the determinant $\det(AA^T)$ of the vertex-facet incidence matrix A of the candidate complex. Clearly, $\det(AA^T)$ is invariant under relabeling vertices or facets.

If the f-vectors and the Altshuler-Steinberg determinants of two candidate complexes coincide, then one possibility for a combinatorial equivalence between these two vertex-transitive complexes is that they are mapped onto each other by an outer automorphism of the acting group. Since many group actions have the cyclic group $\mathbb{Z}_n$, generated by the cycle $(1, 2, 3, \ldots, n)$, as a transitively acting subgroup, we restrict our attention to *multiplications* $k \mapsto (m \cdot k) \bmod n$ with $m \in \{1, 2, 3, \ldots, (n-1)\}$ and $\gcd(m, n) = 1$.

In the above example, the generating simplices of the orbits a, b, c, and d are $1234_{\,7}$, $1235_{\,14}$, $1245_{\,7}$, and $1246_{\,7}$, respectively (*the lower index indicates the size of the corresponding orbit*). The union of orbits $a+c$ is mapped to $a+d$ by multiplication with 2, and to $c+d$ by multiplication with 3. Thus, there is, up to combinatorial equivalence, a unique combinatorial 3-manifold with

7 vertices and vertex-transitive D_7-action. This manifold is (of course) the boundary complex $\partial C_4(7)$ of the cyclic 4-polytope $C_4(7)$ with 7 vertices.

If the f-vectors and Altshuler-Steinberg determinants of two candidate complexes are equal, but the complexes are not multiplication isomorphic, then we take one simplex of the first complex and test for all possible ways it can be mapped to the generating simplices of the orbits of the second complex whether this map can be extended to a simplicial isomorphism of the two complexes. (By strong connectivity, a combinatorial isomorphism between two simplicial manifolds is already determined by its action on one simplex.)

Alternatively, one can use McKay's (fast!) graph isomorphism testing program `nauty` [71] to determine whether the vertex-facet incidence graphs of the two complexes are isomorphic or not.

Steps 2 to 4 (Integration of the Combinatorial Steps): Whenever we find a candidate complex in Step 2, we immediately perform Steps 3 to 4 for this complex before we continue with the backtracking of Step 2.

An integration of Steps 2 to 4 has the advantage that we do not need to store complexes that would be ruled out by Steps 3 to 4 all along the backtracking. In fact, the basic combinatorial tests of Step 3 are sufficient to reject most of the candidates that are not manifolds (at least for vertex-transitive triangulations with $n \leq 15$ vertices). If $d \leq 3$, then all the resulting complexes after Step 3 are indeed manifolds: The vertex-links in a combinatorial 2-manifold are circles, whereas the vertex-links in a triangulated 3-manifold are combinatorial 2-spheres. These are recognized by the combinatorial tests of Step 3.

Our GAP-program MANIFOLD_VT [61] is an implementation of the Steps 1 to 4 above. All candidate complexes that remain after the backtracking of the integrated Steps 2 to 4 (together with the vertex-link of one vertex v_0 for each complex if $d \geq 4$) are printed to a file.

Step 5 (Homology Computation): Remove complexes from the list of candidates for which their homology groups do not obey Poincaré duality (with respect to $\mathbb{Z}_2$-coefficients) or for which the homology of the vertex-link differs from the homology of a $(d-1)$-sphere.

We made use of the C-program `homology` by Heckenbach [39] to compute the homology groups for the candidate complexes and their vertex-links. Alternatively, one can use the (proposed) GAP share package *Simplicial Homology* [31] by Dumas, Heckenbach, Saunders, and Welker (see [30] for a description) or the `TOPAZ` module of the `polymake` system [35] of Gawrilow and Joswig to compute the homology groups.

In the case of vertex-transitive triangulations with $n \leq 15$ vertices, the candidates that remained after Step 5 are all combinatorial manifolds, as we verified in Step 6.

Step 6 (Recognition of the Vertex-Links): Use bistellar flips as a heuristics to recognize the vertex-links as combinatorial spheres.

We used the program BISTELLAR [62] (cf. also [11]) to check whether the link of a candidate is bistellarly equivalent and therefore PL homeomorphic to the boundary of a simplex. With this heuristic, it was possible to show that all remaining vertex-transitive candidates with $n \leq 15$ vertices are indeed combinatorial manifolds. (A fast implementation of the bistellar flip heuristic due to Nikolaus Witte is accessible via the `TOPAZ` module of the `polymake` system [35].)

Step 7 (Topological Type): Use bistellar flips or topological classification theorems to determine the topological types of the manifolds.

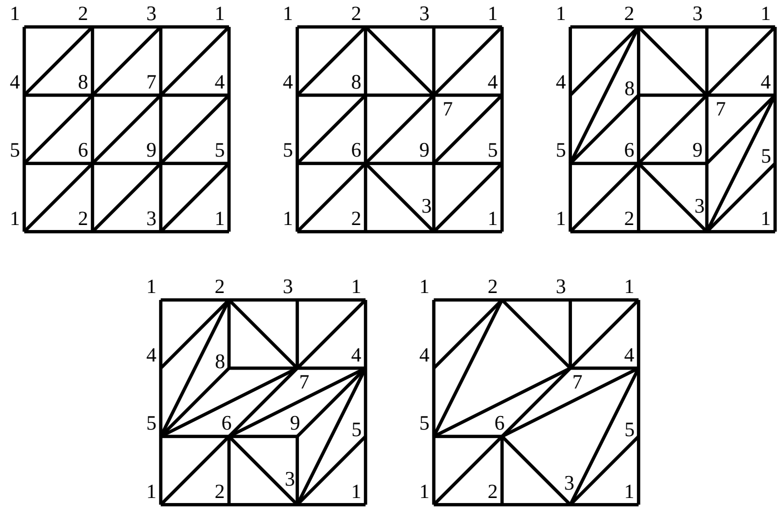
For all but one vertex-transitive combinatorial manifold with $n \leq 15$ vertices that we found in the previous steps it has been possible to determine their homeomorphism types. This was done in most cases with the program BISTELLAR_EQUIVALENT [60], which we used to establish a bistellar equivalence between the test manifold and some reference manifold. As reference manifolds we used small or minimal triangulations of manifolds that have the same homology as the test object, but for which their topological types are known. In a small number of cases, topological classification theorems were used to determine the topological types. For details see Section 3.

Figure 1 displays an application of bistellar flips: We explicitly give a bistellar equivalence between a 9-vertex triangulation of the 2-torus and the unique vertex-minimal 7-vertex triangulation of Möbius [75].

With respect to computation time we remark that the most expensive step in the above procedure is the backtracking of Step 2. More precisely, the computation time crucially depends on the size of the associated matrices. As a consequence, vertex-transitive triangulations for group actions of small size are hard to enumerate. If, on the other side, the acting group has large group order, then there is a good chance to complete the enumeration – even for larger n and d.

The division line between instances that can be computed and those that cannot is sharp: It takes minutes or at most hours to enumerate vertex-transitive triangulations with a dihedral group action on 14 and 15 vertices, but it is hopeless to complete the enumeration (at least with the present techniques) for cyclic actions on 14 and 15 vertices in dimensions $5 \leq d \leq 7$.

9-vertex torus



Möbius' 7-vertex torus

Figure 1: Bistellar flips on the 9-vertex torus to reduce the number of vertices.

2 Enumeration Results

We used the program MANIFOLD_VT [61] to enumerate (candidates for) vertex-transitive triangulated d-manifolds with $n \leq 15$ vertices and $2 \leq d \leq n-2$ for almost all the actions of transitive permutation groups of degree $n \leq 15$. The cases where an enumeration was not possible are for $5 \leq d \leq 7$ the actions ${}^{5}14\,^{1}$, ${}^{6}14\,^{1}$, ${}^{7}14\,^{1}$ of the cyclic group $\mathbb{Z}_{14}$ and the actions ${}^{5}14\,^{2}$, ${}^{6}14\,^{2}$, ${}^{7}14\,^{2}$ of the dihedral group D_7 on 14 vertices as well as for $4 \leq d \leq 8$ the actions ${}^{4}15\,^{1}$, ${}^{5}15\,^{1}$, ${}^{6}15\,^{1}$, ${}^{7}15\,^{1}$, ${}^{8}15\,^{1}$ of the cyclic group $\mathbb{Z}_{15}$ on 15 vertices. For all but these 11 actions we succeeded to complete the enumeration (Steps 2 to 4) and also Steps 5 and 6 of the previous section. All candidates that remained after Step 6 turned out to be combinatorial manifolds.

Theorem 1 *There are at least* 525 *combinatorial manifolds of dimension* $2 \leq d \leq 13$ *with* $n \leq 15$ *vertices that have a vertex-transitive automorphism group.*

According to the Brehm-Kühnel bound [20], every combinatorial manifold with $n < 3\left\lceil \frac{d}{2} \right\rceil + 3$ vertices is a sphere and every combinatorial manifold with $n = 3\left\lceil \frac{d}{2} \right\rceil + 3$ vertices is either a sphere or a manifold 'like a projective plane'. The latter case is only possible for $d = 2, 4, 8$, and 16 (cf. [20] and also [66]). Among the 525 manifolds that we found there are 235 spheres and 290 examples of other topological types. The explicit numbers of spheres and non-spheres are listed for given d and n (together with the Brehm-Kühnel bound for $2 \leq d \leq 8$) in Table 2.

Corollary 2 *There are precisely* 220 *combinatorial manifolds of dimension* $2 \leq d \leq 11$ *with* $n \leq 13$ *vertices that have a vertex-transitive automorphism group:* 110 *spheres and* 110 *manifolds that are not spheres. The* 34 *different homeomorphism types of these manifolds are for*

$d = 2$: S^2, $\mathbf{T}^2$, *the orientable surfaces of genus* 2, 3, 4, 5, 6,
$\mathbb{R}\mathbf{P}^2$, *the non-orientable surfaces of genus* 2, 4, 5, 7, 8, 15,

$d = 3$: S^3, $S^2 \underline{\times} S^1$, $S^2 \times S^1$, $(S^2 \times S^1)^{\#2}$, $\mathbb{R}\mathbf{P}^3$,

$d = 4$: S^4, $\mathbb{C}\mathbf{P}^2$, $S^3 \times S^1$, $S^3 \underline{\times} S^1$, $S^2 \times S^2$, $(S^2 \times S^2)^{\#2}$,

$d = 5$: S^5, $S^4 \underline{\times} S^1$, $SU(3)/SO(3)$,

as well as S^6, S^7, S^8, S^9, S^{10}, *and* S^{11}.

The topological types of the examples were determined according to Step 7 of the previous section. For details see Section 3.

Corollary 3 *There are precisely* 77 *combinatorial* 2*-manifolds with* $n \leq 15$ *vertices that have a vertex-transitive automorphism group. Of these examples* 42 *are orientable and* 35 *are non-orientable. In particular, there are* 18 *different topological types:* S^2, $\mathbf{T}^2$, *the orientable surfaces of genus* 2, 3, 4, 5, 6, 8, $\mathbb{R}\mathbf{P}^2$, *and the non-orientable surfaces of genus* 2, 4, 5, 7, 8, 12, 15, 16, *and* 17.

Corollary 4 *There are exactly* 166 *combinatorial* 3*-manifolds on* $n \leq 15$ *vertices with a vertex-transitive automorphism group;* 52 *of these are spheres, whereas* 114 *are not spheres. The manifolds are of one of* 8 *different topological types:* S^3, $S^2 \underline{\times} S^1$, $S^2 \times S^1$, $(S^2 \times S^1)^{\#2}$, $\mathbb{R}\mathbf{P}^3$, $L(3,1)$, S^3/Q, *and* $\mathbf{T}^3$.

We label every vertex-transitive combinatorial manifold that we found (up to combinatorial equivalence) by our enumeration with a *unique symbol*: The k-th example of a combinatorial manifold of dimension d with n vertices that is listed for the i-th transitive permutation group n^i of degree n is denoted by ${}^d n^i_k$. We list the respective manifolds in the Tables 3 to 12, together with additional information on their topological types and where they appeared previously in the literature – as far as we know.

Corollary 5 *There is no vertex-transitive triangulation of a combinatorial* 5*-manifold, different from the* 5*-sphere, with* 12 *vertices. Also there is no vertex-transitive triangulation of a combinatorial* 6*-manifold, different from the* 6*-sphere, with* 13 *vertices.*

Table 2: Vertex-transitive combinatorial manifolds with $n \leq 15$ vertices; numbers of spheres and non-spheres (in bold).

$n \backslash d$	2	3	4	5	6	7	8	9	10	11	12	13
15	0 **17**	5 **29**	$\geq$ 2 $\geq$ **31**	$\geq$ 23 $\geq$ **12**	$\geq$ 0 $\geq$ **10**	$\geq$ 14 $\geq$ **0**	$\geq$ 0 **1**	5	0	3	0	1
14	0 **13**	20 **35**	0 **23**	$\geq$ 16 $\geq$ **9**	$\geq$ 3 $\geq$ **0**	$\geq$ 6	23	1	1	1	1	
13	0 **4**	6 **9**	0 **5**	17 **2**	0 **0**	6	0	1	0	1		
12	1 **30**	6 **33**	1 **7**	27 **0**	0	4	1	1	1			
11	0 **1**	3 **3**	0 **1**	3	0	1	0	1				
10	0 **3**	6 **4**	4 **0**	1	1	1	1					
9	0 **3**	1 **1**	0 **1**	2	0	1						
8	0 **1**	2	0	1	1							
7	0 **1**	1	0	1								
6	1 **1**	1	1									
5	0	1										
4	1											

Nonetheless, there are asymmetric triangulations of $S^3\times S^2$ with 12 vertices and of $S^3\times S^3$ with 13 vertices from which it follows that the Brehm-Kühnel bound is sharp for $d=5$ and $d=6$, respectively (see [66]). For $d=2,3,4,8$, the 6-vertex triangulation ${}^{2}6^{12}_{1}$ of the real projective plane, Walkup's [83] 9-vertex triangulation ${}^{3}9^{3}_{2}$ of $S^2\,\underline{\times}\,S^1$, Kühnel's [49] 9-vertex triangulation ${}^{4}9^{13}_{1}$ of $\mathbb{C}\mathbf{P}^2$, and Brehm and Kühnel's A_5-invariant triangulation ${}^{8}15^{5}_{1}$ of a manifold $\sim\mathbb{H}\mathbf{P}^2$ like the quaternionic projective plane with 15 vertices are vertex-transitive examples of combinatorial manifolds that are vertex-minimal by the Brehm-Kühnel bound in the respective dimensions.

Theorem 6 *There are two vertex-minimal, vertex-transitive triangulations* ${}^{4}12^{2}_{1}$ *and* ${}^{4}12^{2}_{2}$ *of* $(S^2\times S^2)\#(S^2\times S^2)$ *with* 12 *vertices.*

Proof. Vertex-minimality for these two triangulations follows from Kühnel's bound [47, 4.1] which states that $\binom{n-4}{3}\geq 10\,(\chi(M)-2)$ for every combinatorial 4-manifold M with n vertices. $\square$

3 Topological Types

Various of the 525 vertex-transitive combinatorial manifolds that we found with $n\leq 15$ vertices appeared previously in the literature. For many of these examples, their topological types were determined in the respective papers; see the references cited in the Tables 4 to 12. For all but one of the examples their types can also be recognized as described in the following.

Since the topological type of a 2-dimensional manifold can be determined from its Euler characteristic and its orientation character, the actual work for recognizing topological types starts with dimension 3: All vertex-transitive 3-manifolds with $n\leq 15$ vertices from our enumeration turned out to be either triangulations of Seifert manifolds or triangulations of the connected sum $(S^2\times S^1)^{\#2}$. Reference triangulations for Seifert manifolds are available via the program SEIFERT [63] (for a description of the program see [22] and [68]). The connected sum $(S^2\times S^1)^{\#2}$ can be composed combinatorially by taking two disjoint copies of a triangulation of $S^2\times S^1$, then removing a simplex each, and finally glueing both parts together. By using these reference triangulations, it was possible to recognize the topological types of all the vertex-transitive 3-manifolds with $n\leq 15$ vertices with the bistellar flip program BISTELLAR_EQUIVALENT [60].

The 4-dimensional combinatorial manifolds that we found for $n\leq 15$ with a vertex-transitive group action are of the topological types S^4, $\mathbb{C}\mathbf{P}^2$, $S^3\times S^1$, $S^3\,\underline{\times}\,S^1$, $S^2\times S^2$, $(S^2\times S^2)^{\#2}$, and $(S^3\,\underline{\times}\,S^1)\,\#\,(\mathbb{C}\mathbf{P}^2)^{\#5}$. Reference triangulations for the (twisted) sphere products $S^3\times S^1$, $S^2\times S^2$, and $S^3\,\underline{\times}\,S^1$ can be obtained as (twisted) product triangulations; see [68]. As a reference triangulation for S^4 we can take the boundary of the 5-simplex, and the connected sum $(S^2\times S^2)^{\#2}$ can be composed as described above. Some of the vertex-transitive triangulations of 4-manifolds of these topological types were known before. However, it was also possible to recognize all these examples with the program BISTELLAR_EQUIVALENT. The 9-vertex triangulation of Kühnel of $\mathbb{C}\mathbf{P}^2$ is discussed in [49]. Thus it remained to determine the type of the example ${}^4 15\,{}^4_1$.

Theorem 7 *There is a vertex-transitive triangulation* ${}^4 15\,{}^4_1$ *of the manifold* $(S^3 \underline{\times}\, S^1)\#\,(\mathbb{C}\mathbf{P}^2)^{\#5}$ *with* 15 *vertices.*

Proof. The triangulation ${}^4 15\,{}^4_1$ has infinite cyclic fundamental group, which we computed with the group algebra package GAP [37]. By the classification of Wang [84] of closed non-orientable 4-manifolds with infinite cyclic fundamental group, $(S^3 \underline{\times}\, S^1)\#\,(\mathbb{C}\mathbf{P}^2)^{\#5}$ is the only such 4-manifold with homology $H_* = (\mathbb{Z}, \mathbb{Z}, \mathbb{Z}^5, \mathbb{Z}_2, 0)$. $\square$

Conjecture 8 *The combinatorial manifold* ${}^4 15\,{}^4_1$ *is the unique vertex-minimal triangulation of* $(S^3 \underline{\times}\, S^1)\#\,(\mathbb{C}\mathbf{P}^2)^{\#5}$ *with* 15 *vertices.*

The vertex-transitive combinatorial 5-manifolds that we found with $n \leq 15$ vertices are of the topological types S^5, $S^4 \underline{\times}\, S^1$, $SU(3)/SO(3)$, and $S^3 \times S^2$. All the corresponding triangulations of S^5 and $S^4 \underline{\times}\, S^1$ could be recognized with bistellar flips.

Theorem 9 *There is a* 3*-neighborly triangulation* ${}^5 13\,{}^3_2$ *of the simply connected homogeneous* 5*-dimensional manifold* $SU(3)/SO(3)$ *with homology* $H_* = (\mathbb{Z}, 0, \mathbb{Z}_2, 0, 0, \mathbb{Z})$. *This triangulation with* $f = (13, \underline{78}, \underline{286}, 533, 468, 156)$ *has a vertex-transitive action of the affine group* $13\!:\!3$.

Proof. Since the triangulation ${}^5 13\,{}^3_2$ is 3-neighborly, the corresponding manifold is simply connected. According to the classification of all simply connected 5-manifolds by Barden [10], there is only one simply connected 5-manifold with homology $H_* = (\mathbb{Z}, 0, \mathbb{Z}_2, 0, 0, \mathbb{Z})$, which he denoted by X_{-1}. In fact, it is the well-known simply connected homogeneous 5-manifold $SU(3)/SO(3)$; cf. the classification of compact homogeneous manifolds of low dimension by Gorbatsevich [36] as well as the exposition on homogeneous manifolds in [44]. $\square$

Conjecture 10 *The* 3*-neighborly triangulation* ${}^5 13\,{}^3_2$ *is the unique vertex-minimal triangulation of* $SU(3)/SO(3)$ *with* 13 *vertices.*

There are two further vertex-transitive triangulations ${}^5 14\,{}^4_2$ and ${}^5 14\,{}^4_6$ of $SU(3)/SO(3)$ that are bistellarly equivalent to the triangulation ${}^5 13\,{}^3_2$. However, we did not find another triangulation of $SU(3)/SO(3)$ with 13 vertices.

Altogether, our enumeration yielded four vertex-transitive triangulations ${}^5 14\,{}^3_8$, ${}^5 14\,{}^3_{13}$, ${}^5 14\,{}^3_{14}$, and ${}^5 14\,{}^3_{15}$ of $S^3 \times S^2$ with $n \leq 15$ vertices: For all four examples we used the program BISTELLAR [62] to obtain vertex-minimal 3-neighborly triangulations with 12 vertices and $f = (12, \underline{66}, \underline{220}, 390, 336, 112)$; see [66]. In particular, it follows that the four examples are simply-connected.

By the classification of Barden [10], there are precisely two simply connected 5-manifolds with the homology of $S^3 \times S^2$, namely $M_\infty = S^3 \times S^2$ with trivial and X_∞ with non-vanishing second Stiefel-Whitney class. The first example, ${}^5 14\,{}^3_8$, is centrally symmetric (cf. [65]) and is therefore embedded in the 6-dimensional boundary complex ∂C_7^Δ of the 7-dimensional crosspolytope C_7^Δ with 14 vertices. Since ${}^5 14\,{}^3_8$ is a codimension 1-submanifold in the sphere ∂C_7^Δ, it divides ∂C_7^Δ into two connected components by Alexander duality, both parts having ${}^5 14\,{}^3_8$ as their common boundary. By a theorem of Pontrjagin, the Stiefel-Whitney numbers of a d-manifold that is the boundary of a smooth compact $(d+1)$-manifold are all zero (cf. [74, 4.9]). Hence, ${}^5 14\,{}^3_8$ is a triangulation of $S^3 \times S^2$. For the other three triangulations ${}^5 14\,{}^3_{13}$, ${}^5 14\,{}^3_{14}$, and ${}^5 14\,{}^3_{15}$ we computed their Stiefel-Whitney classes with the `TOPAZ` module of the `polymake` system [35]. In all three cases the Stiefel-Whitney classes vanish and the examples are therefore triangulations of $S^3 \times S^2$.

The vertex-transitive combinatorial 6-manifolds with $n \leq 15$ vertices that we obtained by our enumeration are of three topological types, S^6, $S^5 \times S^1$, and $S^3 \times S^3$. The triangulations of S^6 were recognized with bistellar flips and the one triangulation ${}^6 15\,{}^2_1$ of $S^5 \times S^1$ is a member of a series of sphere products of Kühnel [46, M^6] (see also [54, M_5^6]).

