

Contents

Introduction: Wolfgang Dahmen's mathematical work	1
Ronald A. DeVore and Angela Kunoth	
1 Introduction	1
2 The early years: Classical approximation theory	2
3 Bonn, Bielefeld, Berlin, and multivariate splines	2
3.1 Computer aided geometric design	3
3.2 Subdivision and wavelets	4
4 Wavelet and multiscale methods for operator equations	5
4.1 Multilevel preconditioning	5
4.2 Compression of operators	5
5 Adaptive solvers	6
6 Construction and implementation	7
7 Hyperbolic partial differential equations and conservation laws ...	8
8 Engineering collaborations	9
9 The present	9
10 Final remarks	10
Publications by Wolfgang Dahmen (as of summer 2009)	10
The way things were in multivariate splines: A personal view	19
Carl de Boor	
1 Tensor product spline interpolation	19
2 Quasiinterpolation	20
3 Multivariate B-splines	21
4 Kergin interpolation	23
5 The recurrence for multivariate B-splines	25
6 Polyhedral splines	27
7 Box splines	28
8 Smooth multivariate piecewise polynomials and the B-net	31
References	34

On the efficient computation of high-dimensional integrals and the approximation by exponential sums	39
Dietrich Braess and Wolfgang Hackbusch	
1 Introduction	39
2 Approximation of completely monotone functions by exponential sums	41
3 Rational approximation of the square root function	43
3.1 Heron's algorithm and Gauss' arithmetic-geometric mean	43
3.2 Heron's method and best rational approximation	44
3.3 Extension of the estimate (19)	48
3.4 An explicit formula	49
4 Approximation of $1/x^\alpha$ by exponential sums	50
4.1 Approximation of $1/x$ on finite intervals	50
4.2 Approximation of $1/x$ on $[1, \infty)$	51
4.3 Approximation of $1/x^\alpha$, $\alpha > 0$	54
5 Applications of $1/x$ approximations	55
5.1 About the exponential sums	55
5.2 Application in quantum chemistry	55
5.3 Inverse matrix	56
6 Applications of $1/\sqrt{x}$ approximations	58
6.1 Basic facts	58
6.2 Application to convolution	58
6.3 Modification for wavelet applications	60
6.4 Expectation values of the H-atom	60
7 Computation of the best approximation	62
8 Rational approximation of $\sqrt{x}$ on small intervals	63
9 The arithmetic-geometric mean and elliptic integrals	65
10 A direct approach to the infinite interval	67
11 Sinc quadrature derived approximations	68
References	73
Adaptive and anisotropic piecewise polynomial approximation	75
Albert Cohen and Jean-Marie Mirebeau	
1 Introduction	75
1.1 Piecewise polynomial approximation	75
1.2 From uniform to adaptive approximation	77
1.3 Outline	79
2 Piecewise constant one-dimensional approximation	80
2.1 Uniform partitions	81
2.2 Adaptive partitions	83
2.3 A greedy refinement algorithm	85
3 Adaptive and isotropic approximation	87
3.1 Local estimates	88
3.2 Global estimates	90
3.3 An isotropic greedy refinement algorithm	91

3.4	The case of smooth functions.	95
4	Anisotropic piecewise constant approximation on rectangles	100
4.1	A heuristic estimate	100
4.2	A rigorous estimate	103
5	Anisotropic piecewise polynomial approximation	108
5.1	The shape function	108
5.2	Algebraic expressions of the shape function	109
5.3	Error estimates	111
5.4	Anisotropic smoothness and cartoon functions	112
6	Anisotropic greedy refinement algorithms	116
6.1	The refinement algorithm for piecewise constants on rectangles	119
6.2	Convergence of the algorithm	121
6.3	Optimal convergence	124
6.4	Refinement algorithms for piecewise polynomials on triangles	128
	References	134
	Anisotropic function spaces with applications	137
	Shai Dekel and Pencho Petrushev	
1	Introduction	137
2	Anisotropic multiscale structures on $\mathbb{R}^n$	139
2.1	Anisotropic multilevel ellipsoid covers (dilations) of $\mathbb{R}^n$	139
2.2	Comparison of ellipsoid covers with nested triangulations in $\mathbb{R}^2$	142
3	Building blocks	143
3.1	Construction of a multilevel system of bases	143
3.2	Compactly supported duals and local projectors	145
3.3	Two-level-split bases	146
3.4	Global duals and polynomial reproducing kernels	148
3.5	Construction of anisotropic wavelet frames	151
3.6	Discrete wavelet frames	154
3.7	Two-level-split frames	155
4	Anisotropic Besov spaces (B-spaces)	156
4.1	B-spaces induced by anisotropic covers of $\mathbb{R}^n$	156
4.2	B-spaces induced by nested multilevel triangulations of $\mathbb{R}^2$	158
4.3	Comparison of different B-spaces and Besov spaces	159
5	Nonlinear approximation	160
6	Measuring smoothness via anisotropic B-spaces	162
7	Application to preconditioning for elliptic boundary value problems	164
	References	166

Nonlinear approximation and its applications 169

Ronald A. DeVore

1	The early years	169
2	Smoothness and interpolation spaces	171
2.1	The role of interpolation	172
3	The main types of nonlinear approximation	174
3.1	n -term approximation	174
3.2	Adaptive approximation	178
3.3	Tree approximation	178
3.4	Greedy algorithms	181
4	Image compression	185
5	Remarks on nonlinear approximation in PDE solvers	187
6	Learning theory	189
6.1	Learning with greedy algorithms	193
7	Compressed sensing	195
8	Final thoughts	199
	References	199

Univariate subdivision and multi-scale transforms: The nonlinear case . . 203

Nira Dyn and Peter Oswald

1	Introduction	203
2	Nonlinear multi-scale transforms: Functional setting	210
2.1	Basic notation and further examples	210
2.2	Polynomial reproduction and derived subdivision schemes	215
2.3	Convergence and smoothness	217
2.4	Stability	223
2.5	Approximation order and decay of details	227
3	The geometric setting: Case studies	230
3.1	Geometry-based subdivision schemes	231
3.2	Geometric multi-scale transforms for planar curves	240
	References	245

Rapid solution of boundary integral equations by wavelet Galerkin schemes 249

Helmut Harbrecht and Reinhold Schneider

1	Introduction	249
2	Problem formulation and preliminaries	252
2.1	Boundary integral equations	252
2.2	Parametric surface representation	253
2.3	Kernel properties	255
3	Wavelet bases on manifolds	256
3.1	Wavelets and multiresolution analyses	256
3.2	Refinement relations and stable completions	258
3.3	Biorthogonal spline multiresolution on the interval	259
3.4	Wavelets on the unit square	261

3.5	Patchwise smooth wavelet bases	266
3.6	Globally continuous wavelet bases	267
4	The wavelet Galerkin scheme	271
4.1	Historical notes	272
4.2	Discretization	273
4.3	A-priori compression	274
4.4	Setting up the compression pattern	275
4.5	Computation of matrix coefficients	277
4.6	A-posteriori compression	279
4.7	Wavelet preconditioning	279
4.8	Numerical results	281
4.9	Adaptivity	284
	References	290
	Learning out of leaders	295
	<i>G�rard Kerkycharian, Mathilde Mougeot, Dominique Picard and Karine Tribouley</i>	
1	Introduction	295
2	Various learning algorithms in Wolfgang Dahmen's work	298
2.1	Greedy learning algorithms	298
2.2	Tree thresholding procedures	299
3	Learning out leaders: LOL	303
3.1	Gaussian regression model	303
3.2	LOL procedure	304
3.3	Sparsity conditions on the target function f	305
3.4	Results	306
3.5	Discussion	307
3.6	Restricted LOL	308
4	Practical performances of the LOL procedure	309
4.1	Experimental design	309
4.2	Algorithm	310
4.3	Simulation results	312
4.4	Quality reconstruction	313
4.5	Discussion	314
5	Proofs	316
5.1	Preliminaries	316
5.2	Concentration lemma 5.4	318
5.3	Proof of Theorem 3.2	319
	References	323
	Optimized wavelet preconditioning	325
	<i>Angela Kunoth</i>	
1	Introduction	325
2	Systems of elliptic partial differential equations (PDEs)	329
2.1	Abstract operator systems	329
2.2	A scalar elliptic boundary value problem	330

2.3	Saddle point problems involving essential boundary conditions	331
2.4	PDE-constrained control problems: Distributed control . .	334
2.5	PDE-constrained control problems: Dirichlet boundary control	336
3	Wavelets	337
3.1	Basic properties	337
3.2	Norm equivalences and Riesz maps	340
3.3	Representation of operators	341
3.4	Multiscale decomposition of function spaces	342
4	Problems in wavelet coordinates	356
4.1	Elliptic boundary value problems	356
4.2	Saddle point problems	358
4.3	Control problems: Distributed control	361
4.4	Control problems: Dirichlet boundary control	365
5	Iterative solution	367
5.1	Finite systems on uniform grids	368
5.2	Numerical examples	372
	References	376
	Multiresolution schemes for conservation laws	379
	Siegfried Müller	
1	Introduction	379
2	Governing equations and finite volume schemes	381
3	Multiscale analysis	383
4	Multiscale-based spatial grid adaptation	387
5	Adaptive multiresolution finite volume schemes	389
5.1	From the reference scheme to an adaptive scheme	389
5.2	Approximate flux and source approximation strategies . .	390
5.3	Prediction strategies	392
5.4	Multilevel time stepping	393
5.5	Error analysis	397
6	Numerical results	397
6.1	The solver Quadflow	398
6.2	Application	398
7	Conclusion and trends	402
	References	405
	Theory of adaptive finite element methods: An introduction	409
	Ricardo H. Nochetto, Kunibert G. Siebert and Andreas Veiser	
1	Introduction	410
1.1	Classical vs adaptive approximation in 1d	410
1.2	Outline	411
2	Linear boundary value problems	413
2.1	Sobolev spaces	413
2.2	Variational formulation	416