For the remaining vertex-transitive combinatorial 6-manifolds we made use of the classification of 2-connected topological 6-manifolds by Žubr [86] (cf. also [45] and [56]): The topological type of a 2-connected 6-manifold is determined by its Euler characteristic. We used two approaches to show that the examples are 2-connected. In both approaches, we first computed the homology groups of the examples, which are the homology groups of $S^3 \times S^3$. Then, in the first approach, we computed the fundamental group of the examples with the program GAP, which turned out to be trivial in each case. According to the theorem of Hurewicz (cf. [70, p. 80]), every simply connected space with trivial second homology is 2-connected. Thus, by the classification of Žubr it follows that the examples are triangulations of $S^3 \times S^3$. Another way to show the 2-connectedness is by using bistellar flips to reduces the examples to vertex-minimal triangulations with 13 vertices. The resulting f-vector that we obtained in each of the cases is $f = (13, \underline{78}, \underline{286}, \underline{715}, 1014, 728, 208)$. From the f-vector it can be read of that these 13-vertex triangulations are 4-neighborly and therefore 2-connected.

Apart from the combinatorial manifold ${}^8 15\,{}^5_1$, all vertex-transitive triangulations of manifolds of dimension $7 \leq d \leq 13$ that we found with $n \leq 15$ vertices are spheres, as we recognized with bistellar flips.

The vertex-transitive example ${}^8 15\,{}^5_1$ was first discovered by Brehm and Kühnel [21, M_{15}^8]. It has homology $H_* = (\mathbb{Z}, 0, 0, 0, \mathbb{Z}, 0, 0, 0, \mathbb{Z})$. According to the Brehm-Kühnel bound [20], every combinatorial 8-manifold with 15 vertices is either a sphere or a manifold like the quaternionic projective plane (in the sense of [34]). There are infinitely many such manifolds that can be distinguished by their first Pontrjagin class. However, it is unclear how to explicitely compute the first Pontrjagin class combinatorially for simplicial complexes of the size of ${}^{8}15\,{}^{5}_{1}$. We denote the topological type of the example ${}^{8}15\,{}^{5}_{1}$ by $\sim\mathbb{H}\mathbf{P}^{2}$ to indicate that, most likely, it is homeomorphic to $\mathbb{H}\mathbf{P}^{2}$.

With the exception of 8-manifolds that possibly have the cyclic permutation group $15^{\,1}$ as vertex-transitive automorphism group, we were able to enumerate all vertex-transitive 8-manifolds with $n \leq 15$ vertices (cf. Table 9). Besides ${}^{8}15\,{}^{5}_{1}$, all the respective examples are spheres. The combinatorial manifold ${}^{8}15\,{}^{5}_{1}$ of Brehm and Kühnel has the group A_5 as its vertex-transitive automorphism group and is the only example with this group action.

Corollary 11 (Brehm [17]) *There is exactly one vertex-transitive triangulation of a manifold like the quaternionic projective plane with* 15 *vertices.*

Proof. Manifolds like the quaternionic projective plane are 3-connected and have Euler characteristic $\chi = 3$. By [48, 4.7] it follows that a triangulation of a manifold like the quaternionic projective plane with 15 vertices is 5-neighborly. By the Dehn-Sommerville equations the Euler characteristic of a 5-neighborly combinatorial 8-manifold completely determines the f-vector (see the discussion in [21] and in the proof of Theorem 4.17 of [48]). In the case of $n = 15$ vertices and Euler characteristic $\chi = 3$ the resulting f-vector is $f = (15, \underline{105}, \underline{455}, \underline{1365}, \underline{3003}, 4515, 4230, 2205, 490)$.

It remains to rule out that there are triangulations of manifolds like the quaternionic projective plane with 15 vertices and the above f-vector that are invariant under the action of the cyclic group $15^{\,1}$ with generator $(1, 2, 3, \ldots, 15)$. This was done by Brehm [17]: The cyclic action $15^{\,1}$ has 335 orbits of 9-tuples and, by complementarity (cf. [48, p 75]), also 335 orbits of 6-tuples, 333 of size 15 and 2 of size 5 in both cases. In order to compose an 8-manifold with 490 8-simplices, both small orbits of 9-tuples of size 5 have to be used. These orbits are generated by the simplices 123678 11 12 13 and 124679 11 12 14. On the other hand, the two orbits of 6-tuples of size 5 cannot be taken as 5-faces since $f_5 = 4515$. But these two orbits are generated by the simplices 1267 11 12 and 1368 11 13 and are thus included as subfaces in the above orbits. Contradiction. $\square$

4 Tables of Manifolds

We list the combinatorial manifolds that we found with a vertex-transitive group action on $n \leq 15$ vertices in the Tables 3 to 12. For every example we give the lexicographically smallest simplices of the respective orbits of facets as orbit representatives. All examples can be rebuilt from their orbit representatives by using GAP commands as follows:

```
gap> G:=TransitiveGroup(7,4);
gap> facets:=[ ];
gap> UniteSet(facets,Orbit(G,[1,2,4],OnSets));
gap> Print(facets,"\n");
[ [ 1, 2, 4 ], [ 1, 2, 6 ], [ 1, 3, 4 ], [ 1, 3, 7 ],
  [ 1, 5, 6 ], [ 1, 5, 7 ], [ 2, 3, 5 ], [ 2, 3, 7 ],
  [ 2, 4, 5 ], [ 2, 6, 7 ], [ 3, 4, 6 ], [ 3, 5, 6 ],
  [ 4, 5, 7 ], [ 4, 6, 7 ] ]
```

This example is Möbius' 7-vertex torus ${}^{2}7^{\,4}_{\,1}$ with vertex-transitive automorphism group $7^{\,4}$. The triangle 124 (we omit brackets and commas in the tables to save space) is listed as the generating triangle for the orbit, since it is the lexicographically smallest triangle in the orbit. Examples with more than one orbit of facets can be built similarly by uniting their orbits of facets with additional `UniteSet` commands. The lower index attached to every generating simplex in the tables indicates the size of the corresponding orbit. For Möbius' torus the orbit $124_{\,14}$ has the above 14 triangles.

For every transitive permutation group of degree $n \leq 15$ that occurs as the automorphism group of one of the combinatorial manifolds in the Tables 3 to 12, we list the generators of the group in Table 13, with the following exceptions: The cyclic group actions, 10^1, 11^1, 12^1, 13^1, 14^1, and 15^1, and the dihedral group actions, 7^2, 8^6, 9^3, 10^3, 11^2, 12^{12}, 13^2, 14^3, and 15^2, with generators $a_n = (1, 2, 3, \ldots, n)$ and $b_n = (1, 2\lfloor\frac{n}{2}\rfloor)(2, 2\lfloor\frac{n}{2}\rfloor - 1) \ldots (\lfloor\frac{n}{2}\rfloor, \lfloor\frac{n}{2}\rfloor + 1)$, such that $\mathbb{Z}_n = \langle a_n \rangle$ and $D_n = \langle a_n, b_n \rangle$, are not listed in Table 13. Also the automorphism groups of the boundary complexes of $(d+1)$-simplices, i.e., the symmetric groups S_{d+2} of $d+2$ elements, where $2 \leq d \leq 13$, are omitted from Table 13.

Corollary 12 *The transitive permutation groups of Table* 13 *together with the additional groups above are precisely all permutation groups that occur as vertex-transitive automorphism groups of combinatorial manifolds with* $n \leq 15$ *vertices.*

Kimmerle and Kouzoudi [43] determined that the boundary complex of the 3-simplex with 4 vertices, the real projective plane ${}^{2}6^{\,11}_{\,1}$, triangulated minimally with 6 vertices, and Möbius' vertex-minimal 7-vertex torus ${}^{2}7^{\,4}_{\,1}$ are the only combinatorial surfaces that admit a *doubly transitive* automorphism group.

Not all of the transitive group actions from Table 13 have systematic names. We use the GAP terminology for these groups: e.g., $t8n15(32)$ is the transitive permutation group on 8 vertices with number 15 (we add the size of the group in brackets). There are finite groups that have more than one representation as a transitive permutation group on n vertices. For example, the permutation groups $t12n8(24)$ and $t12n9(24)$ are different representations of the symmetric group S_4.

Further symbols and abbreviations that are frequently used in the Tables 3 to 12 are:

$\partial\,\Delta_{d+1}$	–	boundary complex of the $(d+1)$-simplex,
$C_{d+1}(n)$	–	cyclic $(d+1)$-polytope with n vertices,
C^{Δ}_{d+1}	–	$(d+1)$-dimensional cross-polytope,
$BiC(p,q;n)$	–	bicyclic 4-polytope
$TriC(p,q,r;n)$	–	tricyclic 6-polytope
$k*k$	–	join product of two k-gons,
$K_1 * K_2$	–	join product of two complexes,
$K_1 \wr K_2$	–	wreath product of two complexes [42],
minimal	–	triangulation is vertex-minimal,
tight	–	triangulation is tight [55],
nncs	–	nearly neighborly centrally symmetric sphere [65],
no flip	–	example has no non-trivial bistellar flip [56].

Table 3: Vertex-transitive combinatorial 2-manifolds.

n	Or.	Gen.	f-vector	Group	Type	List of orbits	Remarks
4	+	0	($\underline{6}$,$\underline{4}$)	S_4	${}^{2}4^{5}_{1}$	123_{4}	tetrahedron, regular
6	+	0	(12,8)	$[2^3]S_3 = 2wrS_3$	${}^{2}6^{11}_{1}$	123_{8}	octahedron, regular
	−	1	($\underline{15}$,10)	A_5	${}^{2}6^{12}_{1}$	123_{10}	$\mathbb{R}\mathbf{P}^2_6$, regular
7	+	1	($\underline{21}$,14)	$7\!:\!6$	${}^{2}7^{4}_{1}$	124_{14}	Möbius' torus, [29], [46, M^2], [53], [54, M^2_1], [75]
8	+	1	(24,16)	$t8n15(32)$	${}^{2}8^{15}_{1}$	123_{16}	[54, $M^2_1(8)$]
9	+	1	(27,18)	D_9	${}^{2}9^{3}_{1}$	124_{18}	[54, $M^2_1(9)$]
				$\mathbb{Z}_3^2\!:\!D_6$	${}^{2}9^{18}_{1}$	136_{18}	regular, [1], [16], [28, Ch. 8], [85]
	−	5	($\underline{36}$,24)	$S_3 \times \mathbb{Z}_3$	${}^{2}9^{4}_{1}$	124_{18} 138_{6}	[6, $N(9,2)$], [27]
10	+	1	(30,20)	D_{10}	${}^{2}10^{3}_{2}$	124_{20}	[54, $M^2_1(10)$]
	−	2	(30,20)	D_{10}	${}^{2}10^{3}_{1}$	123_{10} 137_{10}	
		7	($\underline{45}$,30)	A_5	${}^{2}10^{7}_{1}$	123_{30}	[6, $N(10,13)$], [15, 380, (i)], [27]
11	+	1	(33,22)	D_{11}	${}^{2}11^{2}_{1}$	124_{22}	[54, $M^2_1(11)$]
12	+	0	(30,20)	$[2]A_5$	${}^{2}12^{76}_{1}$	126_{20}	icosahedron, regular
		1	(36,24)	D_{12}	${}^{2}12^{12}_{1}$	124_{24}	[54, $M^2_1(12)$]
					${}^{2}12^{12}_{2}$	125_{24}	
				$D_4 \times S_3$	${}^{2}12^{28}_{1}$	126_{24}	
				$S_4 \times S_3$	${}^{2}12^{83}_{1}$	123_{24}	regular, [1], [16], [28, Ch. 8], [85]
		2	(42,28)	$\mathbb{Z}_{12}$	${}^{2}12^{1}_{3}$	123_{12} 137_{12} 159_{4}	
					${}^{2}12^{1}_{10}$	126_{12} 127_{12} 159_{4}	

Table 3: Vertex-transitive combinatorial 2-manifolds (continued).

n	Or.	Gen.	f-vector	Group	Type	List of orbits	Remarks
				D_6	$^{2}12^{3}_{3}$	124_{12} 145_{12} 159_{4}	
		3	(48,32)	A_4	$^{2}12^{4}_{6}$	124_{12} 127_{12} 138_{4} $16\,11_{4}$	
				$t12n8(24) = S_4$	$^{2}12^{8}_{1}$	124_{24} 137_{8}	
				$t12n113(192)$	$^{2}12^{113}_{1}$	124_{32}	Dyck's regular map, [12], [14], [18], [33], [32], [79], [85]
		4	(54,36)	D_6	$^{2}12^{3}_{1}$	124_{12} 137_{12} 145_{12}	
					$^{2}12^{3}_{2}$	124_{12} 137_{12} $14\,12_{12}$	
				A_4	$^{2}12^{4}_{4}$	123_{12} 125_{12} 138_{4} 159_{4} $16\,11_{4}$	
				$A_4(12) \times \mathbb{Z}_2$	$^{2}12^{6}_{1}$	123_{24} 125_{12}	
		5	(60,40)	$\mathbb{Z}_{12}$	$^{2}12^{1}_{1}$	123_{12} 136_{12} 148_{12} 159_{4}	
					$^{2}12^{1}_{2}$	123_{12} 136_{12} 149_{12} 159_{4}	
					$^{2}12^{1}_{6}$	124_{12} 126_{12} 136_{12} 159_{4}	
				$\frac{1}{2}[3\!:\!2]4$	$^{2}12^{5}_{1}$	123_{12} 124_{12} 145_{12} 159_{4}	
		6	($\underline{66}$,44)	A_4	$^{2}12^{4}_{5}$	124_{12} 127_{12} 138_{4} 159_{4} $15\,11_{12}$	[5, N^{12}_{58}]
	$-$	4	(42,28)	A_4	$^{2}12^{4}_{2}$	123_{12} 124_{12} 138_{4}	
		8	(54,36)	$\mathbb{Z}_{12}$	$^{2}12^{1}_{4}$	124_{12} 125_{12} 137_{12}	
					$^{2}12^{1}_{5}$	124_{12} 125_{12} 139_{12}	
					$^{2}12^{1}_{7}$	124_{12} 127_{12} $13\,10_{12}$	
					$^{2}12^{1}_{8}$	125_{12} 127_{12} 148_{12}	
					$^{2}12^{1}_{9}$	126_{12} 127_{12} 135_{12}	

Table 3: Vertex-transitive combinatorial 2-manifolds (continued).

n	Or.	Gen.	f-vector	Group	Type	List of orbits	Remarks
				$A_4(6)\times\mathbb{Z}_2$	${}^{2}12^{7}_{1}$	124_{24} 137_{12}	
				$t12n9(24) = S_4$	${}^{2}12^{9}_{1}$	123_{24} 137_{12}	
		10	(60,40)	A_4	${}^{2}12^{4}_{1}$	123_{12} 124_{12} 137_{12} 159_{4}	
					${}^{2}12^{4}_{3}$	123_{12} 125_{12} 137_{12} $16\,11_{4}$	
				$t12n8(24) = S_4$	${}^{2}12^{8}_{2}$	124_{24} $13\,10_{12}$ $15\,10_{4}$	
13	$+$	1	(39,26)	D_{13}	${}^{2}13^{2}_{1}$	124_{26}	[54, $M^2_1(13)$]
				$13\!:\!6$	${}^{2}13^{5}_{1}$	125_{26}	
	$-$	15	($\underline{78}$,52)	$\mathbb{Z}_{13}$	${}^{2}13^{1}_{1}$	123_{13} 137_{13} 148_{13} 149_{13}	
				$13\!:\!3$	${}^{2}13^{3}_{1}$	123_{39} 138_{13}	
14	$+$	1	(42,28)	D_{14}	${}^{2}14^{3}_{2}$	124_{28}	[54, $M^2_1(14)$]
					${}^{2}14^{3}_{3}$	125_{28}	
		8	(84,56)	$2[\frac{1}{2}]7\!:\!6$	${}^{2}14^{4}_{1}$	123_{42} 137_{14}	
					${}^{2}14^{4}_{2}$	124_{42} 137_{14}	
					${}^{2}14^{4}_{3}$	124_{42} $13\,11_{14}$	
				$7\!:\!3\times\mathbb{Z}_2$	${}^{2}14^{5}_{1}$	123_{42} 137_{14}	
					${}^{2}14^{5}_{2}$	124_{42} $13\,11_{14}$	
	$-$	2	(42,28)	D_{14}	${}^{2}14^{3}_{1}$	123_{14} 139_{14}	
		16	(84,56)	$\mathbb{Z}_{14}$	${}^{2}14^{1}_{1}$	123_{14} 136_{14} 149_{14} 159_{14}	
					${}^{2}14^{1}_{2}$	123_{14} 136_{14} $14\,10_{14}$ 159_{14}	

Table 3: Vertex-transitive combinatorial 2-manifolds (continued).

n	Or.	Gen.	f-vector	Group	Type	List of orbits	Remarks
					${}^2 14^{1}_{3}$	124_{14} 126_{14} 137_{14} 149_{14}	
					${}^2 14^{1}_{4}$	124_{14} 127_{14} $13\,12_{14}$ 159_{14}	
					${}^2 14^{1}_{5}$	124_{14} $12\,10_{14}$ $13\,12_{14}$ 159_{14}	
15	+	1	(45,30)	D_{15}	${}^2 15^{2}_{1}$	124_{30}	[54, $M_1^2(15)$]
					${}^2 15^{2}_{2}$	127_{30}	
				$D_5 \times S_3$	${}^2 15^{7}_{1}$	125_{30}	
					${}^2 15^{7}_{2}$	126_{30}	
		6	(75,50)	$[5^2\!:\!2]S_3$	${}^2 15^{18}_{1}$	123_{50}	regular, [85]
	−	7	(60,40)	A_5	${}^2 15^{5}_{1}$	125_{30} 179_{10}	
		12	(75,50)	$\mathbb{Z}_{15}$	${}^2 15^{1}_{1}$	123_{15} 136_{15} $14\,10_{15}$ $16\,11_{5}$	
					${}^2 15^{1}_{2}$	123_{15} 137_{15} $15\,10_{15}$ $16\,11_{5}$	
					${}^2 15^{1}_{3}$	123_{15} 137_{15} $15\,11_{15}$ $16\,11_{5}$	
					${}^2 15^{1}_{6}$	125_{15} 126_{15} $14\,10_{15}$ $16\,11_{5}$	
				$\mathbb{Z}_5 \times S_3$	${}^2 15^{4}_{2}$	123_{30} 138_{15} $16\,11_{5}$	
		17	(90,60)	$\mathbb{Z}_{15}$	${}^2 15^{1}_{4}$	123_{15} 138_{15} 149_{15} $14\,10_{15}$	
					${}^2 15^{1}_{5}$	123_{15} 138_{15} $14\,10_{15}$ $14\,11_{15}$	
				$D_5 \times \mathbb{Z}_3$	${}^2 15^{3}_{1}$	123_{30} 136_{30}	
					${}^2 15^{3}_{2}$	123_{30} 139_{15} 147_{15}	
				$\mathbb{Z}_5 \times S_3$	${}^2 15^{4}_{1}$	123_{30} 136_{30}	
				S_5	${}^2 15^{10}_{1}$	$12\,10_{60}$	

Table 4: Vertex-transitive combinatorial 3-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
5	S^3	$(\underline{10},\underline{10},\underline{5})$	S_5	${}^{3}5^{\,5}_{1}$	$1234_{\,5}$	$\partial\,\Delta_4$, regular
6	S^3	$(\underline{15},18,9)$	S_3wr2	${}^{3}6^{\,13}_{1}$	$1234_{\,9}$	$\partial C_4(6) = 3 * 3$, [52, I_6]
7	S^3	$(\underline{21},28,14)$	D_7	${}^{3}7^{\,2}_{1}$	$1234_{\,7}$ $1245_{\,7}$	$\partial C_4(7)$, [52, I_7]
8	S^3	$(24,32,16)$	$2wrS_4$	${}^{3}8^{\,44}_{1}$	$1234_{\,16}$	$\partial C_4^{\Delta} = \partial BiC(1,3;8)$ $= 4 * 4$, regular, nncs
		$(\underline{28},40,20)$	D_8	${}^{3}8^{\,6}_{1}$	$1234_{\,8}$ $1245_{\,8}$ $1256_{\,4}$	$\partial C_4(8)$, [52, I_8]
9	S^3	$(\underline{36},54,27)$	D_9	${}^{3}9^{\,3}_{1}$	$1234_{\,9}$ $1245_{\,9}$ $1256_{\,9}$	$\partial C_4(9)$, [52, I_9]
	$S^2 \mathbin{\underline{\times}} S^1$	$(\underline{36},54,27)$	D_9	${}^{3}9^{\,3}_{2}$	$1235_{\,18}$ $1245_{\,9}$	minimal, tight, [8, N^9_{51}], [46, M^3], [52, II_9], [54, M^3_2], [55], [83]
10	S^3	$(35,50,25)$	D_5wr2	${}^{3}10^{\,21}_{1}$	$1234_{\,25}$	$\partial BiC(1,4;10) = 5 * 5$
		$(40,60,30)$	$\frac{1}{2}[5\!:\!4]2$	${}^{3}10^{\,4}_{1}$	$1235_{\,20}$ $1245_{\,5}$ $1289_{\,5}$	
			$S_5 \times \mathbb{Z}_2$	${}^{3}10^{\,22}_{1}$	$1234_{\,30}$	$\partial BiC(1,3;10)$, nncs, [38, p. 116]
		$(\underline{45},70,35)$	$\mathbb{Z}_{10}$	${}^{3}10^{\,1}_{1}$	$1235_{\,10}$ $1236_{\,10}$ $1246_{\,10}$ $1368_{\,5}$	[3], [4, N^{10}_{3574}], [52, 1_{10}], non-polytopal
			D_{10}	${}^{3}10^{\,3}_{1}$	$1234_{\,10}$ $1245_{\,10}$ $1256_{\,10}$ $1267_{\,5}$	$\partial C_4(10)$, [4, N^{10}_4], [52, I_{10}]
			$\frac{1}{2}[5\!:\!4]2$	${}^{3}10^{\,4}_{2}$	$1245_{\,5}$ $1246_{\,20}$ $1267_{\,10}$	[4, N^{10}_{425}], [13], non-polytopal
	$S^2 \times S^1$	$(40,60,30)$	D_{10}	${}^{3}10^{\,3}_{2}$	$1235_{\,20}$ $1245_{\,10}$	minimal, [54, $M^3_2(10)$], [83]
		$(\underline{45},70,35)$	$\mathbb{Z}_{10}$	${}^{3}10^{\,1}_{2}$	$1236_{\,10}$ $1237_{\,10}$ $1257_{\,10}$ $1368_{\,5}$	minimal, [4, N^{10}_{3611}], [52, 2_{10}]

Table 4: Vertex-transitive combinatorial 3-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
	$S^2 \underline{\times} S^1$	($\underline{45}$,70,35)	D_{10}	$^3 10^3_3$	1236_{20} 1256_{10} 1368_5	[4, N^{10}_{3629}], [52, $\tilde{\mathrm{II}}_{10}$]
				$^3 10^3_4$	1246_{20} 1249_{10} 1267_5	[4, N^{10}_{3631}], [52, II_{10}]
11	S^3	(44,66,33)	D_{11}	$^3 11^2_2$	1234_{11} 1245_{11} 1259_{11}	$\partial BiC(1,3;11)$
		($\underline{55}$,88,44)	$\mathbb{Z}_{11}$	$^3 11^1_1$	1234_{11} 1248_{11} 1258_{11} $125\,10_{11}$	[52, 1_{11}]
			D_{11}	$^3 11^2_1$	1234_{11} 1245_{11} 1256_{11} 1267_{11}	$\partial C_4(11)$, [52, I_{11}]
	$S^2 \underline{\times} S^1$	(44,66,33)	D_{11}	$^3 11^2_4$	1235_{22} 1245_{11}	[54, $M^3_2(11)$]
		($\underline{55}$,88,44)	$\mathbb{Z}_{11}$	$^3 11^1_2$	1235_{11} 1239_{11} 1246_{11} 1256_{11}	[52, 2_{11}]
			D_{11}	$^3 11^2_3$	1234_{11} 1248_{22} 1268_{11}	[52, II_{11}]
12	S^3	(48,72,36)	$[S_3^2]D_4$ $= D_6 wr 2$	$^3 12^{125}_1$	1234_{36}	$\partial BiC(1,5;12) = 6*6$
		(60,96,48)	$\mathbb{Z}_{12}$	$^3 12^1_1$	1234_{12} 1246_{12} $126\,11_{12}$ $135\,10_{12}$	nncs
			D_6	$^3 12^3_4$	1235_{12} $123\,12_3$ 1246_{12} $124\,11_6$ 1256_6 1368_6 $1\,10\,11\,12_3$	
				$^3 12^3_5$	1236_{12} $123\,12_3$ 1245_{12} $124\,11_6$ 1256_6 1368_6 $1\,10\,11\,12_3$	
			$t12n13(24)$	$^3 12^{13}_1$	1235_{24} 1245_{12} $124\,11_{12}$	nncs
		($\underline{66}$,108,54)	D_{12}	$^3 12^{12}_1$	1234_{12} 1245_{12} 1256_{12} 1267_{12} 1278_6	$\partial C_4(12)$, [52, I_{12}]
	$S^2 \times S^1$	(48,72,36)	D_{12}	$^3 12^{12}_2$	1235_{24} 1245_{12}	[54, $M^3_2(12)$]
		(54,84,42)	$S_3 \times \mathbb{Z}_2^2$	$^3 12^{10}_1$	1234_{12} 1236_{24} 1458_6	
			$S_3 \times \mathbb{Z}_4$	$^3 12^{11}_1$	1237_{24} 1238_{12} 1379_6	
			D_{12}	$^3 12^{12}_3$	1237_{24} 1267_{12} 1379_6	
				$^3 12^{12}_4$	1245_{12} $124\,10_{24}$ 1379_6	
			$t12n13(24)$	$^3 12^{13}_3$	1237_{24} 1267_{12} 1379_6	
		(60,96,48)	$\mathbb{Z}_{12}$	$^3 12^1_3$	1235_{12} $123\,10_{12}$ 1246_{12} 1256_{12}	

Table 4: Vertex-transitive combinatorial 3-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
		($\underline{66}$,108,54)	$\mathbb{Z}_{12}$	${}^{3}12^{1}_{2}$	1234_{12} 1248_{12} 1268_{12} $126\,11_{12}$ 1379_{6}	[52, 1_{12}]
				${}^{3}12^{1}_{4}$	1236_{12} 1239_{12} 1256_{12} $128\,10_{12}$ 1379_{6}	[52, 2_{12}]
				${}^{3}12^{1}_{8}$	1247_{12} $124\,11_{12}$ 1257_{12} $125\,11_{12}$ 1379_{6}	[52, 6_{12}]
				${}^{3}12^{1}_{9}$	1248_{12} $124\,10_{12}$ 1258_{12} $125\,10_{12}$ 1379_{6}	[52, 7_{12}]
			$\mathbb{Z}_3 \times \mathbb{Z}_2^2$	${}^{3}12^{2}_{2}$	1245_{12} 1247_{12} 1258_{12} 1278_{6} $13\,10\,12_{6}$ 1458_{6}	
			$t12n8(24)$ $= S_4$	${}^{3}12^{8}_{1}$	1246_{12} 1249_{12} $127\,10_{24}$ $128\,10_{6}$	
				${}^{3}12^{8}_{2}$	1249_{12} $124\,10_{24}$ 1279_{12} $128\,10_{6}$	
			$t12n13(24)$	${}^{3}12^{13}_{5}$	1245_{12} $124\,10_{24}$ $125\,10_{12}$ 1379_{6}	
				${}^{3}12^{13}_{6}$	1245_{12} $124\,11_{12}$ $125\,11_{24}$ 1379_{6}	
	$(S^2\times S^1)^{\#2}$	($\underline{66}$,108,54)	$\mathbb{Z}_{12}$	${}^{3}12^{1}_{7}$	1247_{12} 1248_{12} 1278_{6} 1357_{12} 1369_{12}	[52, 5_{12}]
				${}^{3}12^{1}_{10}$	1248_{12} $124\,10_{12}$ 1278_{6} $127\,10_{12}$ 1357_{12}	[52, 8_{12}]
			$\mathbb{Z}_3 \times \mathbb{Z}_2^2$	${}^{3}12^{2}_{1}$	1245_{12} 1247_{12} 1256_{12} $127\,10_{12}$ $13\,10\,12_{6}$	
				${}^{3}12^{2}_{3}$	1245_{12} $124\,10_{12}$ 1256_{12} $139\,12_{12}$ $13\,10\,12_{6}$	
			$t12n13(24)$	${}^{3}12^{13}_{4}$	1245_{12} 1248_{24} 1278_{6} 1357_{12}	no flip
	$\mathbb{R}\mathbf{P}^3$	(60,96,48)	$t12n13(24)$	${}^{3}12^{13}_{2}$	1235_{24} $125\,10_{12}$ $12\,10\,11_{12}$	24-cell/$\mathbb{Z}_2$
	$S^2 \mathbin{\underline{\times}} S^1$	(54,84,42)	$D_4 \times S_3$	${}^{3}12^{28}_{1}$	1258_{24} $125\,10_{12}$ 1278_{6}	
			$S_4 \times S_3$	${}^{3}12^{83}_{2}$	1247_{24} $124\,11_{18}$	
		(60,96,48)	$t12n54(96)$	${}^{3}12^{54}_{1}$	$124\,10_{48}$	
		($\underline{66}$,108,54)	$\mathbb{Z}_{12}$	${}^{3}12^{1}_{5}$	1245_{12} 1247_{12} $125\,11_{12}$ 1278_{6} $128\,10_{12}$	[52, 3_{12}]
				${}^{3}12^{1}_{6}$	1245_{12} 1248_{12} $125\,11_{12}$ 1278_{6} $127\,10_{12}$	[52, 4_{12}]
			D_6	${}^{3}12^{3}_{1}$	1234_{12} 1239_{12} 1246_{12} 1346_{6} 1357_{6} $167\,12_{3}$ $1\,10\,11\,12_{3}$	

Table 4: Vertex-transitive combinatorial 3-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{3}12^{3}_{2}$	1234_{12} $123\,10_{12}$ 1247_{6} 1256_{6} 1267_{12} $167\,12_{3}$ $1\,10\,11\,12_{3}$	
				${}^{3}12^{3}_{3}$	1235_{12} $123\,10_{12}$ 1346_{6} 1357_{6} 1367_{12} $167\,12_{3}$ $1\,10\,11\,12_{3}$	
			$\frac{1}{2}[3\!:\!2]4$	${}^{3}12^{5}_{1}$	1236_{12} 1237_{12} 1278_{3} 1367_{12} $145\,10_{12}$ $147\,10_{3}$	
				${}^{3}12^{5}_{2}$	1236_{12} 1237_{12} 1278_{3} $136\,12_{12}$ $137\,12_{12}$ $147\,10_{3}$	
			$S_4 \times S_3$	${}^{3}12^{83}_{1}$	1246_{36} $124\,11_{18}$	[52, II_{12}]
13	S^3	(52,78,39)	$13\!:\!4$	${}^{3}13^{4}_{1}$	1234_{26} $124\,12_{13}$	$\partial BiC(1,5;13)$
		(65,104,52)	$\mathbb{Z}_{13}$	${}^{3}13^{1}_{1}$	1234_{13} 1247_{13} $127\,12_{13}$ 1369_{13}	
			D_{13}	${}^{3}13^{2}_{2}$	1234_{13} 1245_{13} 1256_{13} $126\,10_{13}$	$\partial BiC(1,3;13)$
		($\underline{78}$,130,65)	$\mathbb{Z}_{13}$	${}^{3}13^{1}_{3}$	1234_{13} 1248_{13} $126\,10_{13}$ $126\,12_{13}$ $128\,10_{13}$	[52, 2_{13}]
				${}^{3}13^{1}_{5}$	1235_{13} 1236_{13} 1249_{13} 1269_{13} 1358_{13}	[52, 4_{13}]
			D_{13}	${}^{3}13^{2}_{1}$	1234_{13} 1245_{13} 1256_{13} 1267_{13} 1278_{13}	$\partial C_4(13)$, [52, I_{13}]
	$S^2 \mathbin{\underline{\times}} S^1$	(52,78,39)	D_{13}	${}^{3}13^{2}_{6}$	1235_{26} 1245_{13}	[54, $M^3_2(13)$]
		(65,104,52)	$\mathbb{Z}_{13}$	${}^{3}13^{1}_{7}$	1235_{13} $123\,11_{13}$ 1246_{13} 1256_{13}	
			D_{13}	${}^{3}13^{2}_{3}$	1234_{13} 1248_{26} $137\,10_{13}$	
				${}^{3}13^{2}_{4}$	1234_{13} 1249_{26} 1279_{13}	
		($\underline{78}$,130,65)	$\mathbb{Z}_{13}$	${}^{3}13^{1}_{2}$	1234_{13} 1248_{13} 1268_{13} $126\,12_{13}$ 1379_{13}	[52, 1_{13}]
				${}^{3}13^{1}_{4}$	1234_{13} 1249_{13} 1267_{13} $126\,12_{13}$ $127\,10_{13}$	[52, 3_{13}]
				${}^{3}13^{1}_{6}$	1235_{13} $123\,10_{13}$ 1246_{13} 1257_{13} 1267_{13}	[52, 5_{13}]
				${}^{3}13^{1}_{8}$	1236_{13} 1237_{13} 1257_{13} $1\underline{3}68_{13}$ 1379_{13}	[52, 6_{13}]
			D_{13}	${}^{3}13^{2}_{5}$	1234_{13} $124\,12_{13}$ $136\,10_{26}$ $137\,10_{13}$	[52, II_{13}]

Table 4: Vertex-transitive combinatorial 3-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
14	S^3	(63,98,49)	D_{14}	${}^{3}14^{3}_{4}$	1234_{14} 1245_{14} $125\,12_{14}$ $148\,11_{7}$	$\partial BiC(1,4;14)$
			D_7wr2	${}^{3}14^{20}_{1}$	1234_{49}	$\partial BiC(1,6;14) = 7*7$
		(70,112,56)	D_{14}	${}^{3}14^{3}_{2}$	1234_{14} 1245_{14} 1256_{14} $126\,11_{14}$	$\partial BiC(1,3;14)$
		(77,126,63)	$\mathbb{Z}_{14}$	${}^{3}14^{1}_{2}$	1234_{14} 1246_{14} $126\,13_{14}$ $135\,12_{14}$ $138\,10_{7}$	
				${}^{3}14^{1}_{3}$	1234_{14} 1248_{14} 1258_{14} $125\,13_{14}$ $148\,11_{7}$	
				${}^{3}14^{1}_{13}$	1235_{14} 1236_{14} 1246_{14} 1368_{14} $138\,10_{7}$	
				${}^{3}14^{1}_{28}$	1237_{14} 1238_{14} 1268_{14} 1357_{14} $138\,10_{7}$	
		(84,140,70)	$\mathbb{Z}_{14}$	${}^{3}14^{1}_{1}$	1234_{14} 1245_{14} $125\,10_{14}$ $126\,10_{14}$ $126\,12_{14}$	nncs
			D_7	${}^{3}14^{2}_{1}$	1234_{7} 1236_{14} $124\,13_{7}$ 1267_{14}	
					1278_{7} 1289_{7} $145\,14_{7}$ $167\,14_{7}$	
				${}^{3}14^{2}_{3}$	1234_{7} 1237_{14} $124\,12_{14}$ 1256_{7}	
					$126\,11_{7}$ $127\,10_{7}$ $147\,10_{7}$ $15\,10\,14_{7}$	
		($\underline{91}$,154,77)	$\mathbb{Z}_{14}$	${}^{3}14^{1}_{4}$	1234_{14} 1248_{14} 1268_{14} $126\,13_{14}$ 1357_{14} $138\,10_{7}$	[52, 1_{14}]
				${}^{3}14^{1}_{7}$	1234_{14} 1248_{14} $126\,11_{14}$ $126\,13_{14}$ 1289_{7} $129\,11_{14}$	[52, 7_{14}]
				${}^{3}14^{1}_{8}$	1234_{14} 1248_{14} $128\,13_{14}$ 1357_{14} $138\,10_{7}$ $138\,11_{14}$	[52, 4_{14}]
				${}^{3}14^{1}_{11}$	1234_{14} 1249_{14} 1269_{14} $126\,13_{14}$ $147\,10_{14}$ $148\,11_{7}$	[52, 8_{14}]
				${}^{3}14^{1}_{14}$	1235_{14} 1238_{14} 1246_{14} 1257_{14} 1268_{14} $138\,10_{7}$	[52, 9_{14}]
				${}^{3}14^{1}_{17}$	1236_{14} 1237_{14} 1247_{14} 1248_{14} 1258_{14} $148\,11_{7}$	[52, 10_{14}]
				${}^{3}14^{1}_{18}$	1236_{14} 1237_{14} 1257_{14} 1357_{14} 1368_{14} $138\,10_{7}$	[52, 11_{14}]
				${}^{3}14^{1}_{26}$	1237_{14} 1238_{14} 1246_{14} 1248_{14} $135\,12_{14}$ $138\,10_{7}$	[52, 16_{14}]
				${}^{3}14^{1}_{27}$	1237_{14} 1238_{14} 1258_{14} $125\,13_{14}$ $126\,13_{14}$ $148\,11_{7}$	[52, 17_{14}]
			D_{14}	${}^{3}14^{3}_{1}$	1234_{14} 1245_{14} 1256_{14} 1267_{14} 1278_{14} 1289_{7}	$\partial C_4(14)$, [52, I_{14}]
	$S^2 \times S^1$	(56,84,42)	D_{14}	${}^{3}14^{3}_{6}$	1235_{28} 1245_{14}	[54, $M^3_2(14)$]
		(63,98,49)	$\mathbb{Z}_{14}$	${}^{3}14^{1}_{33}$	1238_{14} 1239_{14} 1279_{14} $138\,10_{7}$	

Table 4: Vertex-transitive combinatorial 3-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
		(70,112,56)	$\mathbb{Z}_{14}$	$^{3}14^{1}_{16}$	1235_{14} $123\,12_{14}$ 1246_{14} 1256_{14}	
			D_{14}	$^{3}14^{3}_{8}$	1237_{28} 1267_{14} 1357_{14}	
				$^{3}14^{3}_{9}$	1237_{28} 1267_{14} 1379_{14}	
				$^{3}14^{3}_{10}$	1237_{28} 1267_{14} $137\,11_{14}$	
		(77,126,63)	$\mathbb{Z}_{14}$	$^{3}14^{1}_{12}$	1234_{14} 1249_{14} 1279_{14} $127\,13_{14}$ $138\,10_{7}$	
				$^{3}14^{1}_{29}$	1237_{14} 1238_{14} 1268_{14} 1379_{14} $138\,10_{7}$	
		(84,140,70)	$\mathbb{Z}_{14}$	$^{3}14^{1}_{15}$	1235_{14} $123\,11_{14}$ 1246_{14} 1257_{14} 1267_{14}	
				$^{3}14^{1}_{22}$	1236_{14} $123\,11_{14}$ 1256_{14} $12\,10\,12_{14}$ 1357_{14}	
				$^{3}14^{1}_{23}$	1236_{14} $123\,11_{14}$ 1256_{14} $12\,10\,12_{14}$ 1379_{14}	
				$^{3}14^{1}_{24}$	1236_{14} $123\,11_{14}$ 1256_{14} $12\,10\,12_{14}$ $137\,11_{14}$	
		$(\underline{91},154,77)$	$\mathbb{Z}_{14}$	$^{3}14^{1}_{5}$	1234_{14} 1248_{14} 1268_{14} $126\,13_{14}$ 1379_{14} $138\,10_{7}$	[52, 2_{14}]
				$^{3}14^{1}_{9}$	1234_{14} 1248_{14} $128\,13_{14}$ 1379_{14} $138\,10_{7}$ $138\,11_{14}$	[52, 5_{14}]
				$^{3}14^{1}_{19}$	1236_{14} 1237_{14} 1257_{14} 1368_{14} 1379_{14} $138\,10_{7}$	[52, 12_{14}]
	$L(3,1)$	(77,126,63)	$\mathbb{Z}_{14}$	$^{3}14^{1}_{30}$	1237_{14} 1238_{14} 1268_{14} $137\,11_{14}$ $138\,10_{7}$	
		$(\underline{91},154,77)$	$\mathbb{Z}_{14}$	$^{3}14^{1}_{6}$	1234_{14} 1248_{14} 1268_{14} $126\,13_{14}$ $137\,11_{14}$ $138\,10_{7}$	[52, 3_{14}]
				$^{3}14^{1}_{10}$	1234_{14} 1248_{14} $128\,13_{14}$ $137\,11_{14}$ $138\,10_{7}$ $138\,11_{14}$	[52, 6_{14}]
				$^{3}14^{1}_{20}$	1236_{14} 1237_{14} 1257_{14} 1368_{14} $137\,11_{14}$ $138\,10_{7}$	[52, 13_{14}]
				$^{3}14^{1}_{31}$	1237_{14} 1238_{14} $126\,12_{14}$ $128\,12_{14}$ $137\,10_{14}$ $138\,10_{7}$	[52, 18_{14}]
	$S^2 \underline{\times} S^1$	(63,98,49)	D_{14}	$^{3}14^{3}_{11}$	1238_{28} 1278_{14} $138\,10_{7}$	
				$^{3}14^{3}_{13}$	1259_{28} $125\,12_{14}$ 1289_{7}	
		(70,112,56)	D_{14}	$^{3}14^{3}_{5}$	1234_{14} $124\,10_{28}$ $127\,10_{14}$	
		(77,126,63)	$\mathbb{Z}_{14}$	$^{3}14^{1}_{32}$	1237_{14} $123\,10_{14}$ 1267_{14} $129\,11_{14}$ $138\,10_{7}$	
				$^{3}14^{1}_{36}$	1246_{14} $124\,13_{14}$ $126\,13_{14}$ 1358_{14} $148\,11_{7}$	
			D_{14}	$^{3}14^{3}_{7}$	1236_{28} 1256_{14} 1368_{14} $138\,10_{7}$	

Table 4: Vertex-transitive combinatorial 3-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
		(84,140,70)	D_7	${}^3 14\,^2_2$	1234_{7} 1237_{14} $124\,11_{14}$ 1268_{14} 1278_{7} 1458_{7} $145\,14_{7}$	
				${}^3 14\,^2_4$	1234_{7} 1237_{14} $124\,12_{14}$ 1258_{14} 1278_{7} 1458_{7} $145\,14_{7}$	
				${}^3 14\,^2_5$	1234_{7} 1238_{14} $124\,11_{14}$ 1256_{7} 1258_{14} $147\,12_{7}$ $167\,12_{7}$	
			D_{14}	${}^3 14\,^3_3$	1234_{14} 1245_{14} $125\,10_{28}$ $127\,10_{14}$	
		($\underline{91}$,154,77)	$\mathbb{Z}_{14}$	${}^3 14\,^1_{21}$	1236_{14} $123\,10_{14}$ 1256_{14} $129\,11_{14}$ $12\,10\,12_{14}$ $138\,10_{7}$	[52, 14_{14}]
				${}^3 14\,^1_{25}$	1236_{14} $123\,11_{14}$ 1257_{14} 1267_{14} 1368_{14} $138\,10_{7}$	[52, 15_{14}]
				${}^3 14\,^1_{34}$	1245_{14} 1248_{14} 1256_{14} $126\,13_{14}$ 1289_{7} $129\,11_{14}$	[52, 19_{14}]
				${}^3 14\,^1_{35}$	1245_{14} 1249_{14} 1256_{14} $126\,13_{14}$ 1289_{7} $128\,11_{14}$	[52, 20_{14}]
			D_{14}	${}^3 14\,^3_{12}$	1245_{14} $124\,10_{28}$ $125\,12_{14}$ $127\,10_{14}$ $148\,11_{7}$	[52, II_{14}]
15	S^3	(75,120,60)	$D_5 \times S_3$	${}^3 15\,^7_1$	1234_{30} 1245_{30}	$\partial BiC(1,4;15)$
		(90,150,75)	$D_5 \times \mathbb{Z}_3$	${}^3 15\,^3_1$	1234_{30} 1248_{15} 1258_{30}	
		($\underline{105}$,180,90)	$\mathbb{Z}_{15}$	${}^3 15\,^1_3$	1234_{15} 1248_{15} $125\,12_{15}$ $125\,14_{15}$ $128\,12_{15}$ $147\,11_{15}$	[52, 3_{15}]
				${}^3 15\,^1_{13}$	1234_{15} $124\,12_{15}$ 1258_{15} $125\,14_{15}$ $128\,12_{15}$ $137\,11_{15}$	[52, 9_{15}]
			D_{15}	${}^3 15\,^2_1$	1234_{15} 1245_{15} 1256_{15} 1267_{15} 1278_{15} 1289_{15}	$\partial C_4(15)$, [52, I_{15}]
	$\mathbf{T}^3$	($\underline{105}$,180,90)	$5{:}4 \times S_3$	${}^3 15\,^{11}_1$	1248_{30} $124\,12_{60}$	[51], [52, III_{15}], [53], [54, M^3_1]
	$\mathbb{R}\mathbf{P}^3$	(90,150,75)	$D_5 \times S_3$	${}^3 15\,^7_2$	1234_{30} $124\,14_{30}$ $136\,13_{15}$	
		($\underline{105}$,180,90)	$\mathbb{Z}_{15}$	${}^3 15\,^1_2$	1234_{15} 1247_{15} $127\,14_{15}$ 1369_{15} $148\,11_{15}$ $148\,12_{15}$	[52, 2_{15}]
	S^3/Q	(90,150,75)	$[3]A_5$ $= GL(2,4)$	${}^3 15\,^{15}_1$	1235_{60} $123\,15_{15}$	[19, M^3_p]
		($\underline{105}$,180,90)	$\mathbb{Z}_{15}$	${}^3 15\,^1_{10}$	1234_{15} $124\,10_{15}$ $126\,10_{15}$ $126\,14_{15}$ $138\,12_{15}$ $139\,12_{15}$	[52, 7_{15}]
				${}^3 15\,^1_{12}$	1234_{15} $124\,10_{15}$ $127\,10_{15}$ $127\,14_{15}$ $148\,11_{15}$ $148\,12_{15}$	[52, 8_{15}]