2.3	The inf-sup theory	419
2.4	Two special problem classes	423
2.5	Applications	427
2.6	Problems	430
3	The Petrov-Galerkin method and finite element bases	432
3.1	Petrov-Galerkin solutions	433
3.2	Finite element spaces	438
3.3	Problems	445
4	Mesh refinement by bisection	447
4.1	Subdivision of a single simplex	447
4.2	Mesh refinement by bisection	451
4.3	Basic properties of triangulations	454
4.4	Refinement algorithms	457
4.5	Complexity of refinement by bisection	462
4.6	Problems	468
5	Piecewise polynomial approximation	469
5.1	Quasi-interpolation	469
5.2	A priori error analysis	472
5.3	Principle of error equidistribution	474
5.4	Adaptive approximation	476
5.5	Problems	479
6	A posteriori error analysis	480
6.1	Error and residual	481
6.2	Global upper bound	482
6.3	Lower bounds	487
6.4	Problems	495
7	Adaptivity: Convergence	497
7.1	The adaptive algorithm	497
7.2	Density and convergence	499
7.3	Properties of the problem and the modules	501
7.4	Convergence	503
7.5	Problems	511
8	Adaptivity: Contraction property	512
8.1	The modules of AFEM for the model problem	513
8.2	Properties of the modules of AFEM	514
8.3	Contraction property of AFEM	518
8.4	Example: Discontinuous coefficients	520
8.5	Problems	522
9	Adaptivity: Convergence rates	524
9.1	Approximation class	525
9.2	Cardinality of $\mathcal{M}_k$	530
9.3	Quasi-optimal convergence rates	533
9.4	Marking vs optimality	534
9.5	Problems	538
	References	539

Adaptive wavelet methods for solving operator equations:

An overview	543
Rob Stevenson	
1 Introduction	543
1.1 Non-adaptive methods	543
1.2 Adaptive methods	545
1.3 Best N -term approximation and approximation classes	546
1.4 Structure of the paper	547
1.5 Some properties of the (quasi-) norms $\ \cdot\ _{\mathcal{A}^s}$	547
2 Well-posed linear operator equations	549
2.1 Reformulation as a bi-infinite matrix vector equation	549
2.2 Some model examples	550
3 Adaptive wavelet schemes I: Inexact Richardson iteration	552
3.1 Richardson iteration	552
3.2 Practical scheme	553
3.3 The routines COARSE and APPLY	558
3.4 Non-coercive B	562
3.5 Alternatives for the Richardson iteration	564
4 Adaptive wavelet schemes II: The Adaptive wavelet-Galerkin method	565
4.1 The adaptive wavelet-Galerkin method (AWGM) in a idealized setting	565
4.2 Practical scheme	568
4.3 Discussion	574
5 The approximation of operators in wavelet coordinates by computable sparse matrices	574
5.1 Near-sparsity of partial differential operators in wavelet coordinates	575
5.2 The approximate computation of the significant entries	580
5.3 Trees	583
6 Adaptive frame methods	585
6.1 Introduction	585
6.2 Frames	585
6.3 The adaptive solution of an operator equation in frame coordinates	586
6.4 An adaptive Schwarz method for aggregated wavelet frames	588
7 Adaptive methods based on tensor product wavelet bases	590
7.1 Tensor product wavelets	590
7.2 Non-adaptive approximation	590
7.3 Best N -term approximation and regularity	591
7.4 s^* -computability	592
7.5 Truly sparse stiffness matrices	592
7.6 Problems in space high dimension	592
7.7 Non-product domains	593

7.8	Other, non-elliptic problems	594
	References	594

Optimal multilevel methods for $H(\text{grad})$, $H(\text{curl})$, and $H(\text{div})$ systems on graded and unstructured grids 599

Jinchao Xu, Long Chen, and Ricardo H. Nochetto

1	Introduction	599
2	The method of subspace corrections	601
	2.1 Iterative methods	602
	2.2 Space decomposition and method of subspace correction	604
	2.3 Sharp convergence identities	607
3	Multilevel methods on quasi-uniform grids	609
	3.1 Finite element methods	609
	3.2 Multilevel space decomposition and multigrid method	611
	3.3 Stable decomposition and optimality of BPX preconditioner	612
	3.4 Uniform convergence of V-cycle multigrid	616
	3.5 Systems with strongly discontinuous coefficients	618
4	Multilevel methods on graded grids	621
	4.1 Bisection methods	622
	4.2 Compatible bisections	624
	4.3 Decomposition of bisection grids	625
	4.4 Generation of compatible bisections	627
	4.5 Node-oriented coarsening algorithm	629
	4.6 Space decomposition on bisection grids	630
	4.7 Strengthened Cauchy-Schwarz inequality	634
	4.8 BPX preconditioner and multigrid on graded bisection grids	636
5	Multilevel methods for $H(\text{curl})$ and $H(\text{div})$ systems	636
	5.1 Preliminaries	638
	5.2 Space decomposition and multigrid methods	646
	5.3 Stable decomposition	648
6	The auxiliary space method and HX preconditioner for unstructured grids	651
	6.1 The auxiliary space method	651
	6.2 HX preconditioner	653
	References	655