Table 4: Vertex-transitive combinatorial 3-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
	$S^2 \underline{\times} S^1$	(60,90,45)	D_{15}	$^{3}15^{2}_{7}$	1235_{30} 1245_{15}	[54, $M_2^3(15)$]
		(75,120,60)	$\mathbb{Z}_{15}$	$^{3}15^{1}_{18}$	1235_{15} $123\,13_{15}$ 1246_{15} 1256_{15}	
			D_{15}	$^{3}15^{2}_{3}$	1234_{15} 1249_{30} $138\,11_{15}$	
				$^{3}15^{2}_{4}$	1234_{15} $124\,10_{30}$ $128\,10_{15}$	
		(90,150,75)	$\mathbb{Z}_{15}$	$^{3}15^{1}_{6}$	1234_{15} 1248_{15} $128\,14_{15}$ 1379_{15} $139\,12_{15}$	
				$^{3}15^{1}_{7}$	1234_{15} 1249_{15} 1269_{15} $126\,14_{15}$ $148\,11_{15}$	
				$^{3}15^{1}_{8}$	1234_{15} 1249_{15} 1279_{15} $127\,14_{15}$ $138\,10_{15}$	
				$^{3}15^{1}_{11}$	1234_{15} $124\,10_{15}$ 1278_{15} $127\,14_{15}$ $128\,11_{15}$	
				$^{3}15^{1}_{17}$	1235_{15} $123\,12_{15}$ 1246_{15} 1257_{15} 1267_{15}	
				$^{3}15^{1}_{21}$	1237_{15} 1238_{15} 1268_{15} 1379_{15} $138\,10_{15}$	
			D_{15}	$^{3}15^{2}_{2}$	1234_{15} 1245_{15} $125\,11_{30}$ $127\,11_{15}$	
				$^{3}15^{2}_{6}$	1234_{15} $124\,14_{15}$ $136\,12_{30}$ $137\,12_{15}$	
		(<u>105</u>,180,90)	$\mathbb{Z}_{15}$	$^{3}15^{1}_{1}$	1234_{15} 1245_{15} $125\,10_{15}$ 1278_{15} $127\,13_{15}$ $128\,11_{15}$	[52, 1_{15}]
				$^{3}15^{1}_{4}$	1234_{15} 1248_{15} 1268_{15} $126\,14_{15}$ 1379_{15} $138\,10_{15}$	[52, 4_{15}]
				$^{3}15^{1}_{5}$	1234_{15} 1248_{15} $128\,14_{15}$ $136\,10_{15}$ $136\,12_{15}$ $137\,12_{15}$	[52, 5_{15}]
				$^{3}15^{1}_{9}$	1234_{15} $124\,10_{15}$ $125\,10_{15}$ $125\,11_{15}$ $127\,11_{15}$ $127\,14_{15}$	[52, 6_{15}]
				$^{3}15^{1}_{14}$	1234_{15} $124\,14_{15}$ $136\,10_{15}$ 1379_{15} $137\,13_{15}$ $139\,12_{15}$	[52, 10_{15}]
				$^{3}15^{1}_{15}$	1235_{15} $123\,11_{15}$ 1246_{15} 1257_{15} 1268_{15} 1278_{15}	[52, 11_{15}]
				$^{3}15^{1}_{16}$	1235_{15} $123\,11_{15}$ $124\,11_{15}$ $125\,10_{15}$ $135\,10_{15}$ $138\,11_{15}$	[52, 12_{15}]
				$^{3}15^{1}_{19}$	1236_{15} 1237_{15} 1257_{15} 1368_{15} 1379_{15} $138\,10_{15}$	[52, 13_{15}]
				$^{3}15^{1}_{20}$	1237_{15} 1238_{15} 1248_{15} 1249_{15} 1269_{15} $148\,11_{15}$	[52, 14_{15}]
				$^{3}15^{1}_{22}$	1237_{15} 1238_{15} $126\,14_{15}$ $128\,14_{15}$ 1379_{15} $139\,12_{15}$	[52, 15_{15}]
			D_{15}	$^{3}15^{2}_{5}$	1234_{15} $124\,14_{15}$ 1368_{15} $137\,11_{30}$ $137\,12_{15}$	[52, II_{15}]

Table 5: Vertex-transitive combinatorial 4-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
6	S^4	$(\underline{15},\underline{20},\underline{15},\underline{6})$	S_6	${}^{4}6^{16}_{1}$	12345_{6}	$\partial\,\Delta_5$, regular
9	$\mathbb{C}\mathbf{P}^2$	$(\underline{36},\underline{84},90,36)$	$\mathbb{Z}_3^2\!:\!\mathbb{Z}_6$	${}^{4}9^{13}_{1}$	12345_{9} 12347_{27}	$\mathbb{C}\mathbf{P}^2_9$, minimal, tight, [48], [49], [50], [55]
10	S^4	$(40,80,80,32)$	$2wrS_5$	${}^{4}10^{39}_{1}$	12345_{32}	∂C_5^{Δ}, regular, nncs
		$(\underline{45},100,105,42)$	$\mathbb{Z}_5$	${}^{4}10^{1}_{1}$	12345_{10} 12356_{10} 12367_{10} 12379_{10} 13579_{2}	
			D_5	${}^{4}10^{2}_{1}$	12345_{10} 12356_{10} 12367_{10} 12379_{10} 13579_{2}	
			$\frac{1}{2}[5\!:\!4]2$	${}^{4}10^{4}_{1}$	12345_{10} 12347_{20} 12359_{10} 13579_{2}	
11	$S^3\times S^1$	$(\underline{55},110,110,44)$	D_{11}	${}^{4}11^{2}_{1}$	12346_{22} 12356_{22}	min., tight, [46, M^4], [54, M^4_3], [55]
12	S^4	$(60,140,150,60)$	$A_5\times\mathbb{Z}_2$	${}^{4}12^{75}_{1}$	12469_{60}	$\mathbb{R}\mathbf{P}^2_6 *_{\Delta} \mathbb{R}\mathbf{P}^2_6$, nncs, deleted join [77], [78]
	$S^3\times S^1$	$(60,120,120,48)$	D_{12}	${}^{4}12^{12}_{1}$	12346_{24} 12356_{24}	[54, $M^4_3(12)$]
	$S^2\times S^2$	$(60,160,180,72)$	$S_3\times\mathbb{Z}_4$	${}^{4}12^{11}_{1}$	12345_{24} 12356_{24} $1236\,11_{12}$ 12569_{12}	[57], [80, M_1]
			$S_3\times D_4$	${}^{4}12^{28}_{1}$	12345_{24} $1235\,10_{24}$ $1236\,10_{24}$	[57], [80, M_3]
			$[2]A_5\!:\!2$	${}^{4}12^{124}_{1}$	12469_{60} $1247\,11_{12}$	[80, $M=M_2$], [81]
	$(S^2\times S^2)^{\#2}$	$(\underline{66},204,240,96)$	$\mathbb{Z}_3\times\mathbb{Z}_2^2$	${}^{4}12^{2}_{1}$	12346_{12} $1234\,10_{12}$ 12367_{12} $1237\,10_{12}$ 12467_{12} 12478_{12} $124\,10\,12_{12}$ $128\,10\,12_{12}$	minimal
				${}^{4}12^{2}_{2}$	12347_{12} $1234\,10_{12}$ 12357_{12} $1235\,11_{12}$ $123\,10\,11_{12}$ $1247\,11_{12}$ 12578_{12} $1258\,12_{12}$	minimal
	$S^3\,\underline{\times}\,S^1$	$(\underline{66},144,150,60)$	$t12n54(96)$	${}^{4}12^{54}_{1}$	$1245\,10_{48}$ $145\,10\,12_{12}$	

Table 5: Vertex-transitive combinatorial 4-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
13	$S^3 \times S^1$	(65,130,130,52)	D_{13}	${}^4 13^{2}_{1}$	12346_{26} 12356_{26}	[54, $M^4_3(13)$]
		($\underline{78}$,182,195,78)	$\mathbb{Z}_{13}$	${}^4 13^{1}_{1}$	12346_{13} $1234\,11_{13}$ 12356_{13} $123\,10\,11_{13}$ 12457_{13} 12467_{13}	
			D_{13}	${}^4 13^{2}_{2}$	12346_{26} 12357_{26} 12367_{26}	
				${}^4 13^{2}_{3}$	12347_{26} 12367_{26} 12457_{26}	
				${}^4 13^{2}_{4}$	12356_{26} $1235\,11_{26}$ $1246\,12_{26}$	
14	$S^3 \times S^1$	(70,140,140,56)	D_{14}	${}^4 14^{3}_{2}$	12346_{28} 12356_{28}	[54, $M^4_3(14)$]
		(77,168,175,70)	D_{14}	${}^4 14^{3}_{5}$	12349_{28} $1238\,10_{14}$ $1249\,10_{28}$	
		(84,196,210,84)	$\mathbb{Z}_{14}$	${}^4 14^{1}_{1}$	12346_{14} $1234\,12_{14}$ 12356_{14} $123\,11\,12_{14}$ 12457_{14} 12467_{14}	
			D_{7}	${}^4 14^{2}_{5}$	12345_{14} 12358_{14} 12368_{14} $1236\,13_{14}$ $1245\,13_{14}$ $124\,11\,12_{14}$	
			D_{14}	${}^4 14^{3}_{3}$	12346_{28} 12357_{28} 12367_{28}	
				${}^4 14^{3}_{4}$	12347_{28} 12367_{28} 12457_{28}	
		($\underline{91}$,224,245,98)	$\mathbb{Z}_{14}$	${}^4 14^{1}_{2}$	12348_{14} 12349_{14} 12379_{14} $1248\,10_{14}$ $1249\,10_{14}$ $1268\,13_{14}$ $1278\,13_{14}$	
				${}^4 14^{1}_{4}$	12349_{14} $1234\,10_{14}$ 12378_{14} $1237\,10_{14}$ $1249\,11_{14}$ $124\,10\,11_{14}$ $1278\,13_{14}$	
			D_{7}	${}^4 14^{2}_{1}$	12345_{14} 12356_{14} 12368_{14} $1238\,13_{14}$ $1246\,11_{14}$ $1249\,11_{14}$ $1367\,10_{14}$	
				${}^4 14^{2}_{2}$	12345_{14} 12356_{14} $1236\,11_{14}$ $123\,11\,13_{14}$ $1357\,10_{14}$ $1358\,10_{14}$ $1367\,10_{14}$	
				${}^4 14^{2}_{3}$	12345_{14} 12358_{14} 12367_{14} $1236\,13_{14}$ 12378_{14} $1245\,13_{14}$ $124\,11\,12_{14}$	

Table 5: Vertex-transitive combinatorial 4-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{4}14^{2}_{4}$	12345_{14} 12358_{14} 12368_{14} $1236\,13_{14}$ $1245\,13_{14}$ $1249\,11_{14}$ $1249\,12_{14}$	
				${}^{4}14^{2}_{6}$	12345_{14} 12358_{14} $1238\,13_{14}$ 12458_{14} $1248\,13_{14}$ $1358\,10_{14}$ $136\,11\,14_{14}$	
				${}^{4}14^{2}_{7}$	12345_{14} $1235\,13_{14}$ 12457_{14} 12468_{14} $1246\,13_{14}$ 12478_{14} $124\,11\,12_{14}$	
				${}^{4}14^{2}_{8}$	12345_{14} $1235\,13_{14}$ 12457_{14} $1247\,12_{14}$ $1257\,12_{14}$ $1357\,12_{14}$ $135\,10\,12_{14}$	
				${}^{4}14^{2}_{10}$	12347_{14} 12367_{14} $1236\,14_{14}$ $123\,10\,11_{14}$ $123\,10\,12_{14}$ 12457_{14} $1245\,13_{14}$	
				${}^{4}14^{2}_{11}$	12347_{14} 12367_{14} $1236\,14_{14}$ $123\,11\,12_{14}$ 12457_{14} $1245\,12_{14}$ $1367\,14_{14}$	
			D_{14}	${}^{4}14^{3}_{1}$	12345_{14} $1235\,10_{28}$ $1238\,10_{14}$ $1245\,10_{14}$ $1249\,10_{28}$	
	$S^3 \underline{\times} S^1$	(77,168,175,70)	D_{14}	${}^{4}14^{3}_{7}$	12378_{28} 12379_{28} $1238\,10_{14}$	
		(84,196,210,84)	D_{7}	${}^{4}14^{2}_{9}$	12347_{14} 12357_{14} $1235\,13_{14}$ $123\,12\,13_{14}$ 12458_{14} 12478_{14}	
		($\underline{91}$,224,245,98)	$\mathbb{Z}_{14}$	${}^{4}14^{1}_{3}$	12348_{14} $1234\,10_{14}$ 12378_{14} $1239\,11_{14}$ $123\,10\,11_{14}$ $1248\,10_{14}$ $1268\,13_{14}$	
				${}^{4}14^{1}_{5}$	12378_{14} 12379_{14} $1238\,11_{14}$ $1239\,11_{14}$ $1267\,13_{14}$ $1268\,13_{14}$ $1278\,13_{14}$	
			D_{14}	${}^{4}14^{3}_{6}$	12367_{28} 12368_{28} 12379_{28} $1238\,10_{14}$	

Table 5: Vertex-transitive combinatorial 4-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
15	S^4	(90,230,255,102)	A_5	${}^4 15^5_2$	$1235\ 12_{30}$ $1235\ 13_{30}$ $123\ 12\ 15_{15}$ $123\ 13\ 15_{15}$ $12\ 10\ 11\ 13_{6}$ $139\ 10\ 14_{6}$	
		(<u>105</u>,290,330,132)	A_5	${}^4 15^5_1$	$1235\ 12_{30}$ $1235\ 13_{30}$ $123\ 12\ 13_{60}$ $12\ 10\ 11\ 13_{6}$ $139\ 10\ 14_{6}$	
	$S^3 \times S^1$	(75,150,150,60)	D_{15}	${}^4 15^2_1$	12346_{30} 12356_{30}	[54, $M^4_3(15)$]
		(90,210,225,90)	D_{15}	${}^4 15^2_2$	12346_{30} 12357_{30} 12367_{30}	
				${}^4 15^2_4$	12347_{30} 12367_{30} 12457_{30}	
				${}^4 15^2_{13}$	12356_{30} $1235\ 13_{30}$ $1246\ 14_{30}$	
				${}^4 15^2_{16}$	12367_{30} $1236\ 12_{30}$ $1257\ 11_{30}$	
				${}^4 15^2_{22}$	12457_{30} $1247\ 13_{30}$ $1257\ 11_{30}$	
			$D_5 \times S_3$	${}^4 15^7_2$	12378_{60} $1237\ 11_{30}$	
			$S_5 \times S_3$	${}^4 15^{29}_2$	12458_{60} $124\ 10\ 13_{30}$	
		(<u>105</u>,270,300,120)	D_{15}	${}^4 15^2_3$	12346_{30} 12357_{30} 12368_{30} 12378_{30}	
				${}^4 15^2_5$	12347_{30} 12367_{30} 12458_{30} 12478_{30}	
				${}^4 15^2_6$	12347_{30} 12367_{30} $1247\ 14_{30}$ $124\ 11\ 13_{30}$	
				${}^4 15^2_7$	12347_{30} 12368_{30} 12378_{30} 12457_{30}	
				${}^4 15^2_8$	12348_{30} 12378_{30} 12458_{30} $124\ 12\ 13_{30}$	
				${}^4 15^2_9$	12348_{30} 12378_{30} 12468_{30} $1246\ 14_{30}$	
				${}^4 15^2_{10}$	12348_{30} 12378_{30} $1248\ 10_{30}$ $124\ 10\ 12_{30}$	
				${}^4 15^2_{11}$	12349_{30} 12378_{30} 12379_{30} $1249\ 11_{30}$	
				${}^4 15^2_{12}$	12356_{30} $1235\ 13_{30}$ $1246\ 12_{30}$ $1358\ 10_{30}$	
				${}^4 15^2_{14}$	12367_{30} 12368_{30} $1237\ 11_{30}$ $1268\ 12_{30}$	
				${}^4 15^2_{15}$	12367_{30} $1236\ 12_{30}$ $1257\ 10_{30}$ $125\ 10\ 11_{30}$	

Table 5: Vertex-transitive combinatorial 4-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{4}15^{\,2}_{\,17}$	$12367_{\,30}$ $1236\,12_{\,30}$ $1257\,13_{\,30}$ $136\,10\,12_{\,30}$	
				${}^{4}15^{\,2}_{\,18}$	$12368_{\,30}$ $1236\,12_{\,30}$ $12378_{\,30}$ $1257\,11_{\,30}$	
				${}^{4}15^{\,2}_{\,19}$	$12457_{\,30}$ $12478_{\,30}$ $1248\,10_{\,30}$ $124\,10\,13_{\,30}$	
				${}^{4}15^{\,2}_{\,20}$	$12457_{\,30}$ $1247\,10_{\,30}$ $124\,10\,11_{\,30}$ $124\,11\,13_{\,30}$	
				${}^{4}15^{\,2}_{\,21}$	$12457_{\,30}$ $1247\,13_{\,30}$ $1257\,10_{\,30}$ $125\,10\,11_{\,30}$	
				${}^{4}15^{\,2}_{\,23}$	$12458_{\,30}$ $12478_{\,30}$ $1247\,13_{\,30}$ $1257\,11_{\,30}$	
				${}^{4}15^{\,2}_{\,24}$	$1245\,10_{\,30}$ $1247\,10_{\,30}$ $1247\,14_{\,30}$ $125\,10\,11_{\,30}$	
			$\mathbb{Z}_5 \times S_3$	${}^{4}15^{\,4}_{\,2}$	$12458_{\,30}$ $12478_{\,30}$ $1247\,13_{\,30}$ $125\,10\,11_{\,30}$	
			$5\!:\!4[\frac{1}{2}]S_3$	${}^{4}15^{\,6}_{\,1}$	$12458_{\,30}$ $1245\,11_{\,60}$ $124\,10\,13_{\,30}$	
			$D_5 \times S_3$	${}^{4}15^{\,7}_{\,1}$	$12348_{\,60}$ $12378_{\,60}$	
			$S_5 \times S_3$	${}^{4}15^{\,29}_{\,1}$	$12458_{\,60}$ $124\,10\,12_{\,60}$	
	$(S^3 \underline{\times} S^1)$ $\#\,(\mathbb{C}\mathbf{P}^{\,2})^{\#5}$	$(\underline{105},320,375,150)$	$\mathbb{Z}_5 \times S_3$	${}^{4}15^{\,4}_{\,1}$	$1236\,12_{\,30}$ $1236\,13_{\,15}$ $1237\,11_{\,15}$ $1237\,13_{\,30}$ $1238\,11_{\,30}$ $1238\,12_{\,15}$ $1368\,11_{\,15}$	tight, [55]
	....	....	$\mathbb{Z}_{15}$	....	....	

Table 6: Vertex-transitive combinatorial 5-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
7	S^5	$(\underline{21},\underline{35},\underline{35},\underline{21},\underline{7})$	S_7	${}^{5}7^{7}_{1}$	123456_{7}	$\partial\,\Delta_6$, regular
8	S^5	$(\underline{28},\underline{56},68,48,16)$	S_4wr2	${}^{5}8^{47}_{1}$	123456_{16}	$\partial C_6(8) = (\partial\,\Delta_3)^{*2}$ $= \partial\Delta_1 \wr \partial C_2(4)$ [42]
9	S^5	$(\underline{36},81,108,81,27)$	S_3wrS_3	${}^{5}9^{31}_{1}$	123467_{27}	$3*3*3$ $= \partial TriC(1,2,4;9)$
		$(\underline{36},\underline{84},117,90,30)$	D_9	${}^{5}9^{3}_{1}$	123456_{9} 123467_{18} 124578_{3}	$\partial C_6(9)$
10	S^5	$(\underline{45},\underline{120},185,150,50)$	D_{10}	${}^{5}10^{3}_{1}$	123456_{10} 123467_{20} 123478_{10} 124578_{10}	$\partial C_6(10)$
11	S^5	$(\underline{55},154,242,198,66)$	D_{11}	${}^{5}11^{2}_{2}$	123456_{11} 123467_{22} 123479_{11} 123678_{11} $12457\,10_{11}$	$\partial TriC(1,2,4;11)$
		$(\underline{55},\underline{165},275,231,77)$	$\mathbb{Z}_{11}$	${}^{5}11^{1}_{1}$	123456_{11} 123467_{11} 123478_{11} $12348\,10_{11}$ $12378\,10_{11}$ $12379\,10_{11}$ $12468\,10_{11}$	
			D_{11}	${}^{5}11^{2}_{1}$	123456_{11} 123467_{22} 123478_{22} 124578_{11} 124589_{11}	$\partial C_6(11)$
12	S^5	$(60,160,240,192,64)$	$2wrS_6$	${}^{5}12^{293}_{1}$	$12468\,10_{64}$	$\partial C_6^{\Delta} = 4*4*4$, $= \partial TriC(1,3,5;12)$ regular, nncs
		$(\underline{66},196,318,264,88)$	D_6	${}^{5}12^{3}_{10}$	123456_{6} $12345\,10_{12}$ $12346\,11_{12}$ $1234\,10\,11_{12}$ $12356\,10_{12}$ $1236\,10\,11_{12}$ $1245\,10\,11_{6}$ $12469\,11_{6}$ $12569\,10_{2}$ $14589\,12_{2}$ $145\,10\,11\,12_{6}$	
				${}^{5}12^{3}_{11}$	123456_{6} $12345\,11_{12}$ $12346\,11_{12}$ $12356\,10_{12}$ $1235\,10\,11_{12}$ $1236\,10\,11_{12}$ $1245\,10\,11_{6}$ $12469\,11_{6}$ $12569\,10_{2}$ $14589\,12_{2}$ $145\,10\,11\,12_{6}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
			$t12n8(24)$ $= S_4$	$^{5}12^{8}_{3}$	123468_{24} $12346\,12_{24}$ $12348\,12_{8}$ $12358\,11_{24}$ $1236\,10\,11_{8}$	
		($\underline{66}$,204,342, 288,96)	$\frac{1}{2}[3\!:\!2]4$	$^{5}12^{5}_{4}$	123457_{12} 123458_{12} 123468_{12} $12346\,12_{12}$ $12347\,12_{12}$ $12358\,11_{12}$ 124578_{12} $1247\,10\,11_{12}$	
			$D_4 \times \mathbb{Z}_3$	$^{5}12^{14}_{1}$	123456_{24} 123467_{24} 123478_{24} $12348\,11_{24}$	
			$t12n15(24)$	$^{5}12^{15}_{1}$	123456_{12} 123458_{24} $12346\,11_{12}$ 123489_{12} $12356\,12_{24}$ 124578_{12}	
			$S_4 \times S_3$	$^{5}12^{83}_{1}$	123456_{72} 123567_{24}	$\partial TriC(1,2,5;12)$
		($\underline{66}$,208,354, 300,100)	D_6	$^{5}12^{3}_{1}$	123456_{6} 123457_{6} 123468_{12} $12347\,11_{12}$ $12348\,12_{12}$ $1234\,11\,12_{6}$ $12356\,10_{12}$ 123579_{12} $12368\,12_{6}$ $12468\,11_{6}$ $12469\,11_{6}$ $12569\,10_{2}$ $13579\,11_{2}$	
				$^{5}12^{3}_{2}$	123456_{6} 123457_{6} 123468_{12} $12347\,12_{12}$ $12348\,11_{12}$ $1234\,11\,12_{6}$ 123579_{12} $12359\,10_{12}$ $12368\,12_{6}$ $12468\,11_{6}$ $12469\,11_{6}$ $12569\,10_{2}$ $13579\,11_{2}$	
				$^{5}12^{3}_{5}$	123456_{6} 123457_{6} $12346\,10_{12}$ $12347\,12_{12}$ $1234\,10\,12_{6}$ $12357\,11_{12}$ $12369\,10_{12}$ $12369\,12_{6}$ $12378\,11_{12}$ $12478\,10_{6}$ $12569\,10_{2}$ $12578\,10_{6}$ $13579\,11_{2}$	
				$^{5}12^{3}_{7}$	123456_{6} 123458_{12} 123468_{12} 123568_{12} 124589_{12} $12459\,10_{12}$ $1245\,10\,11_{6}$ 124689_{12} $12469\,11_{6}$ $12569\,10_{2}$ $14589\,12_{2}$ $145\,10\,11\,12_{6}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{5}12^{3}_{8}$	123456_{6} 123459_{12} 123469_{12} 123569_{12} $12459\,11_{12}$ $1245\,10\,11_{6}$ $12469\,11_{6}$ $12569\,10_{2}$ 134589_{12} $13458\,10_{12}$ $14589\,12_{2}$ $145\,10\,11\,12_{6}$	
				${}^{5}12^{3}_{9}$	123456_{6} $12345\,10_{12}$ $12346\,10_{12}$ $12356\,10_{12}$ $12459\,11_{12}$ $1245\,10\,11_{6}$ $12469\,11_{6}$ $12569\,10_{2}$ 134589_{12} $13458\,10_{12}$ $14589\,12_{2}$ $145\,10\,11\,12_{6}$	
				${}^{5}12^{3}_{12}$	123456_{6} $12345\,11_{12}$ $12346\,11_{12}$ $12356\,11_{12}$ 124589_{12} $12459\,10_{12}$ $1245\,10\,11_{6}$ 124689_{12} $12469\,11_{6}$ $12569\,10_{2}$ $14589\,12_{2}$ $145\,10\,11\,12_{6}$	
		($\underline{66}$,216,378, 324,108)	$\mathbb{Z}_{12}$	${}^{5}12^{1}_{1}$	123456_{12} 123467_{12} 123478_{12} 123489_{12} $12349\,11_{12}$ $12389\,11_{12}$ $1238\,10\,11_{12}$ 124689_{12} $12469\,11_{12}$	
			D_{6}	${}^{5}12^{3}_{3}$	123456_{6} 123457_{6} 123469_{12} $12347\,10_{12}$ $12349\,12_{12}$ $1234\,10\,12_{6}$ $12356\,11_{12}$ 123578_{12} $12369\,12_{6}$ $12468\,11_{6}$ $12469\,11_{6}$ $12478\,11_{6}$ $13468\,10_{6}$	
				${}^{5}12^{3}_{4}$	123456_{6} 123457_{6} 123469_{12} $12347\,12_{12}$ $12349\,10_{12}$ $1234\,10\,12_{6}$ 123578_{12} $12358\,11_{12}$ $12369\,12_{6}$ $12468\,11_{6}$ $12469\,11_{6}$ $12478\,11_{6}$ $13468\,10_{6}$	
				${}^{5}12^{3}_{6}$	123456_{6} 123457_{6} $12346\,11_{12}$ $12347\,12_{12}$ $1234\,11\,12_{6}$ $12357\,10_{12}$ $12368\,11_{12}$ $12368\,12_{6}$ $12379\,10_{12}$ $12478\,10_{6}$ $12478\,11_{6}$ $12578\,10_{6}$ $13468\,10_{6}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
			$\frac{1}{2}[3:2]4$	${}^5 12^{5}_{1}$	123456_{12} 123457_{12} $12346\,11_{12}$ 123478_{12} 123489_{12} $12349\,12_{12}$ $12356\,11_{12}$ 123578_{12} $1239\,10\,12_{12}$	
				${}^5 12^{5}_{2}$	123456_{12} 123457_{12} $12346\,11_{12}$ $12347\,10_{12}$ $12349\,10_{12}$ $12349\,12_{12}$ $12356\,11_{12}$ 123578_{12} $1239\,10\,12_{12}$	
				${}^5 12^{5}_{3}$	123457_{12} 123458_{12} 123468_{12} 123469_{12} $12347\,12_{12}$ $12349\,12_{12}$ $1235\,10\,11_{12}$ $12369\,12_{12}$ 124578_{12}	
			$t12n8(24)$ $= S_4$	${}^5 12^{8}_{1}$	123457_{24} $12345\,12_{24}$ 123478_{24} $12348\,12_{8}$ 123579_{4} 123678_{24}	
				${}^5 12^{8}_{2}$	123458_{24} $12345\,12_{24}$ $12348\,12_{8}$ 123578_{24} 123579_{4} $12369\,10_{24}$	
			$t12n13(24)$	${}^5 12^{13}_{1}$	123457_{24} 123467_{24} 123468_{24} 123489_{24} $12456\,12_{12}$	
				${}^5 12^{13}_{2}$	123457_{24} 123478_{24} $12348\,11_{24}$ 123578_{24} $12456\,12_{12}$	
		$(\underline{66},\underline{220},390,$ $336,112)$	D_{12}	${}^5 12^{12}_{1}$	123456_{12} 123467_{24} 123478_{24} 123489_{12} 124578_{12} 124589_{24} $12569\,10_{4}$	$\partial C_6(12)$
13	S^5	$(\underline{78},247,416,$ $351,117)$	$\mathbb{Z}_{13}$	${}^5 13^{1}_{2}$	123456_{13} 123467_{13} 123478_{13} 123489_{13} $12349\,12_{13}$ $12389\,11_{13}$ $1239\,11\,12_{13}$ $12467\,12_{13}$ $12479\,12_{13}$	
			D_{13}	${}^5 13^{2}_{3}$	123456_{13} 123467_{26} 123479_{26} 123678_{13} 123789_{13} $12457\,12_{13}$ $12479\,12_{13}$	$\partial TriC(1,2,5;13)$

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
		$(\underline{78},260,455,390,130)$	$\mathbb{Z}_{13}$	$^{5}13^{1}_{1}$	123456_{13} 123467_{13} 123478_{13} 123489_{13} $12349\,11_{13}$ $1234\,11\,12_{13}$ $12389\,11_{13}$ $1238\,10\,11_{13}$ $12457\,12_{13}$ $12479\,12_{13}$	
				$^{5}13^{1}_{5}$	123456_{13} 123467_{13} 123479_{13} $12349\,12_{13}$ 123678_{13} 123789_{13} $1238\,10\,11_{13}$ $1239\,11\,12_{13}$ $12467\,12_{13}$ $12479\,12_{13}$	
				$^{5}13^{1}_{7}$	123456_{13} 123467_{13} $12347\,12_{13}$ $12367\,11_{13}$ $1237\,11\,12_{13}$ $12459\,10_{13}$ $12459\,12_{13}$ $12467\,10_{13}$ $12479\,10_{13}$ $12479\,12_{13}$	
			D_{13}	$^{5}13^{2}_{2}$	123456_{13} 123467_{26} 123478_{26} $12348\,10_{13}$ 123789_{13} 124578_{13} $12458\,11_{13}$ $1248\,10\,11_{13}$	$\partial TriC(1,2,4;13)$
			$13:6$	$^{5}13^{5}_{1}$	123456_{39} 123467_{78} $12347\,11_{13}$	$\partial TriC(1,3,4;13)$
		$(\underline{78},273,494,429,143)$	$\mathbb{Z}_{13}$	$^{5}13^{1}_{3}$	123456_{13} 123467_{13} 123478_{13} $12348\,10_{13}$ $1234\,10\,12_{13}$ 123789_{13} $1239\,10\,12_{13}$ $1239\,11\,12_{13}$ $12468\,12_{13}$ $1248\,10\,12_{13}$ $13579\,11_{13}$	
				$^{5}13^{1}_{4}$	123456_{13} 123467_{13} 123478_{13} $12348\,11_{13}$ $1234\,11\,12_{13}$ $12378\,10_{13}$ $1238\,10\,11_{13}$ $12457\,12_{13}$ $12478\,10_{13}$ $12479\,10_{13}$ $12479\,12_{13}$	
				$^{5}13^{1}_{6}$	123456_{13} 123467_{13} $12347\,12_{13}$ 123678_{13} $12368\,11_{13}$ 123789_{13} $12379\,12_{13}$ $1238\,10\,11_{13}$ $1239\,11\,12_{13}$ $12467\,12_{13}$ $1257\,10\,12_{13}$	
			D_{13}	$^{5}13^{2}_{4}$	123456_{13} 123467_{26} $12347\,11_{13}$ $12367\,11_{26}$ $12457\,10_{26}$ $12459\,10_{13}$ $12479\,10_{13}$ $12479\,12_{13}$	
				$^{5}13^{2}_{5}$	123456_{13} 123469_{26} 123567_{13} 123578_{26} 123678_{13} 123689_{26} 124689_{13} $12479\,12_{13}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
			$13:3$	$^5 13^3_1$	123456_{39} 123467_{39} $12347\,11_{13}$ $12348\,11_{39}$ $1238\,11\,12_{13}$	
		$(\underline{78},\underline{286},533,468,156)$	$\mathbb{Z}_{13}$	$^5 13^1_8$	123456_{13} $12346\,12_{13}$ 123568_{13} $12358\,10_{13}$ $1235\,10\,11_{13}$ 123678_{13} 123679_{13} $12369\,11_{13}$ $1236\,11\,12_{13}$ 123789_{13} 124579_{13} $12469\,11_{13}$	
			D_{13}	$^5 13^2_1$	123456_{13} 123467_{26} 123478_{26} 123489_{26} 124578_{13} 124589_{26} $12459\,10_{13}$ $12569\,10_{13}$	$\partial C_6(13)$
				$^5 13^2_6$	123456_{13} 123469_{26} 123567_{13} 123578_{26} 123678_{13} $12368\,11_{26}$ 124689_{13} $12478\,12_{26}$	
			$13:6$	$^5 13^5_2$	123456_{39} $12346\,10_{78}$ $12347\,11_{13}$ 123569_{26}	
	$SU(3)/SO(3)$	$(\underline{78},\underline{286},533,468,156)$	$13:3$	$^5 13^3_2$	123458_{39} 123459_{39} 123479_{39} 123569_{13} $12358\,10_{13}$ $12379\,12_{13}$	tight, [10], [55]
	$S^4 \underline{\times} S^1$	$(\underline{78},195,260,195,65)$	D_{13}	$^5 13^2_7$	123457_{26} 123467_{26} 123567_{13}	min., tight, [46, M^5], [54, M^5_4], [55]
14	S^5	$(84,266,448,378,126)$	$L(2,7) \times \mathbb{Z}_2$	$^5 14^{19}_1$	123456_{84} $12346\,12_{42}$	
		$(84,280,490,420,140)$	$2[\frac{1}{2}]7:6$	$^5 14^4_1$	123456_{21} $12345\,14_{21}$ 123467_{42} $12347\,12_{7}$ $12356\,11_{42}$ $1236\,11\,14_{7}$	
			$7:6 \times \mathbb{Z}_2$	$^5 14^7_1$	123456_{42} $12346\,12_{84}$ $12347\,12_{14}$	nncs
			$S_7 \times \mathbb{Z}_2$	$^5 14^{49}_1$	123456_{140}	$\partial TriC(1,3,5;14)$, nncs
		$(\underline{91},308,539,462,154)$	D_{14}	$^5 14^3_3$	123456_{14} 123467_{28} 123479_{28} $12349\,10_{14}$ 123678_{14} 123789_{14} $12457\,13_{14}$ $12479\,10_{14}$ $1247\,10\,13_{14}$	$\partial TriC(1,2,5;14)$
			$2[\frac{1}{2}]7:6$	$^5 14^4_2$	123456_{21} $12345\,14_{21}$ 123467_{42} $12347\,12_{7}$ $12356\,12_{42}$ $1235\,12\,14_{21}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				$^5 14^4_3$	123456_{21} $12345\,14_{21}$ $12346\,12_{42}$ $12347\,12_{7}$ $12356\,12_{42}$ $1235\,12\,14_{21}$	
				$^5 14^4_4$	$12346\,11_{42}$ $12346\,13_{21}$ $12348\,11_{21}$ $12369\,13_{42}$ $1236\,11\,14_{7}$ $12389\,14_{21}$	
		($\underline{91}$,322,581, 504,168)	D_{14}	$^5 14^3_5$	123456_{14} 123467_{28} $12347\,12_{14}$ $12367\,11_{28}$ 124578_{14} $12458\,12_{14}$ $12478\,12_{28}$ $12569\,10_{14}$ $12569\,12_{14}$	$\partial TriC(1,3,4;14)$
			$L(2,7):2$	$^5 14^{16}_1$	$12345\,12_{84}$ $12367\,10_{56}$ $12367\,14_{28}$	
		($\underline{91}$,336,623, 546,182)	D_{14}	$^5 14^3_6$	123456_{14} 123468_{28} 123489_{28} $12349\,10_{14}$ 123567_{14} 123678_{14} 124679_{28} 124689_{14} $12479\,10_{14}$ $1247\,10\,13_{14}$	
				$^5 14^3_7$	123456_{14} 123469_{28} $12349\,10_{14}$ 123567_{14} 123578_{28} 123678_{14} 123689_{28} 124689_{14} $12479\,10_{14}$ $1247\,10\,13_{14}$	
		($\underline{91}$,350,665, 588,196)	D_{14}	$^5 14^3_2$	123456_{14} 123467_{28} 123478_{28} $12348\,11_{14}$ $12378\,11_{28}$ 124578_{14} $12458\,12_{14}$ $1248\,11\,12_{28}$ $12569\,10_{14}$ $1256\,10\,11_{14}$	
				$^5 14^3_4$	123456_{14} 123467_{28} $12347\,11_{28}$ $12348\,11_{14}$ $12367\,10_{28}$ 124578_{14} $12458\,12_{14}$ $12478\,11_{28}$ $12569\,10_{14}$ $1256\,10\,11_{14}$	
		($\underline{91}$,$\underline{364}$,707, 630,210)	D_{14}	$^5 14^3_1$	123456_{14} 123467_{28} 123478_{28} 123489_{28} 1234910_{14} 124578_{14} 124589_{28} $12459\,10_{28}$ $12569\,10_{14}$ $1256\,10\,11_{14}$	$\partial C_6(14)$
			$2[\frac{1}{2}]7:6$	$^5 14^4_5$	123478_{42} $12347\,12_{7}$ 123489_{42} $12349\,10_{21}$ $12378\,10_{42}$ $12389\,10_{21}$ $123\,10\,11\,12_{21}$ $12489\,11_{14}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
	$S^4 \times S^1$	(84,210,280, 210,70)	D_{14}	${}^5 14^{\,3}_{\,8}$	123457_{28} 123467_{28} 123567_{14}	[54, $M^5_4(14)$]
		($\underline{91}$,252,371, 294,98)	D_{14}	${}^5 14^{\,3}_{\,10}$	123489_{28} $12348\,10_{28}$ 123789_{14} $12489\,11_{14}$ $1248\,10\,11_{14}$	
				${}^5 14^{\,3}_{\,16}$	123789_{14} $12378\,10_{28}$ $12379\,10_{28}$ $12489\,11_{14}$ $1248\,10\,11_{14}$	
	$S^3 \times S^2$	(84,280,490, 420,140)	D_{14}	${}^5 14^{\,3}_{\,9}$	123467_{28} $12346\,12_{28}$ 123567_{14} $12357\,11_{28}$ $12457\,13_{14}$ $1246\,10\,12_{28}$	
		($\underline{91}$,336,623, 546,182)	D_{14}	${}^5 14^{\,3}_{\,14}$	123567_{14} $12356\,12_{28}$ 123579_{28} $12359\,11_{28}$ $1235\,11\,12_{28}$ $12459\,11_{14}$ 124689_{14} $12489\,11_{14}$ $1248\,10\,11_{14}$	
		($\underline{91}$,350,665, 588,196)	D_{14}	${}^5 14^{\,3}_{\,13}$	123567_{14} 123569_{28} $12357\,13_{28}$ $12359\,11_{28}$ 123679_{28} 124589_{28} $12459\,11_{14}$ $12489\,11_{14}$ $1248\,10\,11_{14}$	
		($\underline{91}$,350,665, 588,196)	D_{14}	${}^5 14^{\,3}_{\,15}$	123567_{14} $12356\,13_{28}$ 123579_{28} $12359\,11_{28}$ $1235\,11\,12_{28}$ 124589_{28} $12459\,11_{14}$ $12489\,11_{14}$ $1248\,10\,11_{14}$	
	$S^4 \mathbin{\underline{\times}} S^1$	($\underline{91}$,252,371, 294,98)	D_{14}	${}^5 14^{\,3}_{\,11}$	123489_{28} $12348\,11_{14}$ $12349\,10_{14}$ $12378\,10_{28}$ $12489\,11_{14}$	
				${}^5 14^{\,3}_{\,12}$	$12348\,10_{28}$ $12348\,11_{14}$ $12349\,10_{14}$ $12379\,10_{28}$ $12489\,11_{14}$	
	....	....	$\mathbb{Z}_{14}$, D_7	....	....	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
15	S^5	(90,275,450, 375,125)	$D_5 wr S_3$	$^{5}15^{60}_{1}$	123456_{125}	$5*5*5$ $= \partial TriC(1,4,6;15)$
		($\underline{105}$,350,600, 510,170)	D_{15}	$^{5}15^{2}_{9}$	123456_{15} $12346\,14_{15}$ $12356\,13_{30}$ $12459\,12_{15}$ 124679_{30} $12479\,14_{30}$ $1249\,11\,12_{15}$ $12569\,13_{15}$ $1368\,11\,13_{5}$	
			$\mathbb{Z}_5 \times S_3$	$^{5}15^{4}_{3}$	123456_{30} $12346\,14_{30}$ 123568_{30} $12358\,13_{30}$ $12368\,13_{15}$ $1236\,13\,14_{15}$ $12469\,14_{15}$ $1368\,11\,13_{5}$	
			$D_5 \times S_3$	$^{5}15^{7}_{3}$	123456_{30} $12346\,14_{30}$ $12356\,14_{60}$ 124689_{15} $12469\,14_{30}$ $1368\,11\,13_{5}$	
		($\underline{105}$,365,645, 555,185)	D_{15}	$^{5}15^{2}_{3}$	123456_{15} 123467_{30} 123479_{30} $12349\,11_{15}$ 123678_{15} 123789_{15} $12389\,10_{15}$ $12457\,14_{15}$ $12479\,14_{30}$ $1368\,11\,13_{5}$	
			$D_5 \times \mathbb{Z}_3$	$^{5}15^{3}_{3}$	123458_{30} 123459_{30} 123469_{30} $12346\,14_{30}$ $12347\,14_{30}$ $12469\,14_{30}$ $1368\,11\,13_{5}$	
		($\underline{105}$,380,690, 600,200)	D_{15}	$^{5}15^{2}_{5}$	123456_{15} 123468_{30} 123489_{30} $12349\,11_{15}$ 123567_{15} 123678_{15} $12389\,10_{15}$ 124689_{15} $12469\,14_{30}$ $1249\,11\,12_{15}$ $1368\,11\,13_{5}$	
			$D_5 \times \mathbb{Z}_3$	$^{5}15^{3}_{1}$	123457_{30} 123458_{30} 123468_{30} 123568_{30} $12356\,14_{30}$ 124689_{15} $12469\,14_{30}$ $1368\,11\,13_{5}$	
			$\mathbb{Z}_5 \times S_3$	$^{5}15^{4}_{4}$	123457_{30} 123458_{30} 123468_{30} 123568_{30} $12356\,14_{30}$ 124689_{15} $12469\,14_{15}$ $1249\,12\,14_{15}$ $1368\,11\,13_{5}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				$^{5}15^{4}_{5}$	123457_{30} 123458_{30} 123468_{30} $12358\,13_{30}$ $1235\,13\,14_{30}$ $12368\,13_{15}$ $1236\,13\,14_{15}$ $12469\,14_{15}$ $1368\,11\,13_{5}$	
		($\underline{105}$,395,735, 645,215)	D_{15}	$^{5}15^{2}_{4}$	123456_{15} 123468_{30} 123489_{30} $12349\,11_{15}$ 123567_{15} 123678_{15} $12389\,10_{15}$ 124679_{30} 124689_{15} $12479\,14_{30}$ $1368\,11\,13_{5}$	$\partial TriC(1,2,6;15)$
				$^{5}15^{2}_{6}$	123456_{15} 123469_{30} $12349\,11_{15}$ 123567_{15} 123578_{30} 123678_{15} 123689_{30} $12389\,10_{15}$ 124689_{15} $12479\,14_{30}$ $1368\,11\,13_{5}$	
			$D_5 \times \mathbb{Z}_3$	$^{5}15^{3}_{4}$	123458_{30} 123459_{30} $12347\,14_{30}$ $12349\,14_{30}$ $12358\,13_{30}$ $12369\,14_{30}$ $1247\,12\,14_{30}$ $1368\,11\,13_{5}$	
			$\mathbb{Z}_5 \times S_3$	$^{5}15^{4}_{1}$	123456_{30} 123467_{30} 123478_{30} $12348\,13_{30}$ $1234\,13\,14_{30}$ $12378\,12_{15}$ $1238\,12\,13_{15}$ 124578_{30} $1267\,11\,12_{5}$	
		($\underline{105}$,405,765, 675,225)	D_{15}	$^{5}15^{2}_{2}$	123456_{15} 123467_{30} 123478_{30} 123489_{30} $12349\,11_{15}$ $12389\,10_{15}$ 124578_{15} 124589_{30} $12459\,12_{15}$ $1249\,11\,12_{15}$ $12569\,13_{15}$	$\partial TriC(1,2,4;15)$
		($\underline{105}$,410,780, 690,230)	$D_5 \times \mathbb{Z}_3$	$^{5}15^{3}_{5}$	123467_{30} 123468_{30} 123478_{30} 123568_{30} 123578_{30} 124578_{30} 124689_{15} $12469\,14_{30}$ $1368\,11\,13_{5}$	
			$\mathbb{Z}_5 \times S_3$	$^{5}15^{4}_{8}$	123467_{30} 123468_{30} 123478_{30} 123568_{30} 123578_{30} 124578_{30} 124689_{15} $12469\,14_{15}$ $1249\,12\,14_{15}$ $1368\,11\,13_{5}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
			$D_5 \times S_3$	${}^5 15^{\,7}_{\,1}$	123456_{30} 123467_{60} 123478_{60} $12348\,12_{15}$ $12378\,12_{30}$ 124578_{30} $1267\,11\,12_{5}$	$\partial TriC(1,3,4;15)$
				${}^5 15^{\,7}_{\,2}$	123456_{30} 123467_{60} $12347\,12_{60}$ $12348\,12_{15}$ $12367\,11_{30}$ 124578_{30} $1267\,11\,12_{5}$	
		$(\underline{105},425,825,735,245)$	$D_5 \times \mathbb{Z}_3$	${}^5 15^{\,3}_{\,2}$	123457_{30} $12345\,13_{30}$ $12346\,13_{30}$ $12347\,12_{30}$ $12357\,12_{30}$ $1235\,10\,13_{30}$ $1235\,10\,14_{30}$ $1246\,11\,14_{30}$ $1368\,11\,13_{5}$	
		$(\underline{105},440,870,780,260)$	D_{15}	${}^5 15^{\,2}_{\,8}$	123456_{15} $12346\,10_{30}$ 123567_{15} 123579_{30} 123678_{15} $12368\,11_{30}$ $1236\,10\,11_{30}$ 123789_{15} 124689_{15} $12469\,10_{30}$ $12489\,14_{30}$ $1368\,11\,13_{5}$	
		$(\underline{105},\underline{455},915,825,275)$	D_{15}	${}^5 15^{\,2}_{\,1}$	123456_{15} 123467_{30} 123478_{30} 123489_{30} $12349\,10_{30}$ 124578_{15} 124589_{30} $12459\,10_{30}$ $1245\,10\,11_{15}$ $12569\,10_{15}$ $1256\,10\,11_{30}$ $1267\,11\,12_{5}$	$\partial C_6(15)$
				${}^5 15^{\,2}_{\,7}$	123456_{15} $12346\,10_{30}$ 123567_{15} 123578_{30} 123589_{30} 123678_{15} $12368\,11_{30}$ $1236\,10\,11_{30}$ 124689_{15} $12469\,10_{30}$ $12489\,14_{30}$ $1368\,11\,13_{5}$	
	$SU(3)/SO(3)$	$(\underline{105},410,780,690,230)$	$\mathbb{Z}_5 \times S_3$	${}^5 15^{\,4}_{\,6}$	123459_{30} $12345\,10_{30}$ $12348\,10_{30}$ $12356\,10_{30}$ $12356\,11_{30}$ $12359\,11_{30}$ $12367\,11_{15}$ $12379\,11_{15}$ $1267\,11\,12_{5}$ $1267\,12\,14_{15}$	
		$(\underline{105},440,870,780,260)$	$\mathbb{Z}_5 \times S_3$	${}^5 15^{\,4}_{\,2}$	123456_{30} $12346\,10_{30}$ $12348\,10_{30}$ $12348\,14_{30}$ $12356\,11_{30}$ $12359\,11_{30}$ $12367\,10_{30}$ $12367\,11_{15}$ $12379\,11_{15}$ $1267\,11\,12_{5}$ $1267\,12\,14_{15}$	

Table 6: Vertex-transitive combinatorial 5-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{5}15^{4}_{7}$	$12345\,10_{30}$ $12345\,11_{30}$ $12349\,11_{30}$ $12356\,10_{30}$ $12356\,11_{30}$ $12367\,11_{15}$ $12368\,10_{30}$ $12368\,14_{30}$ $12379\,11_{15}$ $1267\,11\,12_{5}$ $1267\,12\,14_{15}$	
	$S^4 \mathbin{\underline{\times}} S^1$	(90,225,300, 225,75)	D_{15}	${}^{5}15^{2}_{10}$	123457_{30} 123467_{30} 123567_{15}	[54, $M^5_4(15)$]
		($\underline{105}$,300,450, 360,120)	D_{15}	${}^{5}15^{2}_{12}$	$12345\,10_{30}$ $12349\,11_{15}$ $1235\,10\,11_{30}$ $12389\,11_{30}$ $1245\,10\,11_{15}$	
		($\underline{105}$,305,465, 375,125)	D_{15}	${}^{5}15^{2}_{14}$	123678_{15} $12367\,12_{30}$ $12368\,11_{30}$ $1236\,11\,12_{30}$ $1257\,10\,11_{15}$ $1368\,11\,13_{5}$	
			$D_5 \times \mathbb{Z}_3$	${}^{5}15^{3}_{6}$	$12368\,11_{30}$ $12368\,12_{30}$ $1236\,11\,12_{30}$ $12378\,12_{30}$ $1368\,11\,13_{5}$	
			$D_5 \times S_3$	${}^{5}15^{7}_{4}$	123678_{30} $12367\,11_{30}$ $12368\,13_{30}$ $12378\,12_{30}$ $1368\,11\,13_{5}$	
		($\underline{105}$,315,495, 405,135)	D_{15}	${}^{5}15^{2}_{11}$	123458_{30} 123478_{30} 123567_{15} $12356\,14_{30}$ 123578_{30}	
		($\underline{105}$,320,510, 420,140)	D_{15}	${}^{5}15^{2}_{13}$	123678_{15} $12367\,11_{30}$ $12368\,13_{30}$ $12378\,11_{30}$ $1256\,10\,11_{30}$ $1368\,11\,13_{5}$	
				${}^{5}15^{2}_{15}$	123678_{15} $12367\,13_{30}$ $12368\,11_{30}$ $1236\,11\,12_{30}$ $1256\,10\,11_{30}$ $1368\,11\,13_{5}$	
				${}^{5}15^{2}_{16}$	$12368\,11_{30}$ $12368\,12_{30}$ $1236\,11\,12_{30}$ $12378\,12_{30}$ $1257\,10\,11_{15}$ $1368\,11\,13_{5}$	
	....	....	$\mathbb{Z}_{15}$	....	....	

Table 7: Vertex-transitive combinatorial 6-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
8	S^6	$(\underline{28},\underline{56},\underline{70},\underline{56},\underline{28},\underline{8})$	S_8	${}^{6}8^{50}_{1}$	1234567_{8}	$\partial\,\Delta_7$, regular
10	S^6	$(\underline{45},\underline{120},205,222,140,40)$	$2wrD_5$	${}^{6}10^{23}_{1}$	1234567_{40}	$\partial\Delta_1 \wr \partial C_2(5)$ [42]
14	S^6	$(84,280,560,672,448,128)$	$2wrS_7$	${}^{6}14^{57}_{1}$	1234567_{128}	∂C_7^{Δ}, regular, nncs
		$(\underline{91},322,665,812,546,156)$	$2[\frac{1}{2}]7:6$	${}^{6}14^{4}_{1}$	1234567_{42} $123457\,13_{42}$ $123467\,12_{14}$ $12356\,11\,12_{42}$ $12357\,11\,13_{14}$ $13579\,11\,13_{2}$	
		$(\underline{91},\underline{364},875,1190,840,240)$	$2[\frac{1}{2}]7:6$	${}^{6}14^{4}_{2}$	1234568_{42} $123458\,14_{42}$ $12345\,11\,13_{42}$ $123567\,10_{42}$ $123567\,13_{42}$ $12356\,11\,14_{14}$ $12357\,11\,13_{14}$ $13579\,11\,13_{2}$	
	....	....	$\mathbb{Z}_{14}$, D_7	....	....	
15	$S^5\times S^1$	$(\underline{105},315,525,525,315,90)$	D_{15}	${}^{6}15^{2}_{1}$	1234568_{30} 1234578_{30} 1234678_{30}	min., tight, [46, M^6], [54, M^6_5], [55]
	$S^3\times S^3$	$(\underline{105},435,1125,1605,1155,330)$	D_{15}	${}^{6}15^{2}_{5}$	$123458\,10_{30}$ $123458\,11_{30}$ 1234689_{30} $123468\,14_{30}$ $123469\,12_{30}$ 1234789_{30} $123478\,10_{30}$ $123578\,10_{30}$ $12357\,10\,13_{30}$ $12458\,10\,11_{30}$ $12478\,11\,14_{30}$	
				${}^{6}15^{2}_{6}$	$123458\,10_{30}$ $123458\,11_{30}$ 1234689_{30} $123468\,14_{30}$ $123469\,12_{30}$ 1234789_{30} $123478\,10_{30}$ $123578\,10_{30}$ $12357\,10\,13_{30}$ $12458\,10\,13_{30}$ $12478\,10\,14_{30}$	
			$D_5\times S_3$	${}^{6}15^{7}_{2}$	1234589_{60} $123458\,12_{30}$ 1234789_{60} $123479\,11_{60}$ 1235689_{60} $123568\,10_{60}$	
		$(\underline{105},450,1200,1740,1260,360)$	D_{15}	${}^{6}15^{2}_{2}$	1234589_{30} $123458\,10_{30}$ $123459\,11_{30}$ 1234789_{30} 1235689_{30} $123568\,14_{30}$ $123569\,14_{30}$ $12358\,10\,14_{30}$ $1235\,10\,12\,13_{30}$ $124579\,11_{30}$ $12457\,11\,13_{30}$ $12479\,11\,13_{30}$	

Table 7: Vertex-transitive combinatorial 6-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{6}15^{2}_{3}$	1234589_{30} $123458\,10_{30}$ $123459\,11_{30}$ 1234789_{30} $123589\,14_{30}$ $12358\,10\,14_{30}$ $1235\,10\,11\,13_{30}$ $1235\,10\,12\,13_{30}$ $124578\,11_{30}$ $12457\,11\,13_{30}$ $124589\,11_{30}$ $12479\,11\,13_{30}$	
				${}^{6}15^{2}_{4}$	1234589_{30} $123458\,10_{30}$ $123459\,11_{30}$ 1234789_{30} $123589\,14_{30}$ $12358\,10\,14_{30}$ $1235\,10\,11\,13_{30}$ $1235\,10\,12\,13_{30}$ $124589\,11_{30}$ $12458\,10\,13_{30}$ $12478\,11\,13_{30}$ $12479\,11\,13_{30}$	
			$D_5 \times \mathbb{Z}_3$	${}^{6}15^{3}_{1}$	1234589_{30} $123458\,10_{30}$ $123459\,13_{15}$ $12345\,10\,12_{30}$ $12345\,12\,13_{30}$ 1234789_{30} $12348\,10\,12_{15}$ $12349\,11\,13_{30}$ $1234\,11\,12\,13_{30}$ 1235689_{30} $123568\,10_{30}$ $1235\,10\,13\,14_{30}$ $123679\,14_{30}$	
				${}^{6}15^{3}_{2}$	1234589_{30} $123458\,10_{30}$ $123459\,13_{15}$ $12345\,10\,13_{30}$ 1234789_{30} $12348\,10\,12_{15}$ $12349\,12\,13_{30}$ $1234\,10\,12\,13_{30}$ $123589\,14_{15}$ $12358\,10\,14_{30}$ $1235\,10\,13\,14_{30}$ $123678\,14_{30}$ $123789\,14_{30}$ $124578\,13_{15}$	
			$D_5 \times S_3$	${}^{6}15^{7}_{1}$	1234589_{60} $123458\,10_{60}$ $123459\,11_{60}$ 1234789_{60} 1235689_{60} $123568\,10_{60}$	
	....	....	$\mathbb{Z}_{15}$	....	....	

Table 8: Vertex-transitive combinatorial 7-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
9	S^7	$(\underline{36},\underline{84},\underline{126},\underline{126},\underline{84},\underline{36},\underline{9})$	S_9	${}^{7}9^{34}_{1}$	$12345678_{\,9}$	$\partial\,\Delta_8$, regular
10	S^7	$(\underline{45},\underline{120},\underline{210},250,200,100,25)$	S_5wr2	${}^{7}10^{43}_{1}$	$12345678_{\,25}$	$\partial C_8(10) = (\partial\,\Delta_4)^{*2}$
11	S^7	$(\underline{55},\underline{165},\underline{330},451,407,220,55)$	D_{11}	${}^{7}11^{2}_{1}$	$12345678_{\,11}$ $12345689_{\,22}$ $12346789_{\,11}$ $1234679\,10_{\,11}$	$\partial C_8(11)$
12	S^7	$(\underline{66},216,459,648,594,324,81)$	S_3wrS_4	${}^{7}12^{289}_{1}$	$12345678_{\,81}$	$3*3*3*3$
		$(\underline{66},\underline{220},483,708,670,372,93)$	$t12n8(24)$ $= S_4$	${}^{7}12^{8}_{1}$	$1234567\,11_{\,24}$ $1234567\,12_{\,24}$ $123457\,11\,12_{\,24}$ $123459\,11\,12_{\,12}$ $123467\,10\,12_{\,6}$ $1246789\,10_{\,3}$	
		$(\underline{66},\underline{220},486,720,688,384,96)$	$2wrD_6$	${}^{7}12^{193}_{1}$	$1234568\,10_{\,96}$	$\partial\Delta_1 \wr \partial C_2(6)$ [42]
		$(\underline{66},\underline{220},\underline{495},756,742,420,105)$	D_{12}	${}^{7}12^{12}_{1}$	$12345678_{\,12}$ $12345689_{\,24}$ $1234569\,10_{\,12}$ $12346789_{\,12}$ $1234679\,10_{\,24}$ $123467\,10\,11_{\,12}$ $1234789\,10_{\,6}$ $124578\,10\,11_{\,3}$	$\partial C_8(12)$
13	S^7	$(\underline{78},\underline{286},689,1092,1092,624,156)$	$\mathbb{Z}_{13}$	${}^{7}13^{1}_{1}$	$12345678_{\,13}$ $12345689_{\,13}$ $1234569\,10_{\,13}$ $123456\,10\,12_{\,13}$ $123459\,10\,11_{\,13}$ $12345\,10\,11\,12_{\,13}$ $1234678\,12_{\,13}$ $1234689\,10_{\,13}$ $123468\,10\,12_{\,13}$ $1235679\,10_{\,13}$ $123567\,10\,11_{\,13}$ $123579\,10\,11_{\,13}$	
		$(\underline{78},\underline{286},702,1144,1170,676,169)$	D_{13}	${}^{7}13^{2}_{2}$	$12345678_{\,13}$ $12345689_{\,26}$ $1234569\,10_{\,26}$ $1234678\,12_{\,26}$ $1234689\,10_{\,26}$ $123468\,10\,12_{\,13}$ $123567\,10\,11_{\,13}$ $1235689\,10_{\,13}$ $123579\,10\,11_{\,13}$	

Table 8: Vertex-transitive combinatorial 7-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{7}13^{2}_{3}$	12345678_{13} 12345689_{26} $1234569\,11_{13}$ $1234589\,11_{26}$ 12346789_{13} $1234679\,12_{26}$ $1234789\,10_{13}$ $1235689\,11_{26}$ $124578\,10\,11_{13}$	
			$13\!:\!4$	${}^{7}13^{4}_{1}$	12345678_{26} 12345689_{52} $1234569\,10_{52}$ 12346789_{26} $1235689\,10_{13}$	
		$(\underline{78},\underline{286},\underline{715},1196,$ $1248,728,182)$	D_{13}	${}^{7}13^{2}_{1}$	12345678_{13} 12345689_{26} $1234569\,10_{26}$ 12346789_{13} $1234679\,10_{26}$ $123467\,10\,11_{26}$ $123467\,11\,12_{13}$ $1234789\,10_{13}$ $123478\,10\,11_{13}$ $124578\,10\,11_{13}$	$\partial C_8(13)$
			$13\!:\!4$	${}^{7}13^{4}_{2}$	12345678_{26} 12345689_{52} $1234569\,11_{13}$ $1234589\,10_{52}$ 12346789_{26} $1235689\,10_{13}$	
14	S^7	$(\underline{91},350,861,1372,$ $1372,784,196)$	D_7wr2	${}^{7}14^{20}_{1}$	12345678_{49} $1234578\,10_{98}$ $1234789\,10_{49}$	$(\partial C_4(7))^{*2}$
		$(\underline{91},\underline{364},973,1694,$ $1806,1064,266)$	D_{14}	${}^{7}14^{3}_{2}$	12345678_{14} 12345689_{28} $1234569\,10_{28}$ $123456\,10\,11_{14}$ 12346789_{14} $1234679\,10_{28}$ $123467\,10\,12_{28}$ $123467\,12\,13_{14}$ $12346\,10\,11\,12_{28}$ $123479\,10\,12_{14}$ $123569\,10\,11_{28}$ $123678\,11\,12_{14}$ $124578\,10\,13_{14}$	
				${}^{7}14^{3}_{3}$	12345678_{14} 12345689_{28} $1234569\,10_{28}$ $123456\,10\,11_{14}$ $1234678\,13_{28}$ $1234689\,10_{28}$ $123468\,10\,11_{28}$ $123468\,11\,13_{14}$ $123489\,10\,11_{7}$ $123567\,10\,12_{14}$ $1235689\,10_{14}$ $12356\,10\,11\,12_{28}$ $123678\,11\,12_{14}$ $124689\,11\,13_{7}$	

Table 8: Vertex-transitive combinatorial 7-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{7}14^{3}_{4}$	12345678_{14} 12345689_{28} $1234569\,10_{28}$ $123456\,10\,11_{14}$ $1234678\,13_{28}$ $1234689\,11_{28}$ $123468\,11\,13_{14}$ $123469\,10\,11_{28}$ $123489\,10\,11_{7}$ $123567\,10\,12_{14}$ $1235689\,10_{14}$ $12356\,10\,11\,12_{28}$ $123678\,11\,12_{14}$ $124689\,11\,13_{7}$	
				${}^{7}14^{3}_{5}$	12345678_{14} 12345689_{28} $1234569\,12_{14}$ $1234589\,11_{28}$ 12346789_{14} $1234679\,13_{28}$ $123469\,10\,12_{28}$ $123469\,10\,13_{14}$ $1234789\,10_{14}$ $123489\,10\,11_{7}$ $1235689\,12_{28}$ $124578\,10\,12_{28}$ $124578\,10\,13_{14}$ $124589\,11\,12_{7}$	
		($\underline{91}$,$\underline{364}$,$\underline{1001}$,1806, 1974,1176,294)	D_{14}	${}^{7}14^{3}_{1}$	12345678_{14} 12345689_{28} $1234569\,10_{28}$ $123456\,10\,11_{14}$ 12346789_{14} $1234679\,10_{28}$ $123467\,10\,11_{28}$ $123467\,11\,12_{28}$ $123467\,12\,13_{14}$ $1234789\,10_{14}$ $123478\,10\,11_{28}$ $123478\,11\,12_{14}$ $123489\,10\,11_{7}$ $124578\,10\,11_{14}$ $124578\,11\,12_{14}$ $124589\,11\,12_{7}$	$\partial C_8(14)$
	....	....	$\mathbb{Z}_{14}$, D_7	....	....	
15	S^7	($\underline{105}$,450,1260,2250, 2430,1440,360)	$D_5 \times \mathbb{Z}_3$	${}^{7}15^{3}_{1}$	12345678_{30} 12345689_{30} $1234569\,10_{30}$ $123456\,10\,12_{30}$ $123456\,12\,14_{30}$ $123459\,10\,12_{30}$ $123459\,11\,12_{30}$ $12345\,11\,12\,13_{30}$ $12345\,12\,13\,14_{30}$ $1234678\,14_{30}$ $1234689\,10_{30}$ $123489\,10\,11_{30}$	

Table 8: Vertex-transitive combinatorial 7-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{7}15^{3}_{2}$	12345678_{30} 12345689_{30} $1234569\,12_{30}$ $123456\,12\,14_{30}$ $1234589\,11_{30}$ $123459\,11\,12_{30}$ $12345\,11\,12\,13_{30}$ $12345\,12\,13\,14_{30}$ $1234678\,10_{30}$ $1234689\,10_{30}$ $123469\,10\,12_{30}$ $123489\,10\,11_{30}$	
			$5:4\times\mathbb{Z}_3$	${}^{7}15^{8}_{1}$	12345678_{60} 12345689_{60} $1234569\,10_{60}$ $123456\,10\,14_{60}$ $123459\,10\,12_{60}$ $1234678\,14_{60}$	
			$S_5\times S_3$	${}^{7}15^{29}_{1}$	12345678_{360}	
		($\underline{105}$,450,1275,2310, 2520,1500,375)	$5:4[\frac{1}{2}]S_3$	${}^{7}15^{6}_{1}$	12345678_{60} 12345689_{60} $1234569\,12_{60}$ $123456\,12\,14_{60}$ $1234589\,11_{60}$ $123459\,11\,12_{60}$ $1234689\,12_{15}$	
				${}^{7}15^{6}_{2}$	12345678_{60} $1234568\,10_{60}$ $123456\,10\,14_{60}$ 12345789_{60} $12345\,10\,12\,13_{60}$ $12345\,10\,13\,14_{60}$ $12348\,11\,12\,14_{15}$	
		($\underline{105}$,$\underline{455}$,1275,2265, 2435,1440,360)	D_{15}	${}^{7}15^{2}_{6}$	12345678_{15} 12345689_{30} $1234569\,11_{30}$ $1234589\,10_{30}$ $123459\,10\,12_{30}$ 12346789_{15} $1234679\,14_{30}$ $123469\,11\,14_{15}$ $123489\,10\,11_{15}$ $1235689\,13_{30}$ $123569\,11\,13_{30}$ $123589\,10\,13_{30}$ $12359\,10\,12\,13_{30}$ $12359\,11\,12\,13_{15}$ $124579\,12\,14_{15}$	

Table 8: Vertex-transitive combinatorial 7-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
		($\underline{105}$,$\underline{455}$,1290,2325, 2525,1500,375)	D_{15}	$^{7}15^{2}_{2}$	12345678_{15} 12345689_{30} $1234569\,10_{30}$ $123456\,10\,11_{30}$ $1234678\,14_{30}$ $1234689\,10_{30}$ $123468\,10\,12_{30}$ $123468\,12\,14_{15}$ $12346\,10\,11\,12_{30}$ $123567\,10\,11_{30}$ $123567\,11\,12_{15}$ $1235689\,10_{15}$ $123579\,10\,11_{15}$ $12357\,11\,12\,13_{30}$ $12367\,10\,11\,12_{15}$ $12468\,10\,12\,14_{15}$	
				$^{7}15^{2}_{5}$	12345678_{15} 12345689_{30} $1234569\,11_{30}$ $1234589\,10_{30}$ $123459\,10\,12_{30}$ 12346789_{15} $1234679\,14_{30}$ $123469\,11\,14_{15}$ $123489\,10\,11_{15}$ $1235689\,10_{15}$ $123568\,10\,13_{30}$ $123569\,10\,13_{30}$ $123569\,11\,13_{30}$ $12359\,10\,12\,13_{30}$ $12359\,11\,12\,13_{15}$ $124579\,12\,14_{15}$	
				$^{7}15^{2}_{8}$	12345678_{15} 12345689_{30} $1234569\,13_{15}$ $1234589\,10_{30}$ $123458\,10\,12_{30}$ $123459\,10\,12_{30}$ 12346789_{15} $1234679\,14_{30}$ $123469\,11\,13_{30}$ $123469\,11\,14_{15}$ $123489\,10\,11_{15}$ $1235689\,13_{30}$ $123589\,10\,13_{30}$ $12359\,10\,12\,13_{30}$ $12359\,11\,12\,13_{15}$ $124579\,12\,14_{15}$	
		($\underline{105}$,$\underline{455}$,1305,2385, 2615,1560,390)	D_{15}	$^{7}15^{2}_{4}$	12345678_{15} 12345689_{30} $1234569\,10_{30}$ $123456\,10\,12_{15}$ $123459\,10\,11_{30}$ $1234678\,14_{30}$ $1234689\,10_{30}$ $123468\,10\,12_{30}$ $123468\,12\,14_{15}$ $123567\,10\,11_{30}$ $123567\,11\,12_{15}$ $1235689\,10_{15}$ $12356\,10\,11\,12_{30}$ $123579\,10\,11_{15}$ $12357\,11\,12\,13_{30}$ $12367\,10\,11\,12_{15}$ $12468\,10\,12\,14_{15}$	

Table 8: Vertex-transitive combinatorial 7-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				$^{7}15^{2}_{7}$	12345678_{15} 12345689_{30} $1234569\,13_{15}$ $1234589\,10_{30}$ $123458\,10\,12_{30}$ $123459\,10\,12_{30}$ 12346789_{15} $1234679\,14_{30}$ $123469\,11\,13_{30}$ $123469\,11\,14_{15}$ $123489\,10\,11_{15}$ $1235689\,10_{15}$ $123568\,10\,13_{30}$ $123569\,10\,13_{30}$ $12359\,10\,12\,13_{30}$ $12359\,11\,12\,13_{15}$ $124579\,12\,14_{15}$	
		$(\underline{105},\underline{455},1350,2565,$ $2885,1740,435)$	D_{15}	$^{7}15^{2}_{3}$	12345678_{15} 12345689_{30} $1234569\,10_{30}$ $123456\,10\,12_{15}$ $123459\,10\,11_{30}$ $1234678\,12_{30}$ $123467\,12\,13_{30}$ $123467\,13\,14_{15}$ $1234689\,10_{30}$ $123468\,10\,12_{30}$ $123478\,12\,13_{15}$ $123567\,11\,12_{15}$ $1235689\,10_{15}$ $123568\,10\,12_{30}$ $123578\,10\,11_{30}$ $123578\,11\,12_{30}$ $123579\,10\,11_{15}$ $12358\,10\,11\,12_{15}$ $12468\,10\,12\,14_{15}$	
		$(\underline{105},\underline{455},\underline{1365},2625,$ $2975,1800,450)$	D_{15}	$^{7}15^{2}_{1}$	12345678_{15} 12345689_{30} $1234569\,10_{30}$ $123456\,10\,11_{30}$ 12346789_{15} $1234679\,10_{30}$ $123467\,10\,11_{30}$ $123467\,11\,12_{30}$ $123467\,12\,13_{30}$ $123467\,13\,14_{15}$ $1234789\,10_{15}$ $123478\,10\,11_{30}$ $123478\,11\,12_{30}$ $123478\,12\,13_{15}$ $123489\,10\,11_{15}$ $123489\,11\,12_{15}$ $124578\,10\,11_{15}$ $124578\,11\,12_{30}$ $124589\,11\,12_{15}$ $124589\,12\,13_{15}$	$\partial C_{8}(15)$
	....	....	$\mathbb{Z}_{15}$	....	....	

Table 9: Vertex-transitive combinatorial 8-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
10	S^8	$(\underline{45},\underline{120},\underline{210},\underline{252},$ $\underline{210},\underline{120},\underline{45},\underline{10})$	S_{10}	${}^{8}10^{45}_{1}$	123456789_{10}	$\partial\,\Delta_9$, regular
12	S^8	$(\underline{66},\underline{220},492,768,$ $840,624,288,64)$	$S_4 wr S_3$	${}^{8}12^{294}_{1}$	123456789_{64}	$(\partial\,\Delta_3)^{*3}$ $= \partial\Delta_1 \wr \partial C_3^{\Delta}$ [42]
14	S^8	$(\underline{91},\underline{364},987,1862,$ $2408,2032,1008,224)$	$\mathbb{Z}_{14}$	${}^{8}14^{1}_{5}$	123456789_{14} $12345679\,11_{14}$ $1234567\,11\,13_{14}$ $12345689\,10_{14}$ $1234569\,10\,11_{14}$ $123456\,10\,11\,12_{14}$ $123456\,11\,12\,13_{14}$ $1234579\,11\,13_{14}$ $12346789\,11_{14}$ $1234678\,11\,13_{14}$ $1234689\,10\,11_{14}$ $123468\,10\,11\,13_{14}$ $123468\,10\,12\,13_{14}$ $1234789\,11\,13_{14}$ $1235679\,10\,11_{14}$ $123567\,10\,11\,12_{14}$	
			D_7	${}^{8}14^{2}_{14}$	123456789_{14} $12345679\,11_{14}$ $1234567\,11\,13_{14}$ $12345689\,10_{14}$ $1234569\,10\,11_{14}$ $12345789\,14_{14}$ $1234579\,11\,13_{14}$ $1234589\,10\,14_{14}$ $12346789\,11_{14}$ $1234678\,11\,13_{14}$ $1234689\,10\,11_{14}$ $123468\,10\,11\,12_{14}$ $1235679\,10\,11_{14}$ $123567\,10\,11\,12_{14}$ $123568\,10\,12\,14_{14}$ $123578\,10\,12\,14_{14}$	
			$2wrD_7$	${}^{8}14^{38}_{1}$	123456789_{224}	$\partial\Delta_1 \wr \partial C_2(7)$ [42]
		$(\underline{91},\underline{364},994,1904,$ $2506,2144,1071,238)$	D_7	${}^{8}14^{2}_{2}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,12_{14}$ $1234567\,12\,13_{14}$ $12345789\,13_{14}$ $1234579\,10\,13_{14}$ $123457\,10\,11\,13_{14}$ $123457\,11\,12\,13_{14}$ $12346789\,12_{14}$ $1234679\,10\,11_{14}$ $1234679\,11\,12_{14}$ $123469\,10\,11\,12_{14}$ $1235679\,10\,11_{14}$ $1235689\,11\,14_{14}$ $123569\,10\,11\,14_{14}$ $1235789\,11\,13_{14}$	

Table 9: Vertex-transitive combinatorial 8-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				$^{8}14^{2}_{5}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,13_{14}$ $12345689\,10_{14}$ $12345789\,13_{14}$ $1234579\,10\,13_{14}$ $123457\,10\,11\,13_{14}$ $1234589\,10\,14_{14}$ $123459\,10\,11\,13_{14}$ $12346789\,12_{14}$ $1234678\,12\,13_{14}$ $1234679\,10\,12_{14}$ $123467\,10\,11\,12_{14}$ $123568\,10\,11\,14_{14}$ $1235789\,11\,13_{14}$ $123589\,10\,11\,14_{14}$	
				$^{8}14^{2}_{9}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,13_{14}$ $12345689\,11_{14}$ $1234569\,10\,12_{14}$ $12345789\,11_{14}$ $1234578\,11\,13_{14}$ $1234579\,10\,11_{14}$ $123457\,10\,11\,13_{14}$ $123459\,10\,11\,12_{14}$ $12345\,10\,11\,12\,13_{14}$ $1234679\,10\,13_{14}$ $1234689\,11\,12_{14}$ $1235679\,10\,13_{14}$ $1235689\,11\,14_{14}$ $123569\,10\,12\,13_{14}$ $1235789\,11\,13_{14}$	
				$^{8}14^{2}_{10}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,13_{14}$ $12345689\,12_{14}$ $1234568\,11\,12_{14}$ $12345789\,10_{14}$ $1234578\,10\,11_{14}$ $1234578\,11\,13_{14}$ $123457\,10\,11\,13_{14}$ $1234589\,10\,11_{14}$ $1234589\,11\,12_{14}$ $123459\,10\,11\,13_{14}$ $123459\,11\,12\,13_{14}$ $1235679\,10\,13_{14}$ $1235689\,11\,12_{14}$ $1235689\,11\,14_{14}$ $1235789\,11\,13_{14}$	
				$^{8}14^{2}_{11}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,13_{14}$ $12345689\,12_{14}$ $1234568\,11\,12_{14}$ $12345789\,11_{14}$ $1234578\,11\,13_{14}$ $1234579\,10\,11_{14}$ $123457\,10\,11\,13_{14}$ $1234589\,11\,12_{14}$ $123459\,10\,11\,12_{14}$ $123459\,10\,12\,13_{14}$ $12345\,10\,11\,12\,13_{14}$ $1235679\,10\,13_{14}$ $1235689\,11\,12_{14}$ $1235689\,11\,14_{14}$ $1235789\,11\,13_{14}$	

Table 9: Vertex-transitive combinatorial 8-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				$^{8}14^{2}_{12}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,13_{14}$ $12345689\,12_{14}$ $1234568\,11\,12_{14}$ $12345789\,13_{14}$ $1234579\,10\,13_{14}$ $1234589\,12\,13_{14}$ $123458\,11\,12\,13_{14}$ $12346789\,12_{14}$ $1234678\,11\,12_{14}$ $1234679\,10\,12_{14}$ $123467\,10\,11\,12_{14}$ $1235679\,10\,13_{14}$ $1235679\,11\,12_{14}$ $1235679\,11\,13_{14}$ $1235789\,12\,13_{14}$	
				$^{8}14^{2}_{15}$	123456789_{14} $12345679\,13_{14}$ $12345689\,10_{14}$ $1234568\,10\,12_{14}$ $1234569\,10\,12_{14}$ $12345789\,10_{14}$ $1234578\,10\,12_{14}$ $1234578\,12\,14_{14}$ $1234579\,10\,12_{14}$ $1234579\,11\,12_{14}$ $1234579\,11\,13_{14}$ $1234678\,10\,13_{14}$ $1234679\,10\,13_{14}$ $1235679\,10\,13_{14}$ $123568\,10\,12\,14_{14}$ $123569\,10\,12\,13_{14}$ $123578\,10\,12\,14_{14}$	
				$^{8}14^{2}_{16}$	123456789_{14} $12345679\,13_{14}$ $12345689\,10_{14}$ $1234568\,10\,12_{14}$ $1234569\,10\,12_{14}$ $12345789\,10_{14}$ $1234578\,10\,14_{14}$ $1234579\,10\,12_{14}$ $1234579\,11\,13_{14}$ $1234579\,11\,14_{14}$ $1234579\,12\,14_{14}$ $1234678\,10\,13_{14}$ $1234679\,10\,13_{14}$ $1235679\,10\,13_{14}$ $123568\,10\,12\,13_{14}$ $123569\,10\,12\,13_{14}$ $123579\,10\,12\,13_{14}$	
		$(\underline{91},\underline{364},\underline{1001},1946,$ $2604,2256,1134,252)$	$\mathbb{Z}_{14}$	$^{8}14^{1}_{1}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,12_{14}$ $1234567\,12\,13_{14}$ $12345789\,10_{14}$ $1234578\,10\,11_{14}$ $1234578\,11\,12_{14}$ $1234578\,12\,13_{14}$ $1234589\,10\,11_{14}$ $1234589\,11\,12_{14}$ $1234589\,12\,13_{14}$ $123459\,10\,11\,13_{14}$ $123459\,11\,12\,13_{14}$ $1235679\,10\,11_{14}$ $1235679\,11\,13_{14}$ $123569\,10\,11\,13_{14}$ $1235789\,12\,13_{14}$	

Table 9: Vertex-transitive combinatorial 8-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{8}14^{1}_{2}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,13_{14}$ $123456\,10\,11\,12_{14}$ $123456\,11\,12\,13_{14}$ $12345789\,13_{14}$ $1234579\,10\,11_{14}$ $1234579\,11\,13_{14}$ $12346789\,11_{14}$ $1234678\,11\,13_{14}$ $1234678\,12\,13_{14}$ $1234679\,10\,11_{14}$ $1234689\,10\,11_{14}$ $123468\,10\,11\,13_{14}$ $123468\,10\,12\,13_{14}$ $1234789\,11\,13_{14}$ $123567\,10\,11\,12_{14}$	
				${}^{8}14^{1}_{3}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,13_{14}$ $123456\,10\,11\,12_{14}$ $123456\,11\,12\,13_{14}$ $12345789\,13_{14}$ $1234579\,10\,11_{14}$ $1234579\,11\,13_{14}$ $12346789\,13_{14}$ $1234678\,12\,13_{14}$ $1234679\,10\,11_{14}$ $1234679\,11\,13_{14}$ $1234689\,10\,11_{14}$ $1234689\,11\,13_{14}$ $123468\,10\,11\,12_{14}$ $123468\,11\,12\,13_{14}$ $123567\,10\,11\,12_{14}$	
				${}^{8}14^{1}_{4}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,13_{14}$ $123456\,10\,11\,13_{14}$ $123456\,10\,12\,13_{14}$ $12345789\,13_{14}$ $1234579\,10\,11_{14}$ $1234579\,11\,13_{14}$ $123459\,10\,11\,12_{14}$ $12345\,10\,11\,12\,13_{14}$ $1234678\,10\,11_{14}$ $1234678\,11\,13_{14}$ $1234678\,12\,13_{14}$ $123468\,10\,11\,13_{14}$ $123468\,10\,12\,13_{14}$ $1234789\,11\,13_{14}$ $123567\,10\,11\,12_{14}$	
				${}^{8}14^{1}_{6}$	123456789_{14} $12345679\,11_{14}$ $1234567\,11\,13_{14}$ $12345689\,10_{14}$ $1234569\,10\,11_{14}$ $123456\,10\,11\,13_{14}$ $123456\,10\,12\,13_{14}$ $1234579\,11\,13_{14}$ $123459\,10\,11\,12_{14}$ $12345\,10\,11\,12\,13_{14}$ $1234678\,10\,11_{14}$ $1234678\,11\,13_{14}$ $1234679\,10\,11_{14}$ $123468\,10\,11\,13_{14}$ $123468\,10\,12\,13_{14}$ $1234789\,11\,13_{14}$ $1235679\,10\,11_{14}$ $123567\,10\,11\,12_{14}$	

Table 9: Vertex-transitive combinatorial 8-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
			D_7	${}^{8}14^{2}_{1}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,12_{14}$ $1234567\,12\,13_{14}$ $12345789\,10_{14}$ $1234578\,10\,11_{14}$ $1234578\,11\,12_{14}$ $1234578\,12\,13_{14}$ $1234589\,10\,11_{14}$ $1234589\,11\,12_{14}$ $1234589\,12\,13_{14}$ $123459\,10\,11\,13_{14}$ $123459\,11\,12\,13_{14}$ $1235679\,10\,11_{14}$ $1235679\,11\,13_{14}$ $123569\,10\,11\,13_{14}$ $1235789\,12\,13_{14}$	
				${}^{8}14^{2}_{3}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,13_{14}$ $12345689\,10_{14}$ $12345789\,13_{14}$ $1234579\,10\,13_{14}$ $123457\,10\,11\,13_{14}$ $1234589\,10\,11_{14}$ $1234589\,11\,14_{14}$ $123459\,10\,11\,13_{14}$ $12346789\,12_{14}$ $1234678\,12\,13_{14}$ $1234679\,10\,12_{14}$ $123467\,10\,11\,12_{14}$ $123568\,10\,11\,14_{14}$ $1235789\,11\,13_{14}$ $1236789\,12\,13_{14}$	
				${}^{8}14^{2}_{4}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,11_{14}$ $1234567\,11\,13_{14}$ $12345689\,10_{14}$ $12345789\,13_{14}$ $1234579\,10\,13_{14}$ $123457\,10\,11\,13_{14}$ $1234589\,10\,11_{14}$ $1234589\,11\,14_{14}$ $123459\,10\,11\,13_{14}$ $12346789\,13_{14}$ $1234679\,10\,12_{14}$ $1234679\,12\,13_{14}$ $123467\,10\,11\,13_{14}$ $123567\,10\,11\,14_{14}$ $1235789\,11\,13_{14}$ $123578\,10\,11\,14_{14}$	
				${}^{8}14^{2}_{6}$	123456789_{14} $12345679\,10_{14}$ $1234567\,10\,13_{14}$ $12345689\,10_{14}$ $1234568\,10\,11_{14}$ $12345789\,13_{14}$ $1234579\,10\,13_{14}$ $1234589\,10\,11_{14}$ $1234589\,11\,13_{14}$ $123459\,10\,11\,13_{14}$ $12346789\,12_{14}$ $1234678\,12\,13_{14}$ $1234679\,10\,12_{14}$ $123467\,10\,11\,12_{14}$ $123467\,10\,11\,13_{14}$ $123567\,10\,11\,13_{14}$ $1235789\,11\,13_{14}$ $1236789\,12\,13_{14}$	

Table 9: Vertex-transitive combinatorial 8-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
				${}^{8}14^{2}_{7}$	123456789_{14} $12345679\ 10_{14}$ $1234567\ 10\ 13_{14}$ $12345689\ 10_{14}$ $1234568\ 10\ 11_{14}$ $12345789\ 13_{14}$ $1234579\ 10\ 13_{14}$ $1234589\ 10\ 11_{14}$ $1234589\ 11\ 13_{14}$ $123459\ 10\ 11\ 13_{14}$ $12346789\ 13_{14}$ $1234679\ 10\ 12_{14}$ $1234679\ 12\ 13_{14}$ $123567\ 10\ 11\ 13_{14}$ $123567\ 10\ 11\ 14_{14}$ $123568\ 10\ 11\ 14_{14}$ $1235789\ 11\ 13_{14}$ $123578\ 10\ 11\ 14_{14}$	
				${}^{8}14^{2}_{8}$	123456789_{14} $12345679\ 10_{14}$ $1234567\ 10\ 13_{14}$ $12345689\ 11_{14}$ $1234569\ 10\ 11_{14}$ $12345789\ 11_{14}$ $1234578\ 11\ 13_{14}$ $1234579\ 10\ 11_{14}$ $123457\ 10\ 11\ 13_{14}$ $12345\ 10\ 11\ 12\ 13_{14}$ $1234679\ 10\ 13_{14}$ $1234689\ 11\ 12_{14}$ $123469\ 10\ 11\ 12_{14}$ $1234789\ 11\ 12_{14}$ $1235679\ 10\ 13_{14}$ $1235689\ 11\ 14_{14}$ $123569\ 10\ 12\ 13_{14}$ $1235789\ 11\ 13_{14}$	
				${}^{8}14^{2}_{13}$	123456789_{14} $12345679\ 10_{14}$ $1234567\ 10\ 13_{14}$ $12345689\ 13_{14}$ $1234568\ 11\ 12_{14}$ $12345789\ 13_{14}$ $1234579\ 10\ 13_{14}$ $123458\ 11\ 12\ 13_{14}$ $12346789\ 12_{14}$ $1234678\ 11\ 12_{14}$ $1234679\ 10\ 12_{14}$ $123467\ 10\ 11\ 12_{14}$ $123467\ 10\ 11\ 13_{14}$ $1235679\ 10\ 13_{14}$ $1235679\ 11\ 12_{14}$ $1235679\ 11\ 13_{14}$ $1235689\ 12\ 13_{14}$ $1235789\ 12\ 13_{14}$	
15	$\sim\mathbb{H}\mathbf{P}^{2}$	($\underline{105}$,$\underline{455}$,$\underline{1365}$,$\underline{3003}$, 4515,4230,2205,490)	A_5	${}^{8}15^{5}_{1}$	$12345678\ 12_{60}$ $12345678\ 13_{60}$ $1234567\ 12\ 14_{60}$ $1234567\ 13\ 15_{15}$ $1234567\ 14\ 15_{15}$ $12345689\ 12_{30}$ $12345689\ 13_{30}$ $1234569\ 13\ 15_{60}$ $1234569\ 14\ 15_{60}$ $1234578\ 10\ 11_{20}$ $123459\ 10\ 13\ 15_{10}$ $123459\ 10\ 14\ 15_{30}$ $1234689\ 10\ 12_{30}$ $123479\ 11\ 14\ 15_{10}$	minimal, tight, [21, M^{8}_{15}], [48], [55]
	....	....	$\mathbb{Z}_{15}$	....	....	

Table 10: Vertex-transitive combinatorial 9-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
11	S^9	$(\underline{55},\underline{165},\underline{330},\underline{462},\underline{462},$ $\underline{330},\underline{165},\underline{55},\underline{11})$	S_{11}	${}^{9}11^{8}_{1}$	$123456789\,10_{11}$	$\partial\,\Delta_{10}$, regular
12	S^9	$(\underline{66},\underline{220},\underline{495},\underline{792},922,$	$S_6 wr 2$	${}^{9}12^{299}_{1}$	$123456789\,10_{36}$	$\partial C_{10}(12)=(\partial\,\Delta_5)^{*2}$ $=\partial\Delta_1 \wr \partial C_4(6)$ $=\partial\Delta_2 \wr \partial C_2(4)$ [42]
13	S^9	$(\underline{78},\underline{286},\underline{715},\underline{1287},1703,$ $1638,1092,455,91)$	D_{13}	${}^{9}13^{2}_{1}$	$123456789\,10_{13}$ $12345678\,10\,11_{26}$ $12345689\,10\,11_{26}$ $12345689\,11\,12_{13}$ $12346789\,11\,12_{13}$	$\partial C_{10}(13)$
14	S^9	$(\underline{91},\underline{364},\underline{1001},\underline{2002},2954,$ $3136,2254,980,196)$	D_{14}	${}^{9}14^{3}_{1}$	$123456789\,10_{14}$ $12345678\,10\,11_{28}$ $12345678\,11\,12_{14}$ $12345689\,10\,11_{28}$ $12345689\,11\,12_{28}$ $12345689\,12\,13_{14}$ $1234569\,10\,11\,12_{14}$ $12346789\,11\,12_{28}$ $1234679\,10\,11\,12_{14}$ $1234679\,10\,12\,13_{14}$	$\partial C_{10}(14)$
15	S^9	$(\underline{105},450,1305,2673,3915,$ $4050,2835,1215,243)$	$S_3 wr S_5$	${}^{9}15^{93}_{1}$	$123456789\,10_{243}$	3^{*5}
		$(\underline{105},\underline{455},\underline{1365},2985,4775,$ $5400,4050,1800,360)$	$\mathbb{Z}_5 \times S_3$	${}^{9}15^{4}_{1}$	$123456789\,10_{30}$ $12345678\,10\,11_{30}$ $12345678\,11\,12_{30}$ $12345678\,12\,14_{30}$ $1234567\,11\,12\,13_{30}$ $1234567\,12\,13\,14_{30}$ $12345689\,10\,14_{30}$ $1234568\,10\,11\,12_{30}$ $1234568\,10\,12\,14_{30}$ $12345789\,10\,11_{30}$ $1234578\,12\,13\,14_{30}$ $123458\,10\,12\,13\,14_{30}$	

Table 10: Vertex-transitive combinatorial 9-manifolds (continued).

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
			$D_5 \times S_3$	${}^{9}15^{7}_{1}$	123456789 10$_{30}$ 12345678 10 11$_{60}$ 12345678 11 12$_{60}$ 12345689 10 14$_{60}$ 1234568 10 11 12$_{60}$ 1234568 10 12 14$_{30}$ 12345789 10 11$_{30}$ 1234678 12 13 14$_{30}$	
		$(\underline{105},\underline{455},\underline{1365},3000,4850,$ $5550,4200,1875,375)$	$\mathbb{Z}_{15}$	${}^{9}15^{1}_{1}$	123456789 10$_{15}$ 12345678 10 11$_{15}$ 12345678 11 12$_{15}$ 12345678 12 13$_{15}$ 12345678 13 14$_{15}$ 12345689 10 14$_{15}$ 1234568 10 11 12$_{15}$ 1234568 10 12 14$_{15}$ 1234568 12 13 14$_{15}$ 123456 10 11 12 14$_{15}$ 12345789 10 11$_{15}$ 12345789 11 12$_{15}$ 12345789 12 13$_{15}$ 1234579 11 12 13$_{15}$ 1234589 10 11 12$_{15}$ 1234589 10 12 13$_{15}$ 1234589 10 13 14$_{15}$ 123458 10 12 13 14$_{15}$ 1234678 12 13 14$_{15}$ 123489 10 12 13 14$_{15}$ 1235679 10 11 13$_{15}$ 1235679 10 13 14$_{15}$ 1235679 11 12 13$_{15}$ 123567 10 11 13 14$_{15}$ 1235689 10 12 14$_{15}$	
		$(\underline{105},\underline{455},\underline{1365},\underline{3003},4865,$ $5580,4230,1890,378)$	D_{15}	${}^{9}15^{2}_{1}$	123456789 10$_{15}$ 12345678 10 11$_{30}$ 12345678 11 12$_{30}$ 12345689 10 11$_{30}$ 12345689 11 12$_{30}$ 12345689 12 13$_{30}$ 12345689 13 14$_{15}$ 1234569 10 11 12$_{30}$ 1234569 10 12 13$_{15}$ 12346789 11 12$_{30}$ 12346789 12 13$_{15}$ 1234679 10 11 12$_{15}$ 1234679 10 12 13$_{30}$ 1234679 10 13 14$_{30}$ 123467 10 11 12 13$_{30}$ 124578 10 11 13 14$_{3}$	$\partial C_{10}(15)$

Table 11: Vertex-transitive combinatorial 10-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
12	S^{10}	$(\underline{66},\underline{220},\underline{495},\underline{792},\underline{924},\underline{792},\underline{495},\underline{220},\underline{66},\underline{12})$	S_{12}	$^{10}12^{301}_{1}$	123456789 10 11$_{12}$	$\partial\,\Delta_{11}$, regular
14	S^{10}	$(\underline{91},\underline{364},\underline{1001},\underline{2002},2996,3376,2814,1652,616,112)$	$2wrD_7$	$^{10}14^{38}_{1}$	123456789 10 11$_{56}$ 123456789 11 12$_{56}$	$\partial\Delta_1 \wr \partial C_4(7)$ [42]

Table 12: Vertex-transitive combinatorial 11-manifolds.

n	Manifold	f-vector	Group	Type	List of orbits	Remarks
13	S^{11}	$(\underline{78},\underline{286},\underline{715},\underline{1287},\underline{1716},\underline{1716},$ $\underline{1287},\underline{715},\underline{286},\underline{78},\underline{13})$	S_{13}	$^{11}13^{9}_{1}$	123456789 10 11 12$_{13}$	$\partial\,\Delta_{12}$, regular
14	S^{11}	$(\underline{91},\underline{364},\underline{1001},\underline{2002},\underline{3003},3430,$ $2989,1960,931,294,49)$	S_7wr2	$^{11}14^{61}_{1}$	123456789 10 11 12$_{49}$	$\partial C_{12}(14) = (\partial\,\Delta_6)^{*2}$
15	S^{11}	$(\underline{105},\underline{455},\underline{1365},3000,4975,6300,$ $6075,4375,2250,750,125)$	S_5wrS_3	$^{11}15^{102}_{1}$	123456789 10 11 12$_{125}$	$(\partial\,\Delta_4)^{*3}$
		$(\underline{105},\underline{455},\underline{1365},\underline{3003},5000,6390,$ $6255,4590,2403,810,135)$	S_3wrD_5	$^{11}15^{86}_{1}$	123456789 10 11 12$_{135}$	$\partial\Delta_2 \wr \partial C_2(5)$ [42]
		$(\underline{105},\underline{455},\underline{1365},\underline{3003},\underline{5005},6420,$ $6330,4690,2478,840,140)$	D_{15}	$^{11}15^{2}_{1}$	123456789 10 11 12$_{15}$ 123456789 10 12 13$_{30}$ 12345678 10 11 12 13$_{30}$ 12345678 10 11 13 14$_{15}$ 12345689 10 11 12 13$_{15}$ 12345689 10 11 13 14$_{30}$ 12346789 11 12 13 14$_{5}$	$\partial C_{12}(15)$

Table 13: List of generators for the group actions in Table 3 – Table 12.

Action	Group	Generators
6^{11}	$[2^3]S_3 = 2 wr S_3$	(3,6), (1,3,5)(2,4,6), (1,5)(2,4)
6^{12}	A_5	(1,2,3,4,6), (1,4)(5,6)
6^{13}	$[S_3^2]2 = S_3 wr 2$	(2,4,6), (2,4), (1,4)(2,5)(3,6)
7^4	$7:6$	(1,2,3,4,5,6,7), (1,3,2,6,4,5)
8^{15}	$t8n15(32)$	(1,2,3,4,5,6,7,8), (1,5)(3,7), (1,6)(2,5)(3,4)(7,8)
8^{44}	$[2^4]S_4 = 2 wr S_4$	(4,8), (1,8)(4,5), (1,2,3,8)(4,5,6,7)
8^{47}	$[S_4^2]2 = S_4 wr 2$	(1,2,3,8), (2,3), (1,5)(2,6)(3,7)(4,8)
9^4	$S_3 \times \mathbb{Z}_3$	(1,2,9)(3,4,5)(6,7,8), (1,2)(4,5)(7,8), (1,4,7)(2,5,8)(3,6,9)
9^{13}	$\mathbb{Z}_3^2 : \mathbb{Z}_6$	(1,2,9)(3,4,5)(6,7,8), (1,4,7)(2,5,8)(3,6,9), (3,4,5)(6,8,7), (1,2)(3,5)(6,7)
9^{18}	$\mathbb{Z}_3^2 : D_6$	(1,2,9)(3,4,5)(6,7,8), (1,4,7)(2,5,8)(3,6,9), (3,4,5)(6,8,7), (1,2)(3,6)(4,8)(5,7), (1,2)(3,5)(6,7)
9^{31}	$[S_3^3]S_3 = S_3 wr S_3$	(1,2,9), (1,2), (1,4,7)(2,5,8)(3,6,9), (3,6)(4,7)(5,8)
10^2	D_5	(1,3,5,7,9)(2,4,6,8,10), (1,4)(2,3)(5,10)(6,9)(7,8)
10^4	$\frac{1}{2}[5:4]2$	(1,3,5,7,9)(2,4,6,8,10), (1,2,9,8)(3,6,7,4)(5,10)
10^7	A_5	(1,3,5,7,9)(2,4,6,8,10), (1,9)(3,4)(5,10)(6,7)
10^{21}	$[D_5^2]2 = D_5 wr 2$	(2,4,6,8,10), (2,8)(4,6), (1,6)(2,7)(3,8)(4,9)(5,10)
10^{22}	$S_5 \times \mathbb{Z}_2$	(1,3,5,7,9)(2,4,6,8,10), (2,10)(5,7), (1,6)(2,7)(3,8)(4,9)(5,10)
10^{23}	$[2^5]D_5 = 2 wr D_5$	(5,10), (1,3,5,7,9)(2,4,6,8,10), (1,9)(2,8)(3,7)(4,6)
10^{39}	$[2^5]S_5 = 2 wr S_5$	(5,10), (1,3,5,7,9)(2,4,6,8,10), (2,10)(5,7)
10^{43}	$[S_5^2]2 = S_5 wr 2$	(2,4,6,8,10), (2,10), (1,6)(2,7)(3,8)(4,9)(5,10)
12^2	$\mathbb{Z}_3 \times \mathbb{Z}_2^2$	(1,10)(2,5)(3,12)(4,7)(6,9)(8,11), (1,7)(2,8)(3,9)(4,10)(5,11)(6,12), (1,5,9)(2,6,10)(3,7,11)(4,8,12)
12^3	D_6	(1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,11)(2,8)(3,9)(4,6)(5,7)(10,12), (1,12)(2,3)(4,5)(6,7)(8,9)(10,11)
12^4	A_4	(1,9,5)(2,4,3)(6,8,7)(10,12,11), (1,11,6)(2,9,7)(3,10,5)(4,8,12)

Table 13: List of generators (continued).

Action	Group	Generators
12^{5}	$\frac{1}{2}[3{:}2]4$	(1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,7)(2,8)(3,9)(4,10)(5,11)(6,12), (1,8,7,2)(3,6,9,12)(4,11,10,5)
12^{6}	$A_4(12)\times\mathbb{Z}_2$	(1,9,5)(2,4,3)(6,8,7)(10,12,11), (1,11,6)(2,9,7)(3,10,5)(4,8,12), (1,7)(2,11)(3,12)(4,10)(5,8)(6,9)
12^{7}	$A_4(6)\times\mathbb{Z}_2$	(2,8)(3,9)(4,10)(5,11), (1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,12)(2,3)(4,5)(6,7)(8,9)(10,11)
12^{8}	$t12n8(24)=S_4$	(1,2)(3,5)(4,6)(7,9)(8,10), (1,3,6,12)(2,4,7,10)(5,8,11,9)
12^{9}	$t12n9(24)=S_4$	(1,7)(3,9)(4,10)(6,12), (1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,2)(3,12)(4,11)(5,10)(6,9)(7,8)
12^{10}	$S_3\times\mathbb{Z}_2^2$	(1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,5)(2,10)(4,8)(7,11), (1,10)(2,5)(3,12)(4,7)(6,9)(8,11), (1,7)(2,8)(3,9)(4,10)(5,11)(6,12)
12^{11}	$S_3\times\mathbb{Z}_4$	(1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,5)(2,10)(4,8)(7,11), (1,4,7,10)(2,5,8,11)(3,6,9,12)
12^{13}	$t12n13(24)$	(1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,10)(2,5)(3,12)(4,7)(6,9)(8,11), (1,7)(2,8)(3,9)(4,10)(5,11)(6,12), (1,11)(2,10)(3,9)(4,8)(5,7)
12^{14}	$D_4\times\mathbb{Z}_3$	(1,4,7,10)(2,5,8,11)(3,6,9,12), (1,7)(3,9)(5,11), (1,5,9)(2,6,10)(3,7,11)(4,8,12)
12^{15}	$t12n15(24)$	(1,5,9)(2,6,10)(3,7,11)(4,8,12), (2,8)(4,10)(6,12), (1,7)(3,9)(5,11), (1,2)(3,12)(4,11)(5,10)(6,9)(7,8)
12^{28}	$D_4\times S_3$	(1,4,7,10)(2,5,8,11)(3,6,9,12), (1,7)(3,9)(5,11), (1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,5)(2,10)(4,8)(7,11)
12^{54}	$t12n54(96)$	(1,12)(2,3), (1,5,9)(2,6,10)(3,7,11)(4,8,12), (2,3)(4,9)(5,8)(6,10)(7,11), (1,2)(3,12)(4,11)(5,10)(6,9)(7,8)
12^{75}	$A_5\times\mathbb{Z}_2$	(1,3,5,7,9)(2,4,6,8,12), (1,11)(2,8)(3,9)(10,12), (1,12)(2,3)(4,5)(6,7)(8,9)(10,11)
12^{76}	$[2]A_5$	(1,12)(2,3)(4,5)(6,7)(8,9)(10,11), (2,4,6,8,10)(3,5,7,9,11), (4,10)(5,11)(6,8)(7,9), (1,2)(3,12)(4,11)(5,10)
12^{83}	$S_4\times S_3$	(1,4,7,10)(2,5,8,11)(3,6,9,12), (1,10)(2,5)(6,9), (1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,5)(2,10)(4,8)(7,11)
12^{113}	$t12n113(192)$	(1,12)(2,3)(6,7)(8,9), (1,3,5,7,9,11)(2,4,6,8,10,12), (1,3)(2,12)(4,10)(5,11)(6,8)(7,9), (4,10)(5,11)(6,7)(8,9)
12^{124}	$[2]A_5{:}2$	(1,12)(2,3)(4,5)(6,7)(8,9)(10,11), (2,4,6,8,10)(3,5,7,9,11), (1,3,12,2)(4,6,5,7)(8,11,9,10)

Table 13: List of generators (continued).

Action	Group	Generators
12^{125}	$[S_3^2]D_4 = D_6 wr 2$	(2,6,10)(4,8,12), (2,10)(4,8), (1,4,7,10)(2,5,8,11)(3,6,9,12), (1,7)(3,9)(5,11)
12^{193}	$[2^6]D_6 = 2 wr D_6$	(1,12), (1,3,5,7,9,11)(2,4,6,8,10,12), (1,11)(2,8)(3,9)(4,6)(5,7)(10,12)
12^{289}	$[S_3^4]S_4 = S_3 wr S_4$	(4,8,12), (4,8), (1,4,7,10)(2,5,8,11)(3,6,9,12), (1,10)(2,5)(6,9)
12^{293}	$[2^6]S_6 = 2 wr S_6$	(1,12), (1,3)(2,12), (1,3,5,7,9,11)(2,4,6,8,10,12)
12^{294}	$[S_4^3]S_3 = S_4 wr S_3$	(3,6,9,12), (6,9), (1,5,9)(2,6,10)(3,7,11)(4,8,12), (1,5)(2,10)(4,8)(7,11)
12^{299}	$[S_6^2]2 = S_6 wr 2$	(2,12), (2,4,6,8,10,12), (1,12)(2,3)(4,5)(6,7)(8,9)(10,11)
13^{3}	$13{:}3$	(1,2,3,4,5,6,7,8,9,10,11,12,13), (1,3,9)(2,6,5)(4,12,10)(7,8,11)
13^{4}	$13{:}4$	(1,2,3,4,5,6,7,8,9,10,11,12,13), (1,5,12,8)(2,10,11,3)(4,7,9,6)
13^{5}	$13{:}6$	(1,2,3,4,5,6,7,8,9,10,11,12,13), (1,4,3,12,9,10)(2,8,6,11,5,7)
14^{2}	D_7	(1,3,5,7,9,11,13)(2,4,6,8,10,12,14), (1,12)(2,11)(3,10)(4,9)(5,8)(6,7)(13,14)
14^{4}	$2[\frac{1}{2}]7{:}6$	(1,3,5,7,9,11,13)(2,4,6,8,10,12,14), (1,9,11)(2,4,8)(3,13,5)(6,12,10), (1,6)(2,5)(3,4)(7,14)(8,13)(9,12)(10,11)
14^{5}	$7{:}3 \times \mathbb{Z}_2$	(1,3,5,7,9,11,13)(2,4,6,8,10,12,14), (1,9,11)(2,4,8)(3,13,5)(6,12,10), (1,8)(2,9)(3,10)(4,11)(5,12)(6,13)(7,14)
14^{7}	$7{:}6 \times \mathbb{Z}_2$	(1,3,5,7,9,11,13)(2,4,6,8,10,12,14), (1,9,11)(2,4,8)(3,13,5)(6,12,10), (1,13)(2,12)(3,11)(4,10)(5,9)(6,8), (1,8)(2,9)(3,10)(4,11)(5,12)(6,13)(7,14)
14^{16}	$L(2,7){:}2$	(1,13,11,9,7,5,3)(2,4,6,8,10,12,14), (1,9,11)(2,4,8)(3,13,5)(6,12,10), (2,4)(5,13)(6,12)(9,11), (1,8)(2,9)(3,10)(4,11)(5,12)(6,13)(7,14)
14^{19}	$L(2,7) \times \mathbb{Z}_2$	(1,3,5,7,9,11,13)(2,4,6,8,10,12,14), (1,9,11)(2,4,8)(3,13,5)(6,12,10), (2,4)(5,13)(6,12)(9,11), (1,8)(2,9)(3,10)(4,11)(5,12)(6,13)(7,14)
14^{20}	$[D_7^2]2 = D_7 wr 2$	(2,4,6,8,10,12,14), (2,12)(4,10)(6,8), (1,8)(2,9)(3,10)(4,11)(5,12)(6,13)(7,14)
14^{38}	$[2^7]D_7 = 2 wr D_7$	(7,14), (1,3,5,7,9,11,13)(2,4,6,8,10,12,14), (1,13)(2,12)(3,11)(4,10)(5,9)(6,8)

Table 13: List of generators (continued).

Action	Group	Generators
14^{49}	$S_7 \times \mathbb{Z}_2$	(1,3,5,7,9,11,13)(2,4,6,8,10,12,14), (3,5)(10,12), (1,8)(2,9)(3,10)(4,11)(5,12)(6,13)(7,14)
14^{57}	$[2^7]S_7 = 2wrS_7$	(7,14), (1,3,5,7,9,11,13)(2,4,6,8,10,12,14), (3,13,5)(6,12,10), (3,5)(10,12)
14^{61}	$[S_7^2]2 = S_7wr2$	(2,4,6,8,10,12,14), (10,12), (1,8)(2,9)(3,10)(4,11)(5,12)(6,13)(7,14)
15^{3}	$D_5 \times \mathbb{Z}_3$	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15), (1,4)(2,8)(3,12)(6,9)(7,13)(11,14)
15^{4}	$\mathbb{Z}_5 \times S_3$	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15), (1,11)(2,7)(4,14)(5,10)(8,13)
15^{5}	A_5	(1,9,10,3,14)(2,15,7,12,6)(4,5,11,13,8), (1,4,10)(2,5,8)(3,7,11)(6,9,15)(12,14,13)
15^{6}	$5:4[\frac{1}{2}]S_3$	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15), (1,4)(2,8)(3,12)(6,9)(7,13)(11,14), (1,2,4,8)(3,6,12,9)(5,10)(7,14,13,11)
15^{7}	$D_5 \times S_3$	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15), (1,4)(2,8)(3,12)(6,9)(7,13)(11,14), (1,11)(2,7)(4,14)(5,10)(8,13)
15^{8}	$5:4 \times \mathbb{Z}_3$	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15), (1,7,4,13)(2,14,8,11)(3,6,12,9)
15^{10}	S_5	(1,9,10,3,14)(2,15,7,12,6)(4,5,11,13,8), (1,4,10)(2,5,8)(3,7,11)(6,9,15)(12,14,13), (1,4)(2,6)(3,7)(5,15)(8,9)(12,13)
15^{11}	$5:4 \times S_3$	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15), (1,7,4,13)(2,14,8,11)(3,6,12,9), (1,11)(2,7)(4,14)(5,10)(8,13)
15^{15}	$[3]A_5 = GL(2,4)$	(1,9,10,3,14)(2,15,7,12,6)(4,5,11,13,8), (1,4,10)(2,5,8)(3,7,11)(6,9,15)(12,14,13), (1,2,15)(4,5,6)(8,9,10)(12,13,14)
15^{18}	$[5^2:2]S_3$	(1,13,10,7,4)(2,5,8,11,14), (1,6,11)(2,7,12)(3,8,13)(4,9,14)(5,10,15), (1,4)(2,8)(3,12)(6,9)(7,13)(11,14), (1,11)(2,7)(4,14)(5,10)(8,13)
15^{29}	$S_5 \times S_3$	(1,2,3,4,5,6,7,8,9,10,11,12,13,14,15), (1,4)(6,9)(11,14), (1,11)(2,7)(4,14)(5,10)(8,13)
15^{60}	$[D_5^3]S_3 = D_5wrS_3$	(3,6,9,12,15), (3,12)(6,9), (1,6,11)(2,7,12)(3,8,13)(4,9,14)(5,10,15), (1,11)(2,7)(4,14)(5,10)(8,13)
15^{86}	$[S_3^5]D_5 = S_3wrD_5$	(5,10,15), (5,10), (1,4,7,10,13)(2,5,8,11,14)(3,6,9,12,15), (1,4)(2,8)(3,12)(6,9)(7,13)(11,14)
15^{93}	$[S_3^5]S_5 = S_3wrS_5$	(5,10,15), (5,10), (1,4,7,10,13)(2,5,8,11,14)(3,6,9,12,15), (1,4)(6,9)(11,14)
15^{102}	$[S_5^3]S_3 = S_5wrS_3$	(3,6,9,12,15), (6,9), (1,6,11)(2,7,12)(3,8,13)(4,9,14)(5,10,15), (1,11)(2,7)(4,14)(5,10)(8,13)

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